· HANDBOOK ·
OF
MATHEMATICS
FOR
ENGINEERS

L. A. WATERBURY

WITH
TABLES

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TAB- LES | ELEC. ENG. | HEAT ENG. | HY- DRAULICS | MECHANICS OF MATERIALS | THEORETICAL MECHANICS | CALCULUS | DIFFERENTIAL CALCULUS | ANALYTIC GEOMETRY | TRIGO- NOMETRY | GEBRA

TAB-LES | ELEC. ENG. | HEAT ENG. | HY-DRAULICS | MECHANICS OF MATERIALS | THEORETICAL MECHANICS | INTEGRAL CALCULUS | DIFFERENTIAL CALCULUS | ANALYTIC GEOMETRY | TRIGO-NOMETRY | ALGEBRA

A VEST-POCKET

HANDBOOK
OF
MATHEMATICS
FOR
ENGINEERS

BY

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WITH SPECIAL SECTIONS

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THIRD EDITION ENLARGED
THIRD PRINTING CORRECTED

NEW YORK
JOHN WILEY AND SONS, Inc.
London: CHAPMAN & HALL, Limited
[1919]

GEBRA
TRIGO-NOMETRY
ANALYTIC GEOMETRY
DIFFERENTIAL CALCULUS
CALCULUS
MECHANICS
OF MATERIALS
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TAB-LES

Stanhope Press
F. H. GILSON COMPANY
BOSTON, U.S.A.

2-24

PREFACE TO THIRD EDITION

The former editions of this handbook have been so well received that the publishers, Messrs. John Wiley and Sons, Inc., suggested the possibility of increasing its usefulness by the addition of material relating to thermodynamics and to electrical engineering. For the preparation of a section on heat engineering, Professor G. A. Goodenough, of the University of Illinois, was selected, while Professor H. H. Higbie, of the University of Michigan, was chosen to prepare a section on electrical engineering. These two new sections and their related tables constitute the principal addition which has been made to the former edition.

L. A. W.

NITRO, W. VA., May, 1918.

PREFACE TO SECOND EDITION.

In preparing the second edition, the errors which have been discovered in the previous edition have been corrected, revisions and alterations have been made throughout the work, and new material has been added, including sections on hydraulics and reinforced concrete, and a table of conversion factors.

L. A. W.

URBANA, ILL., April, 1915.

PREFACE.

This handbook is intended as a reference book, for the use of those who have studied or are studying the branches of mathematics usually taught in engineering courses. It is not intended for a text book, and does not, therefore, attempt to prove many of the formulæ which are given.

Most of the material in this book was obtained from the following sources: algebra from Hall & Knight's Algebra (Macmillan Co.); trigonometry from Bowser's Trigonometry; analytic geometry from Candy's Analytic Geometry; calculus from Taylor's Differential and Integral Calculus; theoretical mechanics from Church's Mechanics of Engineering; and mechanics of materials from Merriman's Mechanics of Materials; to all of which the writer is very much indebted and from all these Authors he has received permission to use the material. The reader is referred to these works for the proof and explanation of the various formulæ.

L. A. W.

TUCSON, ARIZ., March, 1908.

PREFACE TO FIRST EDITION WITH TABLES

In this edition tables of logarithms of numbers, natural and logarithmic sines and cosines, and natural and logarithmic tangents and cotangents have been added to facilitate the solution of problems.

L. A. W.

TUCSON, ARIZ., September, 1909.

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GREEK LETTERS.

Α	α	Alpha	Ν	ν	Nu
Β	β	Beta	Ξ	ξ	Xi
Γ	γ	Gamma	Ο	o	Omicron
Δ	δ	Delta	Π	π	Pi
Ε	ϵ	Epsilon	Ρ	ρ	Rho
Ζ	ζ	Zeta	Σ	σ ς	Sigma
Η	η	Eta	Τ	τ	Tau
Θ	θ ϑ	Theta	Υ	υ	Upsilon
Ι	ι	Iota	Φ	ϕ	Phi
Κ	κ	Kappa	Χ	χ	Chi
Λ	λ	Lambda	Ψ	ψ	Psi
Μ	μ	Mu	Ω	ω	Omega

ALGEBRA.

EXPONENTS AND LOGARITHMS

If $a^m = b$, $m = \log_a b$. $a^m \cdot a^n = a^{m+n}$,
$\therefore \log (x \cdot y) = \log x + \log y$. $a^m \div a^n = a^{m-n}$,
$\therefore \log (x \div y) = \log x - \log y$. $(a^m)^2 =$
$a^m \cdot a^m = a^{2m}$, $\therefore \log x^2 = 2 \cdot \log x$.
$(a^m)^n = a^{m \cdot n}$, $\therefore \log x^n = n \cdot \log x$.
$a^0 = 1$, $\therefore \log (1) = 0$.

For common logarithms the base is 10; log 10 = 1, log 100 = 2, log 1000 = 3, etc., or for any number between 1 and 10, the logarithm will have a value between 0 and 1, and may be found in a table of logarithms. The values of the logarithm of any number may be obtained by adding the proper integer to the proper value obtained from the tables. For example,

$$\begin{aligned}\log (451.7) &= \log (4.517 \times 100)\\ &= \log 4.517 + \log 100\\ &= 0.65485 + 2\\ &= 2.65485.\end{aligned}$$

It may be observed that the integral part of the logarithm, called the characteristic, indicates the location of the decimal point of the number; and that the decimal portion of the logarithm, called the mantissa, determines the sequence of significant figures.

For a number less than unity, the logarithm is negative, but since the tables contain only positive values, the logarithm for such a number is ordinarily used in the form of a positive mantissa with a negative characteristic. For the purpose of involution or evolution the

logarithm may well be used in the negative form. For example,

$$\begin{aligned}\log(0.04517) &= \log(4.517 \div 100)\\ &= \log 4.517 - \log 100\\ &= +0.65485 - 2\\ (\text{or} \quad &= +8.65485 - 10)\\ &= -1.34515.\end{aligned}$$

(The logarithm is usually written $\bar{2}.65485$.)

$$\begin{aligned}\log(0.04517)^{1.6} &= (1.6)\cdot\log(0.04517)\\ &= (1.6)(-1.34515)\\ &= -2.15224\\ &= +0.84776 - 3\\ &= \log(0.007043),\\ \therefore (0.04517)^{1.6} &= 0.007043.\end{aligned}$$

The base of the natural system of logarithms is

$$e = 1 + 1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 5} + \cdots = 2.7182818284.$$

The cologarithm of a number is the logarithm of its reciprocal. $\operatorname{Log}\left(\frac{1}{x}\right) = 0 - \log x$.

To transform a logarithm from base e to base 10, multiply by $\log_{10} e$.

$$\operatorname{Log}_{10} e = 0.43429448.$$

$$\operatorname{Log}_e 10 = 2.30258509.$$

$$\operatorname{Log}_{10} e = \frac{1}{\log_e 10}.$$

PROPORTION.

If $a : b :: c : d$,

$$\frac{a}{b} = \frac{c}{d}, \quad \text{or} \quad \frac{b}{a} = \frac{d}{c},$$

$$ad = bc, \quad \frac{a+b}{b} = \frac{c+d}{d},$$

$$\frac{a-b}{b} = \frac{c-d}{d}, \quad \frac{a+b}{a-b} = \frac{c+d}{c-d}.$$

ARITHMETICAL PROGRESSION.

$$a,\ a+d,\ a+2d,\ \ldots$$

Last term, $L = a + (n-1)\,d$.
Sum of terms,

$$S = \frac{n}{2}(a+L) = \frac{n}{2}[2a + (n-1)\,d].$$

GEOMETRICAL PROGRESSION.

$$a,\ ar,\ ar^2,\ ar^3,\ \ldots$$

Last term, $L = ar^{n-1}$.
Geometric mean, $M = \sqrt{ab}$.

Sum, $$S = \frac{a\,(r^n - 1)}{r-1}$$
$$= \frac{a\,(1-r^n)}{1-r} = \frac{rL - a}{r-1}.$$

For an infinite geometrical series, the sum to infinity is $S = \dfrac{a}{1-r}$.

HARMONIC PROGRESSION.

a, b, c are in harmonic progression if

$$\frac{a}{c} = \frac{a-b}{b-c},$$

or if $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in arithmetical progression.

PERMUTATIONS AND COMBINATIONS.

ab and ba are two permutations but only one combination.

The number of permutations possible of n things taken r at a time is

$${}^nP_r = n\,(n-1)\,(n-2)\ \ldots\ (n-r+1).$$
$${}^nP_n = \underline{|n}.$$
$$(\underline{|n} = 1 \times 2 \times 3 \times 4 \cdots \times n).$$

$${}^nC_r = \frac{{}^nP_r}{\underline{|r}} = \frac{\underline{|n}}{\underline{|r}\,\underline{|n-r}} = {}^nC_{n-r}.$$

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BINOMIAL THEOREM.

$$(a+b)^n = a^n + n \cdot a^{n-1} \cdot b$$
$$+ \frac{n \cdot (n-1)}{\lfloor 2} \cdot a^{n-2} \cdot b^2$$
$$+ \frac{n \cdot (n-1)\,(n-2)}{\lfloor 3} \cdot a^{n-3} \cdot b^3$$
$$+ \cdots \quad \cdots \quad \cdots$$

SERIES.

1. An infinite series in which the terms are alternately positive and negative is convergent if each term is numerically less than the preceding term.

2. An infinite series in which all the terms are of the same sign is divergent if each term is greater than some finite quantity, however small.

3. An infinite series is convergent if from and after some fixed term the ratio of each term to the preceding term is numerically less than unity.

4. An infinite series in which all the terms are of the same sign is divergent if from and after some fixed term the ratio of each term to the preceding term is greater than unity, or is equal to unity.

5. If there are two infinite series in each of which all the terms are positive, and if the ratio of the corresponding terms in the two series is always finite, the two series are both convergent, or both divergent.

DETERMINANTS.

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1.$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \cdot b_2 \cdot c_3 + a_2 \cdot b_3 \cdot c_1 + a_3 \cdot b_1 \cdot c_2 - a_1 \cdot b_3 \cdot c_2 - a_2 \cdot b_1 \cdot c_3 - a_3 \cdot b_2 \cdot c_1.$$

If
$$\begin{cases} a_1x+b_1y+c_1z+d_1=0, \\ a_2x+b_2y+c_2z+d_2=0, \\ a_3x+b_3y+c_3z+d_3=0, \end{cases}$$
then
$$\frac{x}{\begin{vmatrix} b_1c_1d_1 \\ b_2c_2d_2 \\ b_3c_3d_3 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a_1c_1d_1 \\ a_2c_2d_2 \\ a_3c_3d_3 \end{vmatrix}} = \frac{z}{\begin{vmatrix} a_1b_1d_1 \\ a_2b_2d_2 \\ a_3b_3d_3 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} a_1b_1c_1 \\ a_2b_2c_2 \\ a_3b_3c_3 \end{vmatrix}}$$

QUADRATIC EQUATIONS.

$$ax^2+bx+c=0$$
$$x=\frac{-b\pm\sqrt{b^2-4\,ac}}{2\,a}.$$

CUBIC EQUATIONS.*

First Form.

$$x^3+bx+c=0. \tag{1}$$

Let
$$x=y-\frac{b}{3\,y} \tag{2}$$
or
$$y^6+cy^3-\frac{b^3}{27}=0, \tag{3}$$
whence,
$$y^3=-\frac{c}{2}\pm\sqrt{\frac{c^2}{4}+\frac{b^3}{27}}, \tag{4}$$
from which x may be obtained by substituting the value of y in equation (2).

Second Form.

$$x^3+ax^2+c=0. \tag{5}$$

Let
$$x=1/z \tag{6}$$
or
$$z^3+\frac{a}{c}\,z+\frac{1}{c}=0, \tag{7}$$
which may be solved by equations (1) to (4) and the value of x may then be obtained by equation (6).

Third Form.

$$x^3+ax^2+bx+c=0. \tag{8}$$

Let
$$x=z-\frac{a}{3}, \tag{9}$$

* The equations here used follow the method given in Wells' University Algebra.

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which, when substituted in equation (8), will give an equation of the first form, the solution of which will give the value of z, from which x may be obtained by equation (9).

HIGHER EQUATIONS.*

For higher algebraic equations, an approximate numerical solution can be obtained by the method of double position, as follows:

$$f(x) = x^n + ax^{n-1} + bx^{n-2} \cdots = 0. \quad (1)$$

By trial find two numbers one of which when substituted for x makes $f(x)$ positive, and the other when substituted for x makes $f(x)$ negative. Let a and b be the two numbers, and let A and B be the respective corresponding values of $f(x)$. Then, approximately,

$$A : B = (x - a) : (x - b) \quad (2)$$

or

$$x = a + \frac{A\,(b - a)}{A - B}. \quad (3)$$

GRAPHICAL SOLUTION OF EQUATIONS.

To determine the value of x in any equation, $f(x) = 0$, let $y = f(x)$ and compute the values of y for a number of assumed values of x. Using the values of x and y as coördinates, plot the graph of the equation, $y = f(x)$, from which the value of x which will make $f(x)$ become zero can be observed.

For two simultaneous equations, involving two unknowns, the graph of each equation may be plotted with reference to one set of axes. If the two graphs intersect, the points of intersection will have coördinates which are the values of the two unknowns. If the graphs can not be made to intersect, there are no real values of x and y which are common to both equations.

* See Wells' University Algebra.

For any equation, $y = f(x)$, the logarithms of x and y may be plotted instead of the quantities themselves, producing the logarithmic graph of the equation. Logarithmic graphs are particularly useful for equations of the form, $y = ax^b$, for which the graphs are straight lines.

TRIGONOMETRY.

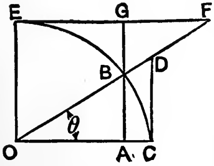

Fig. 1.

Radius = 1.

$AB = \sin\theta.$

$OA = \cos\theta.$

$CD = \tan\theta.$

$EF = \cot\theta.$

$OD = \sec\theta.$

$OF = \operatorname{cosec}\theta.$

$$AC = \operatorname{vers}\theta = 1 - \cos\theta.$$

$$BG = \operatorname{covers}\theta = 1 - \sin\theta.$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}.$$

$$\sin^2\theta + \cos^2\theta = 1.$$

$$\sec^2\theta = 1 + \tan^2\theta.$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta.$$

$$\operatorname{exsec}\theta = \sec\theta - 1.$$

For θ in radians,

$$\sin\theta = \theta - \frac{\theta^3}{\underline{|3}} + \frac{\theta^5}{\underline{|5}} - \frac{\theta^7}{\underline{|7}} + \cdots$$

$$\cos\theta = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{\underline{|4}} - \frac{\theta^6}{\underline{|6}} + \cdots$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\cdot\theta^5}{3\cdot 5} + \frac{17\,\theta^7}{3\cdot3\cdot5\cdot7} + \cdots$$

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B.$$

$$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B.$$

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B.$$

$$\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B.$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}.$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}.$$

$$\sin 2A = 2 \cdot \sin A \cdot \cos A.$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2 \cdot \sin^2 A.\end{aligned}$$

$$\tan 2A = \frac{2 \cdot \tan A}{1 - \tan^2 A}.$$

$$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{1}{2}(1 - \cos A).}$$

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{1}{2}(1 + \cos A).}$$

$$\tan\left(\frac{A}{2}\right) = \frac{1 - \cos A}{\sin A}.$$

$$\sin 3A = 3 \cdot \sin A - 4 \cdot \sin^3 A.$$

$$\cos 3A = 4\cos^3 A - 3\cos A.$$

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}.$$

$$\sin A + \sin B = 2 \cdot \sin\frac{A+B}{2} \cdot \cos\frac{A-B}{2}.$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2} \cdot \sin\frac{A-B}{2}.$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2} \cdot \cos\frac{A-B}{2}.$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2} \cdot \sin\frac{A-B}{2}.$$

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Function.	$90° - \theta$.	$\sin\theta$.	$\cos\theta$.	$\tan\theta$.	$\cot\theta$.	$\sec\theta$.	$\csc\theta$.
$\sin\theta$	$\cos(90°-\theta)$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\pm\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sin(90°-\theta)$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\cot(90°-\theta)$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\tan(90°-\theta)$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
$\sec\theta$	$\operatorname{cosec}(90°-\theta)$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\operatorname{cosec}\theta$	$\sec(90°-\theta)$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

$$\frac{\sin A+\sin B}{\sin A-\sin B}=\frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}.$$

$$\frac{\sin A+\sin B}{\cos A+\cos B}=\tan\tfrac{1}{2}(A+B).$$

$$\frac{\sin A+\sin B}{\cos A-\cos B}=\cot\tfrac{1}{2}(A-B).$$

$$\frac{\sin A-\sin B}{\cos A+\cos B}=\tan\tfrac{1}{2}(A-B).$$

$$\frac{\sin A-\sin B}{\cos A-\cos B}=\cot\tfrac{1}{2}(A+B).$$

$$\frac{\cos A+\cos B}{\cos A-\cos B}=\cot\left(\frac{A+B}{2}\right)\cdot\cot\left(\frac{A-B}{2}\right).$$

PLANE TRIANGLES.

$$A+B+C=180°.$$

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}.$$

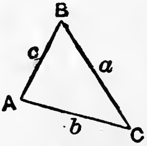

Fig. 2.

$$\tan A=\frac{a\cdot\sin C}{b-a\cdot\cos C}.$$

$$\cos A=\frac{b^2+c^2-a^2}{2\,bc},$$

or

$$a^2=b^2+c^2-2\,bc\cdot\cos A.$$

$$\frac{a+b}{a-b}=\frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}.$$

$$\sin A+\sin B+\sin C$$

$$=4\cdot\cos\frac{A}{2}\cdot\cos\frac{B}{2}\cdot\cos\frac{C}{2}.$$

$$\cos A+\cos B+\cos C$$

$$=1+4\cdot\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}.$$

$$\tan A+\tan B+\tan C=\tan A\cdot\tan B\cdot\tan C.$$

$$\text{Area} = \tfrac{1}{2}\, b \cdot c \cdot \sin A$$

$$= \frac{a^2 \sin B \cdot \sin C}{2 \cdot \sin A}$$

$$= \sqrt{s\,(s-a)\,(s-b)\,(s-c)},$$

where $s = \frac{1}{2}\,(a+b+c)$.

SPHERICAL TRIANGLES.

Center of sphere is at O.

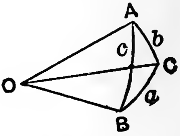

Fig. 3.

Right Spherical Triangles. Let C represent the right angle.

$$\cos c = \cos a \cdot \cos b.$$

$$\sin b = \sin B \cdot \sin c.$$

$$\tan a = \cos B \cdot \tan c.$$

$$\tan a = \tan A \cdot \sin b.$$

$$\tan A \cdot \tan B = \frac{1}{\cos c}.$$

$$\cos A = \sin B \cdot \cos a.$$

Oblique Spherical Triangles.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} = \text{modulus}.$$

$$\cos a = \cos b \cdot \cos c + \sin b \cdot \sin c \cdot \cos A.$$

$$\cos A = -\cos B \cdot \cos C + \sin B \cdot \sin C \cdot \cos a.$$

$$\cot a \cdot \sin b = \cot A \cdot \sin C + \cos C \cdot \cos b.$$

Let $s = \frac{1}{2}\,(a+b+c)$,

$$S = \tfrac{1}{2}\,(A+B+C),$$

then $$\sin\left(\frac{A}{2}\right) = \sqrt{\frac{\sin(s-b)\cdot\sin(s-c)}{\sin b\cdot\sin c}}.$$

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{\sin s\cdot\sin(s-a)}{\sin b\cdot\sin c}}.$$

$$\tan\left(\frac{A}{2}\right) = \sqrt{\frac{\sin(s-b)\cdot\sin(s-c)}{\sin s\cdot\sin(s-a)}}.$$

$$\sin\left(\frac{a}{2}\right) = \sqrt{-\frac{\cos S\cdot\cos(S-A)}{\sin B\cdot\sin C}}.$$

$$\cos\left(\frac{a}{2}\right) = \sqrt{\frac{\cos(S-B)\cdot\cos(S-C)}{\sin B\cdot\sin C}}.$$

$$\tan\left(\frac{a}{2}\right) = \sqrt{-\frac{\cos S\cdot\cos(S-A)}{\cos(S-B)\cdot\cos(S-C)}}.$$

HYPERBOLIC FUNCTIONS.

For the equilateral hyperbola, $x^2 - y^2 = a^2$, a series of functions can be obtained, analogous to the circular functions.

Let x, y be the coördinates of any point P (Fig. 4), let the radius $OP = r$, let $v =$ the arc MP divided by r, and let $OM = a$.

Then,

$\sinh u = y/a.$

$\cosh u = x/a.$

$\tanh u = y/x.$

$\coth u = x/y.$

$\text{sech}\, u = a/x.$

$\text{cosech}\, u = a/y.$

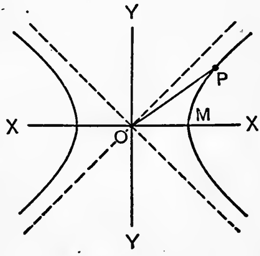

Fig. 4.

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ANALYTIC GEOMETRY.

TRANSFORMATION OF COÖRDINATES.

To transform an equation of a curve from one system of coördinates to another system, substitute for each variable its value in terms of variables of the new system.

Rectangular System. Old Axes Parallel to New Axes.

$$x' = x - h.$$
$$y' = y - k.$$
$$x = x' + h.$$
$$y = y' + k.$$

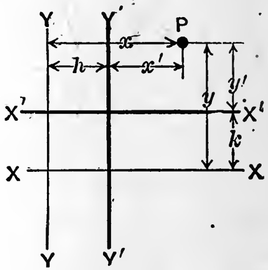

Fig. 5.

Rectangular System. Old Origin Coincident with New Origin.

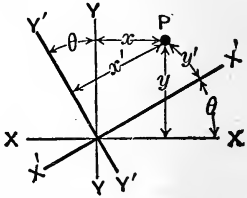

Fig. 6.

$$x' = x \cdot \cos\theta + y \cdot \sin\theta.$$
$$y' = y \cdot \cos\theta - x \cdot \sin\theta.$$
$$x = x' \cdot \cos\theta - y' \cdot \sin\theta.$$
$$y = y' \cdot \cos\theta + x' \cdot \sin\theta.$$

Rectangular System. Old Axes not Parallel to New Axes. Old Origin not Coincident with New Origin.

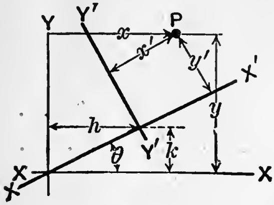

Fig. 7.

$$x' = (x - h)\cos\theta + (y - k)\sin\theta.$$

$$y' = (y - k)\cos\theta - (x - h)\sin\theta.$$

$$x = x'\cdot\cos\theta - y'\cdot\sin\theta + h.$$

$$y = y'\cdot\cos\theta + x'\cdot\sin\theta + k.$$

Polar and Rectangular Systems.

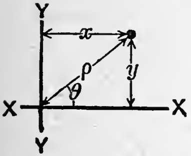

Fig. 8.

$$x = \rho\cdot\cos\theta.$$

$$y = \rho\cdot\sin\theta.$$

$$\rho = \sqrt{x^2 + y^2}.$$

$$\tan\theta = \frac{y}{x}.$$

$$\cos\theta = \frac{x}{\sqrt{x^2 + y^2}}.$$

$$\sin\theta = \frac{y}{\sqrt{x^2 + y^2}}.$$

$$\cot\theta = \frac{x}{y}.$$

$$\sec\theta = \frac{\sqrt{x^2 + y^2}}{x}.$$

$$\operatorname{cosec}\theta = \frac{\sqrt{x^2 + y^2}}{y}.$$

THE STRAIGHT LINE.

Equations of Straight Line. An equation of the first degree containing but two variables can always be represented by a straight line.

The equation of the straight line may assume the following forms, for the rectangular system of coördinates.

$$Ax + By + C = 0 \quad . \quad . \quad . \quad . \quad (1)$$

$$y = mx + k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which m is the value of the tangent of the angle which the line makes with the X-axis, and k is the intercept on the Y-axis between the line and the X-axis.

$$y - y' = A\ (x - x') \quad . \quad . \quad . \quad . \quad (3)$$

in which x', y' are the coördinates of a point of the line, and A is a constant.

$$y - y' = \frac{y' - y''}{x' - x''}(x - x') \quad . \quad . \quad . \quad (4)$$

in which x', y' and x'', y'' are the coördinates of two points of the line.

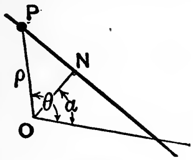

Fig. 9.

The *polar* equation of a straight line is

$$\rho \cdot \cos\,(\theta - \alpha) = k \quad (5)$$

where k is the length of the normal ON.

Distance between Two Points. The distance between two points, x', y' and x'', y'', is equal to

$$\sqrt{(x' - x'')^2 + (y' - y'')^2}.$$

The distance between two points, ρ_1, θ_1, and ρ_2, θ_2, is equal to

$$\sqrt{\rho_1{}^2 + \rho_2{}^2 - 2\,\rho_1 \cdot \rho_2 \cdot \cos\,(\theta_1 - \theta_2)}.$$

Angle between Two Lines. The angle between two lines, $y = m'x + k'$ and $y = m''x + k''$,

is the difference between the two angles whose tangents are m' and m''.

Area of Triangle. The area of the triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is equal to

$$\frac{1}{2} \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \cdot$$

THE CIRCLE.

The most general equation of the circle, for rectangular coördinates, is

$$(x - a)^2 + (y - b)^2 = R^2,$$

in which a, b are the coördinates of the center of the circle, and R is the radius.

The following are special equations of the circle for rectangular and polar systems of coördinates.

$$x^2 + y^2 = R^2.$$
$$\rho = R.$$

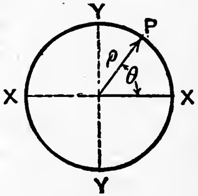

Fig. 10.

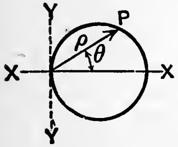

Fig. 11.

$$y^2 = 2\,Rx - x^2.$$
$$\rho = 2\,R \cdot \cos\theta.$$

$$x^2 = 2\,Ry - y^2.$$
$$\rho = 2\,R \cdot \sin\theta.$$

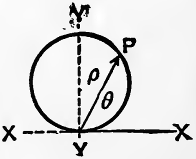

Fig. 12.

Diameter of circle $= 2\,R = D$.
Circumference $= 2\,\pi\,R = \pi\,D$.
Area $= \pi R^2 = \frac{1}{4}\pi D^2$.

THE PARABOLA.

In Fig. 13, F is the focus, $OF = OD = a$, and $L-L$ is the latus rectum $= 4a$.

Eccentricity, $e = \dfrac{FP}{PQ} = 1$.

If the Y-axis coincides with the directrix, DM, then

$$y^2 = 4a(x - a).$$

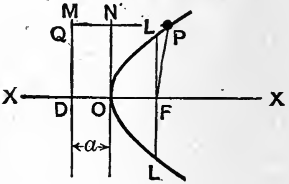

Fig. 13.

If the Y-axis coincides with ON, passing through the vertex, then

$$y^2 = 4ax.$$

For a symmetrical segment of a parabola, the area of the segment is exactly two-thirds of the area of the enclosing rectangle.

THE ELLIPSE.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

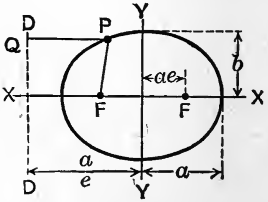

Fig. 14.

F, F are foci.
Eccentricity, $e < 1$.
The area of the ellipse is equal to πab.

THE HYPERBOLA.

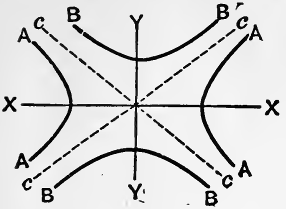

Fig. 15.

$A-A$ = principal hyperbola.
$B-B$ = conjugate hyperbola.
$c-c$ = asymptote.

Principal hyperbola: $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1.$

Asymptotes: $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=0.$

Conjugate hyperbola: $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=-1.$

When referred to the asymptotes as axes, the equations become:

Principal hyperbola: $xy=\dfrac{a^2+b^2}{4}.$

Conjugate hyperbola: $xy=-\left(\dfrac{a^2+b^2}{4}\right).$

$D-D$ is the directrix.

F, F are foci.

$\dfrac{FP}{PQ}=e>1.$

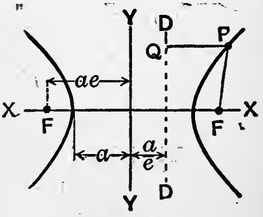

Fig. 16.

For the equilateral hyperbola, $a=b$, for which the equation of the principal hyperbola becomes $x^2-y^2=a^2.$

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THE CYCLOID.

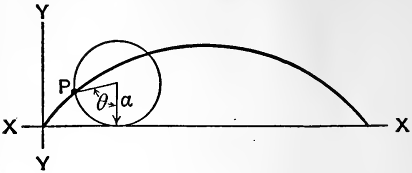

Fig. 17.

$$\begin{cases} x = a\,(\theta - \sin\theta), \\ y = a\,(1 - \cos\theta), \end{cases}$$

or $$x = a \cdot \text{vers}^{-1}\left(\frac{y}{a}\right) - \sqrt{2\,ay - y^2}.$$

THE SPIRAL OF ARCHIMEDES.

$$\rho = k \cdot \theta.$$

THE RECIPROCAL OR HYPERBOLIC SPIRAL.

$$\rho = \frac{k}{\theta}.$$

THE PARABOLIC SPIRAL.

$$\rho^2 = k \cdot \theta.$$

THE LITUUS OR TRUMPET.

$$\rho^2 = \frac{k}{\theta}.$$

THE LOGARITHMIC SPIRAL.

$$\log \rho = k \cdot \theta.$$

If $k = 1$, and logarithms to the base a are employed, then the equation may be written

$$\rho = a^\theta.$$

THE CATENARY.

$$y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right).$$

THE CUBIC PARABOLA.

$$y = kx^3.$$

THE SPHERE.

R = radius, and D = diameter.

For the origin at the center,

$$x^2 + y^2 + z^2 = R^2.$$

Area of surface $= 4\pi R^2 = \pi D^2$.

Volume $= \frac{4}{3}\pi R^3 = \frac{1}{6}\pi D^3$.

CONES.

The equation of the cone generated by the line, $z = mx + c$, rotated about the Z-axis, is

$$x^2 + y^2 = \frac{(z-c)^2}{m^2}.$$

The volume of a cone is $\frac{1}{3}Ah$, where A is the area of the base, and h is the altitude.

OBLATE SPHEROIDS.

The equation of the oblate spheroid generated by the ellipse, $\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$, rotated about its minor axis, is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1.$$

PROLATE SPHEROIDS.

The equation of the prolate spheroid generated by the ellipse, $\frac{x^2}{b^2} + \frac{z^2}{a^2} = 1$, rotated about its major axis, is

$$\frac{x^2}{b^2} + \frac{y^2}{b^2} + \frac{z^2}{a^2} = 1.$$

HYPERBOLOIDS.

The equation of the hyperboloid of one nappe, generated by the hyperbola, $\frac{x^2}{a^2} - \frac{z^2}{b^2} = 1$, rotated about its conjugate axis, is

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} - \frac{z^2}{b^2} = 1.$$

The equation of the hyperboloid of two nappes, generated by the hyperbola, $\frac{x^2}{a^2} - \frac{z^2}{b^2} = 1$, rotated about its transverse axis, is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{b^2} = 1.$$

THE PARABOLOID.

The equation of the paraboloid of revolution generated by the parabola, $x^2 = 4\,az$, rotated about its axis, is

$$x^2 + y^2 = 4\,az.$$

GENERAL EQUATION OF CONIC SECTION.

The general equation of any conic section, for which the Y-axis coincides with the directrix and the X-axis passes through the foci normal to the directrix, is

$$(x-k)^2 + y^2 = e^2x^2,$$

where k is the distance from the directrix to the focus, and e is the eccentricity.

DIFFERENTIAL CALCULUS.

Variables will be represented by u, v, x, y, and z, and constants by a, b, m, and n.

D will be used as the sign for the derivative, and d as the sign for the differential.

$\operatorname{Sin}^{-1} x =$ angle whose sine is x.

$$D\,(fx) = \frac{d\,(fx)}{dx}$$

$$D_x y = \frac{dy}{dx}.$$

$\therefore$ To obtain the derivative of any function, drop the differential of the variable from the differential of the function.

$$D_x(fy) = D_y(fy) \cdot D_x y.$$

$$da = 0.$$

$$d\,(av) = a \cdot dv.$$

$$d\,(u+v+x) = du + dv + dx.$$

$$d\,(x \cdot y) = y \cdot dx + x \cdot dy.$$

$$d\,(u \cdot v \cdot x \cdot y\ \ldots) = (v \cdot x \cdot y\ \ldots)\,du + (u \cdot x \cdot y\ \ldots)\,dv + (u \cdot v \cdot y\ \ldots)\,dx + (u \cdot v \cdot x\ \ldots)\,dy + \ldots$$

$$d\,(\log_e u) = \frac{du}{u}.$$

$$d\,(\log_a u) = \log_a e \cdot \frac{du}{u}.$$

$$d\left(\frac{x}{y}\right) = \frac{y \cdot dx - x \cdot dy}{y^2}.$$

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$$dx^y = y \cdot x^{y-1} \cdot dx + x^y \cdot \log_a x \cdot \frac{dy}{M},$$

where $$M = \log_a e.$$

$$d\,(b^y) = b^y \cdot \log_a b \cdot \frac{dy}{M}.$$

$$dx^a = a \cdot x^{a-1} \cdot dx.$$

$$d\sqrt{x} = \frac{dx}{2\sqrt{x}}.$$

$$d\,(\sin x) = \cos x \cdot dx.$$

$$d\,(\cos x) = -\sin x \cdot dx.$$

$$d\,(\tan x) = \sec^2 x \cdot dx.$$

$$d\,(\cot x) = -\operatorname{cosec}^2 x \cdot dx.$$

$$d\,(\sec x) = \sec x \cdot \tan x \cdot dx.$$

$$d\,(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x \cdot dx.$$

$$d\,(\operatorname{vers} x) = d\,(1 - \cos x) = +\sin x \cdot dx.$$

$$d\,(\operatorname{covers} x) = d\,(1 - \sin x) = -\cos x \cdot dx.$$

$$d\,(\sin^{-1} x) = dx / \sqrt{1 - x^2}.$$

$$d\,(\cos^{-1} x) = -dx / \sqrt{1 - x^2}.$$

$$d\,(\tan^{-1} x) = dx / (1 + x^2).$$

$$d\,(\cot^{-1} x) = -dx / (1 + x^2).$$

$$d\,(\sec^{-1} x) = dx / \left(x\sqrt{x^2 - 1}\right).$$

$$d\,(\operatorname{vers}^{-1} x) = dx / \sqrt{2x - x^2}.$$

$$d\,(\operatorname{covers}^{-1} x) = -dx / \sqrt{2x - x^2}.$$

To differentiate a function:

1. Find the value of the increment of the function in terms of the increments of its variables.

2. Consider the increments to be infinitesimals, and in all sums drop the infinitesimals of higher order than the first, and in the

remaining terms substitute differentials for increments.

For the *maximum* value of a function the first derivative is zero, and the second derivative is negative.

For the *minimum* value of a function the first derivative is zero, and the second derivative is positive.

If $\frac{Fx}{fx}$ assumes the form $\frac{0}{0}$, then

$$\frac{Fx}{fx}=\frac{D\,(Fx)}{D\,(fx)}.$$

Taylor's theorem is

$$f\,(x+h)=fx+h\cdot D\,(fx)+\frac{h^2}{\underline{|2}}\cdot D^2\,(fx)+\cdots$$

$$\cdots+\frac{h^n}{\underline{|n}}\cdot D^n\,(fx).$$

$$fx=f\,(0+x)=f\,(0)+$$

$$x\cdot D\,(f0)+\frac{x^2}{\underline{|2}}\cdot D^2\,(f0)+\cdots$$

The *radius of curvature* for a curve, $y=fx$, is

$$R=\frac{ds}{d\alpha}=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{(dx)^2}}=\frac{(ds)^3}{dx\cdot d^2y},$$

where s is length of curve.

INTEGRAL CALCULUS.

$\int dx = x + C$, where C is the constant of integration. The constant C must be added to all of the following forms.

$$\int (dx + dy + dz \ . \ . \ . \ ) =$$

$$\int dx + \int dy + \int dz + \cdot \cdot \cdot$$

$$\int x^n \cdot dx = \frac{x^{n+1}}{n+1}.$$

$$\int \frac{dx}{x} = \log_e x.$$

$$\int a^x \cdot dx = \frac{a^x}{\log_e a}.$$

$$\int e^x \cdot dx = e^x.$$

$$\int a^x \cdot \log_e a \cdot dx = a^x.$$

$$\int \sin x \cdot dx = -\cos x \text{ or vers } x.$$

$$\int \cos x \cdot dx = \sin x \text{ or} - \text{covers } x.$$

$$\int \sec^2 x \cdot dx = \tan x.$$

$$\int \operatorname{cosec}^2 x \cdot dx = -\cot x.$$

$$\int \sec x \cdot \tan x \cdot dx = \sec x.$$

$$\int \operatorname{cosec} x \cdot \cot x \cdot dx = -\operatorname{cosec} x.$$

$$\int \tan x \cdot dx = \log (\sec x).$$

$$\int \cot x \cdot dx = \log (\sin x).$$

$$\int \operatorname{cosec} x \cdot dx = \log \left(\tan \frac{x}{2}\right)\cdot$$

$$\int \sec x \cdot dx = \log \left[\tan \left(\frac{x}{2} + \frac{\pi}{4}\right)\right]\cdot$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \cdot \tan^{-1} \left(\frac{x}{a}\right), \text{ or}$$

$$= -\frac{1}{a} \cdot \cot^{-1} \left(\frac{x}{a}\right).$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \cdot \log \left(\frac{x-a}{x+a}\right), \text{ or}$$

$$= \frac{1}{2a} \cdot \log \left(\frac{a-x}{a+x}\right).$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \left(\frac{x}{a}\right) = -\cos^{-1} \left(\frac{x}{a}\right).$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log (x + \sqrt{x^2 \pm a^2}).$$

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \cdot \sec^{-1} \left(\frac{x}{a}\right), \text{ or}$$

$$= -\frac{1}{a} \operatorname{cosec}^{-1} \left(\frac{x}{a}\right)\cdot$$

$$\int \frac{dx}{\sqrt{2ax-x^2}} = \operatorname{vers}^{-1} \left(\frac{x}{a}\right), \text{ or}$$

$$= -\operatorname{covers}^{-1} \left(\frac{x}{a}\right)\cdot$$

$$\int f(x)\,dx = Fx + C, \text{ if}$$

$$d\,(Fx) = fx \cdot dx.$$

$$\int a \cdot dx = a \int dx.$$

$$\int 0 = C.$$

$$\int x \cdot dy = xy - \int y \cdot dx.$$

$$\int \frac{x \cdot dx}{a+bx} = \frac{1}{b^2}\,[a+bx-a \cdot \log\,(a+bx)\,].$$

$$\int \frac{x \cdot dx}{(a+bx)^2} = \frac{1}{b^2}\left[\log\,(a+bx) + \frac{a}{a+bx}\right].$$

$$\int \frac{x^2 \cdot dx}{a+bx} = \frac{1}{b^3}\left[\frac{(a+bx)^2}{2} - 2\,a\,(a+bx) + a^2 \cdot \log\,(a+bx)\right].$$

$$\int \frac{x^2 \cdot dx}{(a+bx)^2} = \frac{1}{b^3}\left[a+bx-2\,a \cdot \log\,(a+bx) - \frac{a^2}{a+bx}\right].$$

$$\int \frac{dx}{x\,(a+bx)} = -\frac{1}{a} \cdot \log\left(\frac{a+bx}{x}\right).$$

$$\int \frac{dx}{x\,(a+bx)^2} = \frac{1}{a\,(a+bx)} - \frac{1}{a^2} \cdot \log\left(\frac{a+bx}{x}\right).$$

$$\int \frac{dx}{x^2\,(a+bx)} = -\frac{1}{ax} + \frac{b}{a^2} \cdot \log\left(\frac{a+bx}{x}\right).$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{\sqrt{ab}} \cdot \tan^{-1}\left(x\sqrt{\frac{b}{a}}\right),$$

when $a > 0$ and $b > 0$.

$$\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ab}} \cdot \log \frac{\sqrt{a}+x\sqrt{-b}}{\sqrt{a}-x\sqrt{-b}},$$

when $a>0$ and $b<0$.

$$\int \frac{dx}{(a+bx^2)^2} = \frac{x}{2a\,(a+bx^2)} + \frac{1}{2a}\int \frac{dx}{a+bx^2}.$$

$$\int \frac{dx}{(a+bx^2)^{n+1}} = \frac{1}{2na} \cdot \frac{x}{(a+bx^2)^n} + \frac{2n-1}{2na}\int \frac{dx}{(a+bx^2)^n}.$$

$$\int \frac{x^2 \cdot dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b}\int \frac{dx}{a+bx^2}.$$

$$\int \frac{x^2 \cdot dx}{(a+bx^2)^{n+1}} = \frac{-x}{2nb\,(a+bx^2)^n} + \frac{1}{2nb}\int \frac{dx}{(a+bx^2)^n}.$$

$$\int \frac{dx}{x\,(a+bx^2)} = \frac{1}{2a}\log\left(\frac{x^2}{a+bx^2}\right).$$

$$\int \frac{dx}{x^2\,(a+bx^2)} = -\frac{1}{ax} - \frac{b}{a}\int \frac{dx}{a+bx^2}.$$

$$\int \frac{dx}{x^2\,(a+bx^2)^{n+1}} = \frac{1}{a}\int \frac{dx}{x^2\,(a+bx^2)^n} - \frac{b}{a}\int \frac{dx}{(a+bx^2)^{n+1}}.$$

$$\int x^m \cdot (a+bx^n)^P \cdot dx = \frac{x^{m-n+1} \cdot (a+bx^n)^{P+1}}{b\,(nP+m+1)} - \frac{a\,(m-n+1)}{b\,(nP+m+1)} \cdot \int x^{m-n} \cdot (a+bx^n)^P \cdot dx.$$

$$\text{or} \quad = \frac{x^{m+1} \cdot (a+bx^n)^P}{nP+m+1} + \frac{anP}{nP+m+1} \int x^m \cdot (a+bx^n)^{P-1} \cdot dx$$

$$\text{or} \quad = \frac{x^{m+1} \cdot (a+bx^n)^{P+1}}{a\,(m+1)} - \frac{b\,(nP+m+n+1)}{a\,(m+1)} \int x^{m+n} \cdot (a+bx^n)^P \cdot dx,$$

$$\text{or} \quad = -\frac{x^{m+1} \cdot (a+bx^n)^{P+1}}{an\,(P+1)} + \frac{nP+m+n+1}{an\,(P+1)} \int x^m \cdot (a+bx^n)^{P+1} \cdot dx.$$

$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{4\,ac-b^2}} \cdot \tan^{-1}\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right),$$

$$\text{or} \quad = \frac{1}{\sqrt{b^2-4\,ac}} \cdot \log\left(\frac{2\,ax+b-\sqrt{b^2-4\,ac}}{2\,ax+b+\sqrt{b^2-4\,ac}}\right).$$

$$\int \frac{x \cdot dx}{ax^2+bx+c} = \frac{1}{2\,a} \cdot \log\,(ax^2+bx+c) - \frac{b}{2\,a} \int \frac{dx}{ax^2+bx+c}.$$

$$\int x\sqrt{a+bx} \cdot dx = -\frac{2\,(2\,a-3\,bx)\,(a+bx)^{\frac{3}{2}}}{15\,b^2}.$$

$$\int x^2 \cdot \sqrt{a+bx} \cdot dx = \frac{2\,(8\,a^2-12\,abx+15\,b^2x^2)\,(a+bx)^{\frac{3}{2}}}{105\,b^3}.$$

$$\int \frac{x^n \cdot dx}{\sqrt{a+bx}} = \frac{2\,x^n\sqrt{a+bx}}{(2\,n+1)\,b}$$

$$-\frac{2\,na}{(2\,n+1)\,b}\int \frac{x^{n-1}\cdot dx}{\sqrt{a+bx}}.$$

$$\int \frac{x \cdot dx}{\sqrt{a+bx}} = -\frac{2\,(2\,a-bx)\sqrt{a+bx}}{3\,b^2}.$$

$$\int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}}\cdot \log \frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}},$$

when $a > 0$,

or $$= \frac{2}{\sqrt{-a}}\cdot \tan^{-1}\sqrt{\frac{a+bx}{-a}},$$

when $a < 0$.

$$\int \frac{dx}{x^n\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{(n-1)\,ax^{n-1}}$$

$$-\frac{(2\,n-3)\,b}{(2\,n-2)\,a}\int \frac{dx}{x^{n-1}\sqrt{a+bx}}.$$

$$\int \frac{\sqrt{a+bx}}{x}\cdot dx = 2\sqrt{a+bx}$$

$$+a\int \frac{dx}{x\sqrt{a+bx}}.$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right).$$

$$\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{1}{a}\cdot \log\left(\frac{x}{a+\sqrt{a^2-x^2}}\right).$$

$$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = \frac{-\sqrt{a^2-x^2}}{a^2x}.$$

$$\int \sqrt{a^2-x^2}\cdot dx = \frac{x}{2}\sqrt{a^2-x^2}$$

$$+\frac{a^2}{2}\cdot \sin^{-1}\left(\frac{x}{a}\right).$$

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$$\int x^2\sqrt{a^2-x^2}\cdot dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2}+\frac{a^4}{8}\sin^{-1}\left(\frac{x}{a}\right).$$

$$\int\frac{\sqrt{a^2-x^2}}{x}\cdot dx=\sqrt{a^2-x^2} - a\cdot\log\left(\frac{a+\sqrt{a^2-x^2}}{x}\right).$$

$$\int\frac{\sqrt{a^2-x^2}}{x^2}\cdot dx=\frac{-\sqrt{a^2-x^2}}{x}-\sin^{-1}\left(\frac{x}{a}\right).$$

$$\int\frac{x^2\cdot dx}{\sqrt{a^2-x^2}}=-\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right).$$

$$\int\frac{dx}{(a^2-x^2)^{\frac{3}{2}}}=\frac{x}{a^2\sqrt{a^2-x^2}}.$$

$$\int(a^2-x^2)^{\frac{3}{2}}\cdot dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2}+\frac{3}{8}a^4\cdot\sin^{-1}\left(\frac{x}{a}\right).$$

$$\int\frac{x^2\cdot dx}{(a^2-x^2)^{\frac{3}{2}}}=\frac{x}{\sqrt{a^2-x^2}}-\sin^{-1}\left(\frac{x}{a}\right).$$

$$\int\frac{dx}{\sqrt{x^2\pm a^2}}=\log\left(x+\sqrt{x^2\pm a^2}\right).$$

$$\int\frac{dx}{x\sqrt{x^2-a^2}}=\frac{1}{a}\cdot\sec^{-1}\left(\frac{x}{a}\right).$$

$$\int\frac{dx}{x\sqrt{x^2+a^2}}=\frac{1}{a}\cdot\log\left(\frac{x}{a+\sqrt{x^2+a^2}}\right).$$

$$\int\frac{dx}{x^2\sqrt{x^2\pm a^2}}=\mp\frac{\sqrt{x^2\pm a^2}}{a^2x}.$$

$$\int\frac{dx}{x^3\sqrt{x^2-a^2}}=\frac{\sqrt{x^2-a^2}}{2a^2x^2}+\frac{1}{2a^3}\sec^{-1}\frac{x}{a}.$$

$$\int \frac{dx}{x^3\sqrt{x^2+a^2}} =$$

$$\frac{-\sqrt{x^2+a^2}}{2a^2x^2} + \frac{1}{2a^3}\log\frac{a+\sqrt{x^2+a^2}}{x}.$$

$$\int \sqrt{x^2\pm a^2}\cdot dx =$$

$$\frac{x}{2}\sqrt{x^2\pm a^2} \pm \frac{a^2}{2}\log\left(x+\sqrt{x^2\pm a^2}\right).$$

$$\int x^2\sqrt{x^2\pm a^2}\cdot dx = \frac{x}{8}(2x^2\pm a^2)\sqrt{x^2\pm a^2}$$

$$-\frac{a^4}{8}\log\left(x+\sqrt{x^2\pm a^2}\right).$$

$$\int \frac{\sqrt{x^2-a^2}}{x}\cdot dx = \sqrt{x^2-a^2} - a\cos^{-1}\frac{a}{x}.$$

$$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\cdot\log\frac{a+\sqrt{x^2+a^2}}{x}.$$

$$\int \frac{\sqrt{x^2\pm a^2}}{x^2}\cdot dx =$$

$$\frac{-\sqrt{x^2\pm a^2}}{x} + \log\left(x+\sqrt{x^2\pm a^2}\right).$$

$$\int \frac{x^2\,dx}{\sqrt{x^2\pm a^2}} = \frac{x}{2}\sqrt{x^2\pm a^2} \mp \frac{a^2}{2}\log\left(x+\sqrt{x^2\pm a^2}\right).$$

$$\int \frac{dx}{(x^2\pm a^2)^{\frac{3}{2}}} = \pm\frac{x}{a^2\sqrt{x^2\pm a^2}}.$$

$$\int \frac{x^2\,dx}{(x^2\pm a^2)^{\frac{3}{2}}} = \frac{-x}{\sqrt{x^2\pm a^2}} + \log\left(x+\sqrt{x^2\pm a^2}\right).$$

$$\int (x^2\pm a^2)^{\frac{3}{2}}\,dx = \frac{x}{8}(2x^2\pm 5a^2)\sqrt{x^2\pm a^2}$$

$$-\frac{3a^4}{8}\log\left(x+\sqrt{x^2\pm a^2}\right).$$

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$$\int \frac{dx}{\sqrt{2\,ax - x^2}} = \text{vers}^{-1}\frac{x}{a}.$$

$$\int \frac{x^m\,dx}{\sqrt{2\,ax - x^2}} = -\frac{x^{m-1}\sqrt{2\,ax - x^2}}{m} + \frac{(2\,m-1)\,a}{m}\int \frac{x^{m-1}\cdot dx}{\sqrt{2\,ax - x^2}}.$$

$$\int \frac{dx}{x^m\sqrt{2\,ax - x^2}} = -\frac{\sqrt{2\,ax - x^2}}{(2\,m-1)\,ax^m} + \frac{m-1}{(2\,m-1)\,a}\int \frac{dx}{x^{m-1}\sqrt{2\,ax - x^2}}.$$

$$\int \sqrt{2\,ax - x^2}\cdot dx = \frac{x-a}{2}\sqrt{2\,ax - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x-a}{a}.$$

$$\int x^m\sqrt{2\,ax - x^2}\cdot dx = -\frac{x^{m-1}(2\,ax - x^2)^{\frac{3}{2}}}{m+2} + \frac{(2\,m+1)\,a}{m+2}\int x^{m-1}\cdot\sqrt{2\,ax - x^2}\cdot dx.$$

$$\int \frac{\sqrt{2\,ax - x^2}}{x^m}\cdot dx = \frac{-(2\,ax - x^2)^{\frac{3}{2}}}{(2\,m-3)\,ax^m} + \frac{m-3}{(2\,m-3)\,a}\int \frac{\sqrt{2\,ax - x^2}}{x^{m-1}}\cdot dx.$$

$$\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \frac{1}{\sqrt{a}}\log\left(2\,ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c}\right).$$

$$\int \sqrt{ax^2 + bx + c}\cdot dx = \frac{2\,ax + b}{4\,a}\sqrt{ax^2 + bx + c} - \left(\frac{b^2 - 4\,ac}{8\,a}\right)\int \frac{dx}{\sqrt{ax^2 + bx + c}}.$$

$$\int \frac{dx}{\sqrt{-ax^2+bx+c}} = \frac{1}{\sqrt{a}} \sin^{-1}\left(\frac{2\,ax-b}{\sqrt{b^2+4\,ac}}\right).$$

$$\int \sqrt{-ax^2+bx+c} \cdot dx = \frac{2\,ax-b}{4\,a} \sqrt{-ax^2+bx+c}$$

$$+ \frac{b^2+4\,ac}{8\,a} \int \frac{dx}{\sqrt{-ax^2+bx+c}}.$$

$$\int \frac{x\,dx}{\sqrt{\pm\,ax^2+bx+c}} = \frac{\sqrt{\pm\,ax^2+bx+c}}{\pm\,a}$$

$$\mp \frac{b}{2\,a} \int \frac{dx}{\sqrt{\pm\,ax^2+bx+c}}.$$

$$\int x\sqrt{\pm\,ax^2+bx+c} \cdot dx = \frac{(\pm\,ax^2+bx+c)^{\frac{3}{2}}}{3\,a}$$

$$\mp \frac{b}{2\,a} \int \sqrt{\pm\,ax^2+bx+c} \cdot dx.$$

$$\int \sin^2 x \cdot dx = \frac{x}{2} - \frac{1}{4} \sin(2\,x).$$

$$\int \cos^2 x \cdot dx = \frac{x}{2} + \frac{1}{4} \sin(2\,x).$$

$$\int \sin^2 x \cdot \cos^2 x \cdot dx = \frac{1}{8}\left(x - \frac{1}{4} \sin 4\,x\right).$$

$$\int \sec x \cdot \csc x \cdot dx = \int \frac{dx}{\sin x \cdot \cos x}$$

$$= \log \tan x.$$

$$\int \sec^2 x \cdot \csc^2 x \cdot dx = \int \frac{dx}{\sin^2 x \cdot \cos^2 x}$$

$$= \tan x - \cot x.$$

$$\int \sin^m x \cdot \cos^n x \cdot dx = \frac{-\sin^{m-1} x \cdot \cos^{n+1} x}{m+n}$$

$$+ \frac{m-1}{m+n} \int \sin^{m-2} x \cdot \cos^n x \cdot dx,$$

or
$$= \frac{\sin^{m+1} x \cdot \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cdot \cos^{n-2} x \cdot dx.$$

$$\int \sin^m x \cdot dx = -\frac{\sin^{m-1} x \cdot \cos x}{m} + \frac{m-1}{m} \int \sin^{m-2} x \cdot dx.$$

$$\int \cos^n x \cdot dx = \frac{\sin x \cdot \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \cdot dx.$$

$$\int \frac{\sin^m x}{\cos^n x} dx = \frac{\sin^{m+1} x}{(n-1) \cos^{n-1} x} + \frac{n-m-2}{n-1} \int \frac{\sin^m x \cdot dx}{\cos^{n-2} x}.$$

$$\int \frac{\cos^n x}{\sin^m x} \cdot dx = \frac{-\cos^{n+1} x}{(m-1) \sin^{m-1} x} + \frac{m-n-2}{m-1} \int \frac{\cos^n x \, dx}{\sin^{m-2} x}.$$

$$\int \frac{dx}{\sin^m x} = \frac{-\cos x}{(m-1) \sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{dx}{\sin^{m-2} x}.$$

$$\int \frac{dx}{\cos^n x} = \frac{\sin x}{(n-1) \cos^{n-1} x} + \frac{n-2}{n-1} \int \frac{dx}{\cos^{n-2} x}.$$

$$\int \tan^n x \cdot dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \cdot dx.$$

$$\int \cot^n x \cdot dx = \frac{-\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \cdot dx.$$

$$\int \frac{dx}{a+b\cos x} = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1}\left(\sqrt{\frac{a-b}{a+b}} \cdot \tan \frac{x}{2}\right),$$

if $a^2 > b^2$;

$$= \frac{1}{\sqrt{b^2-a^2}} \cdot \log \frac{\sqrt{b-a}\tan\frac{x}{2}+\sqrt{b+a}}{\sqrt{b-a}\tan\frac{x}{2}-\sqrt{b+a}}.$$

if $a^2<b^2$.

$$\int x^m \cdot \sin x \cdot dx =$$

$$-x^m \cos x + m\int x^{m-1}\cos x\, dx.$$

$$\int x^m \cdot \cos x \cdot dx =$$

$$x^m \cdot \sin x - m\int x^{m-1} \cdot \sin x \cdot dx.$$

$$\int \frac{\sin x}{x}\, dx = x - \frac{x^3}{3\,\underline{|3}} + \frac{x^5}{5\,\underline{|5}} - \frac{x^7}{7\,\underline{|7}} + \cdots.$$

$$\int \frac{\sin x}{x^m}\, dx = \frac{-1}{m-1}\,\frac{\sin x}{x^{m-1}} + \frac{1}{m-1}\int \frac{\cos x\, dx}{x^{m-1}}.$$

$$\int \frac{\cos x}{x}\, dx = \log x - \frac{x^2}{2\,\underline{|2}} + \frac{x^4}{4\,\underline{|4}} - \frac{x^6}{6\,\underline{|6}} + \cdots.$$

$$\int \frac{\cos x}{x^m}\, dx = \frac{-1}{m-1} \cdot \frac{\cos x}{x^{m-1}} - \frac{1}{m-1}\int \frac{\sin x\, dx}{x^{m-1}}.$$

$$\int x \sin^{-1} x \cdot dx =$$

$$\frac{1}{4}\left[(2x^2-1)\sin^{-1} x + x\sqrt{1-x^2}\right].$$

$$\int x^n \sin^{-1} x \cdot dx =$$

$$\frac{x^{n+1}\sin^{-1} x}{n+1} - \frac{1}{n+1}\int \frac{x^{n+1}\, dx}{\sqrt{1-x^2}}.$$

$$\int x^n \cos^{-1} x \cdot dx =$$

$$\frac{x^{n+1}\cos^{-1} x}{n+1} + \frac{1}{n+1}\int \frac{x^{n+1}\mathbf{dx}}{\sqrt{1-x^2}}.$$

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$$\int x^n \tan^{-1} x \cdot dx = \frac{x^{n+1}\tan^{-1}x}{n+1} - \frac{1}{n+1}\int\frac{x^{n+1}\,dx}{1+x^2}.$$

$$\int x^n \log x \cdot dx = x^{n+1}\left[\frac{\log x}{n+1} - \frac{1}{(n+1)^2}\right].$$

$$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx.$$

$$\int \frac{e^{ax}}{x^n}\,dx = \frac{-1}{n-1}\cdot\frac{e^{ax}}{x^{n-1}} + \frac{a}{n-1}\int\frac{e^{ax}}{x^{n-1}}\,dx.$$

$$\int e^{ax}\log x \cdot dx = \frac{e^{ax}\log x}{a} - \frac{1}{a}\int\frac{e^{ax}\,dx}{x}.$$

$$\int e^{ax}\sin(nx)\cdot dx = e^{ax}\left(\frac{a\sin[nx] - n\cos[nx]}{a^2+n^2}\right).$$

$$\int e^{ax}\cos(nx)\,dx = e^{ax}\left[\frac{a\cos(nx)+n\sin(nx)}{a^2+n^2}\right].$$

$$\int\sqrt{\frac{a+x}{b+x}}\cdot dx = \sqrt{(a+x)\,(b+x)} + (a-b)\log(\sqrt{a+x}+\sqrt{b+x}).$$

$$\int\sqrt{\frac{a-x}{b+x}}\,dx = \sqrt{(a-x)\,(b+x)} + (a+b)\sin^{-1}\sqrt{\frac{b+x}{a+b}}.$$

$$\int\frac{dx}{\sqrt{(x-a)\,(b-x)}} = 2\cot^{-1}\sqrt{\frac{b-x}{x-a}} = 2\sin^{-1}\sqrt{\frac{x-a}{b-a}}.$$

$$\int\frac{dx}{x\sqrt{x^n+a^2}} = \frac{1}{an}\log\frac{\sqrt{a^2+x^n}-a}{\sqrt{a^2+x^n}+a}.$$

$$\int\frac{dx}{x\sqrt{x^n-a^2}} = \frac{2}{an}\sec^{-1}\frac{x^{\frac{n}{2}}}{a}.$$

THEORETICAL MECHANICS.

NOTATION.

A = area.

α = angular acceleration.

a = linear acceleration.

a_n = normal acceleration.

a_t = tangential acceleration.

C = component of a force.

F = force.

F_n = normal force.

F_t = tangential force.

g = acceleration due to gravity = 32.2. (The exact value is 32.1808 − 0.0821 cos 2 L, where L is the latitude.)

I_g = moment of inertia referred to center of gravity.

I_{gx} = moment of inertia about an axis through the center of gravity and parallel to the X-axis.

M = moment of a force.

m = mass = weight $\div g$.

R = resultant of a system of forces.

S = space.

v = velocity.

v_0 = initial velocity.

v_t = tangential velocity.

x, y, z = rectangular coördinates of a point.

ρ, θ = polar coördinates of a point.

$\bar{\rho}$ = distance from pole to center of gravity.

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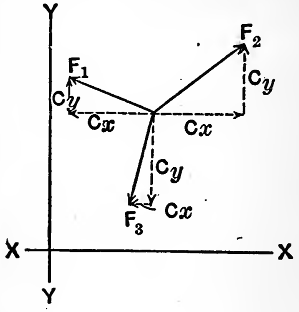
STATICS.

Equilibrium of Forces.

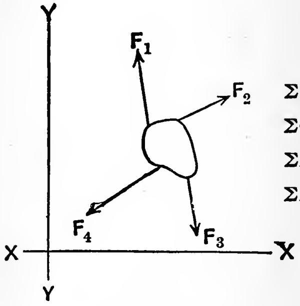
Concurrent Forces in Equilibrium in One Plane.



$$\Sigma C_x = 0.$$
$$\Sigma C_y = 0.$$

Fig. 18.

Non-concurrent Forces in Equilibrium in One Plane.



$$\Sigma C_x = 0.$$
$$\Sigma C_y = 0.$$
$$\Sigma M = 0.$$
$$\Sigma M = \Sigma C_x y + \Sigma C_y x.$$

Fig. 19.

If three forces are in equilibrium they must be concurrent or parallel.

If a system of non-concurrent forces in space is in equilibrium, the plane systems formed by projecting the given system upon

three coördinate planes must each be in equilibrium.

A couple consists of two equal and opposite parallel forces acting on a rigid body at a fixed distance apart.

The moment of a couple is equal to the product of one force by the distance between the two forces.

Centroid of Parallel Forces.

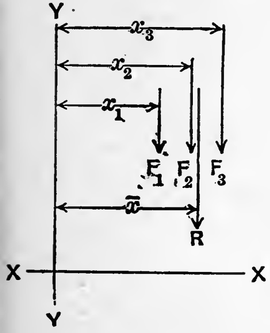


Fig. 20.

$$R = \Sigma F.$$

$$\bar{x} = \frac{\Sigma F x}{\Sigma F}.$$

For a variable pressure,

$$\bar{x} = \frac{\int x F\, dx}{\int F\, dx}.$$

Center of Gravity of an Area.

$$\bar{x} = \frac{\Sigma x \cdot dA}{\Sigma\, dA}$$

$$= \frac{\int\int x\, dx\, dy}{\int\int dx\, dy}.$$

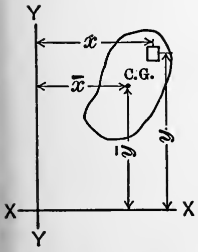


Fig. 21.

$$\bar{y} = \frac{\Sigma y \cdot dA}{\Sigma\, dA}$$

$$= \frac{\int\int y\, dx\, dy}{\int\int dx\, dy}.$$

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If $y_2 - y_1 = fx$,

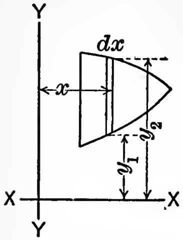

Fig. 22.

$$\bar{x} = \frac{\int x\,(y_2 - y_1)\,dx}{\int (y_2 - y_1)\,dx}.$$

$$= \frac{\int x \cdot fx \cdot dx}{\int fx \cdot dx}.$$

Center of Gravity of a Mass.

For a homogeneous mass,

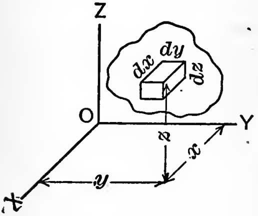

Fig. 23.

$$\bar{x} = \frac{\Sigma x\,dm}{\Sigma\,dm} = \frac{\int\int\int x\,dx\,dy\,dz}{\int\int\int dx\,dy\,dz}.$$

$$\bar{y} = \frac{\Sigma y\,dm}{\Sigma\,dm} = \frac{\int\int\int y\,dx\,dy\,dz}{\int\int\int dx\,dy\,dz}.$$

$$\bar{z} = \frac{\Sigma z\,dm}{\Sigma\,dm} = \frac{\int\int\int z\,dx\,dy\,dz}{\int\int\int dx\,dy\,dz}.$$

Moment of Inertia of an Area.

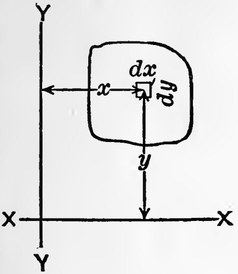

Fig. 24.

$$I_x = \Sigma y^2\, dA$$
$$= \int\int y^2\, dx\, dy.$$
$$I_y = \Sigma y^2\, dA$$
$$= \int\int x^2\, dx\, dy.$$

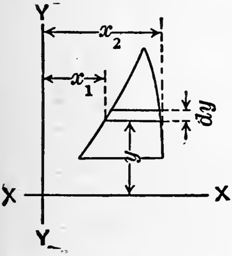

Fig. 25.

$$I_x = \Sigma y^2\, dA$$
$$= \int y^2 \cdot (x_2 - x_1)\, dy$$
$$= \int y^2 \cdot fy \cdot dy.$$

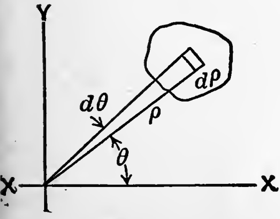

Fig. 26.

$$I_0 = \Sigma \rho^2\, dA$$
$$= \int\int \rho^2 \cdot d\rho \cdot d\theta.$$
$$\rho^2 = x^2 + y^2.$$
$$I_0 = I_x + I_y.$$

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Moment of Inertia of a Mass.

If k is a constant, equal to the density divided by g,

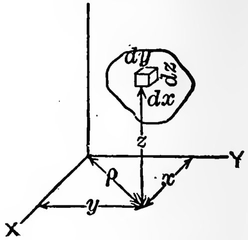

Fig. 27.

$$I_z = \rho^2\, dm$$
$$= k \int\int\int \rho^2\, dx\, dy\, dz$$
$$= k \int\int\int (x^2+y^2)\, dx\, dy\, dz.$$

Product of Inertia of an Area.

$$J = \Sigma xy\, dA = \int\int xy\, dx\, dy.$$

For the principal axes, J is zero.

Radius of Gyration.

$$r = \sqrt{\frac{I}{A}}, \quad \text{or} \quad r = \sqrt{\frac{I}{m}}.$$

Transformation Formulæ.

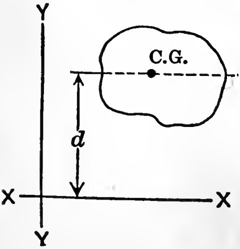

Fig. 28.

$$I_x = I_{gx} + Ad^2$$

or

$$I_x = I_{gx} + md^2.$$
$$J_{xy} = J_{c.g.} + Akh,$$

where h, k are the coördinates of the center of gravity referred to X–X and Y–Y.

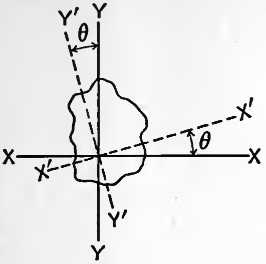

Fig. 29.

$$I'_x = I_x \cos^2\theta + I_y \sin^2\theta - J_{xy} \sin^2 2\theta.$$

$$J'_{xy} = J_{xy} \cos 2\theta + \frac{1}{2}(I_x - I_y)\sin 2\theta.$$

To determine the value of θ which will make X'–X' a principal axis,

$$\tan 2\theta = \frac{2\,J_{xy}}{I_y - I_x}.$$

Ellipsoid of Inertia.

The moments of inertia about all axes through any given point of any rigid body are inversely proportional to the squares of the diameters which they intercept in an imaginary ellipsoid, whose center is the given point, and whose position depends upon the distribution of the mass and the location of the given point. This ellipsoid is the ellipsoid of inertia for the body. The axes which contain the principal diameters of the ellipsoid are called the principal axes of the body for the given point.

Circle of Inertia.*

For any plane figure, lay off OX parallel to X–X,

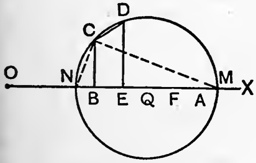

Fig. 30.

$OA = I_x$,

$OB = I_y$,

$BC = J_{xy}$,

$BQ = \frac{1}{2} BA$,

circle through C with center at Q,

* See Maurer's Technical Mechanics, Appendix B, or Civil Engineers' Pocket Book.

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CD parallel to X'–X' (Fig. 29),

DE perpendicular to OX,

$$QF = EQ.$$

Then $OE = I'_x$, and $OF = I'_y$.

$$ED = J'_{xy}.$$

The principal axes for the given point are parallel to CM and CN.

J is positive above and negative below OX.

DYNAMICS.

Velocity and Acceleration.

$$v = \frac{ds}{dt}.$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}.$$

Uniformly Accelerated Motion.

If a is constant,

$$v = v_0 + at.$$

$$S = v_0t + \tfrac{1}{2}\,at^2$$

$$= \frac{v^2 - v_0{}^2}{2\,a}$$

$$= \frac{1}{2}\,(v_0 + v)\ t.$$

$$v\,dv = a\,ds.$$

Falling Bodies.

For a body falling in a vacuum, $a = g$, hence

$$v = v_0 + gt.$$

$$S = v_0t + \frac{1}{2}\,gt^2$$

$$= \frac{v^2 - v_0{}^2}{2\,g}$$

$$= \frac{1}{2}\,(v_0 + v)\ t.$$

Force and Acceleration.

$$F = m \cdot a = \frac{W}{g} \cdot a.$$

Direct Central Impact.

For two inelastic bodies, let

m_1 = mass of first body.

m_2 = mass of second body.

v_1 = original velocity of first body.

v_2 = original velocity of second body.

v = common velocity after impact.

Then $$v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}.$$

For two elastic bodies having velocities k_1 and k_2 after impact,

$$m_1 v_1 + m_2 v_2 = m_1 k_1 + m_2 k_2.$$

The product of mass by its velocity is momentum.

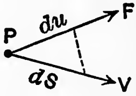
The sum of the momenta before and after impact is constant.

Virtual Velocities.

F = force.

V = direction of motion of P.

du = virtual velocity of force.



$$\frac{du}{dt} = \text{velocity of force.}$$

$$\frac{ds}{dt} = \text{velocity of } P.$$

$$F \cdot du = \text{virtual moment of force.}$$

The virtual moment of a force is equal to the algebraic sum of the virtual moments of its components.

For a system of concurrent forces in equilibrium,

$$\Sigma F \cdot du = 0.$$

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For any small displacement or motion of a rigid body in equilibrium under non-concurrent forces in a plane, with all points of the body moving parallel to this plane,

$$\Sigma F \cdot du = 0.$$

Curvilinear Motion of a Point.

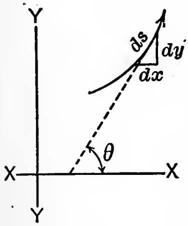

Fig. 31.

$$v_t = \frac{ds}{dt}.$$

$$v_t^2 = \left(\frac{ds}{dt}\right)^2$$

$$= \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2.$$

$$v_t^2 = v_x^2 + v_y^2.$$

$$a_t = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$= a_x \cos\theta + a_y \sin\theta.$$

$$a_n = a_y \cos\theta - a_x \sin\theta = \frac{v_t^2}{r},$$

where r is the radius of curvature.

$$F_n = \frac{m \cdot v_t^2}{r}.$$

$$F_t = m \cdot a_x \cos\theta + m \cdot a_y \sin\theta = m \cdot a_t.$$

$$\frac{v^2 - v_0^2}{2} = \int a_t \, ds.$$

Projectiles.

Neglecting the resistance of the air,

$$x = v_0 \cos\theta \cdot t.$$

$$y = v_0 \sin\theta \cdot t - \frac{1}{2} g t^2,$$

or

$$y = x \tan\theta - \frac{g x^2}{2 v_0^2 \cos^2\theta}.$$

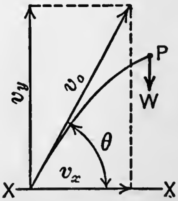

Fig. 32.

Horizontal range,

$$x_r = \frac{v_0^2}{g} \sin 2\,\theta,$$

which is a maximum for $\theta = 45°$.

The greatest height of ascent is

$$y_m = \frac{v_0^2}{2\,g} \sin^2 \theta.$$

Translation of a Rigid Body.

$$dF_x = a_x \cdot dm.$$

$$R_x = \int a_x \cdot dm.$$

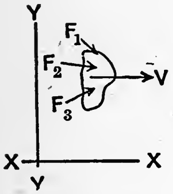

Fig. 33.

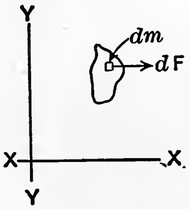

Fig. 34.

The resultant force must act in a line through the center of gravity and parallel to the direction of motion.

Rotation of a Rigid Body.

Let O be the axis of rotation.

θ = angular space passed over by any line from O.

α = angular acceleration.

ω = angular velocity.

Then

$$\omega = \frac{d\theta}{dt}.$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}.$$

$$\omega\, d\omega = \alpha\, d\theta,$$

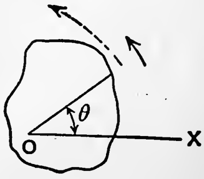

Fig. 35.

For uniform acceleration, $\alpha = k$, $\therefore$

$$\omega = \omega_0 + kt.$$

$$\theta = \omega_0 t + \frac{1}{2} kt^2$$

$$= \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$= \frac{\omega_0 + \omega}{2} \cdot t.$$

For a point ρ distant from O,

$$v_t = \rho \cdot \omega.$$

$$a = \rho \cdot \alpha.$$

$$s = \rho \cdot \theta.$$

$$dF = dm \cdot a$$

$$= \rho\,\alpha \cdot dm.$$

$$dM_0 = \rho \cdot dF.$$

$$dM_0 = \rho^2 \alpha\, dm.$$

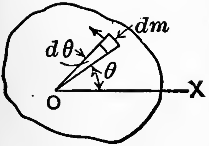

Fig. 36.

$$M_0 = \int \rho^2 \cdot \alpha \cdot dm$$

$$= \alpha \int \rho^2\, dm$$

$$= \alpha \cdot I.$$

For a mass m concentrated ρ distant from O,

$$M_0 = \alpha \rho^2 m.$$

Precessional Rotation.

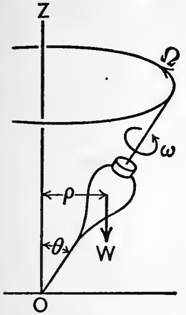

Fig. 37.

ω = velocity about axis of spin (OP) in radians per sec.

Ω = velocity about axis of precession (OZ) in radians per sec.

I = moment of inertia about axis of spin.

T = torque ($= W\rho$ for equilibrium).

$$T = \omega\Omega I \sin\theta.$$

For $\theta = 90°$, $T = \omega\Omega I.$

Center of Percussion or Oscillation.

If an unsupported bar upon being struck at a begins to rotate about b, then a is the center of percussion for b as a center, and b is the center of instantaneous rotation.

$$Fh = \int \rho^2 \cdot \alpha \cdot dm$$

$$= \alpha I_b.$$

$$dF = \alpha \cdot \rho \cdot dm.$$

$$F = \alpha \int \rho \cdot dm$$

$$= \alpha \cdot \bar{\rho} \cdot m.$$

$$h = \frac{I_b}{\bar{\rho} m} = \frac{r^2}{\bar{\rho}}.$$

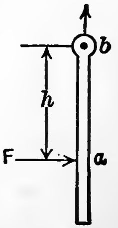

Fig. 38.

Pendulum.

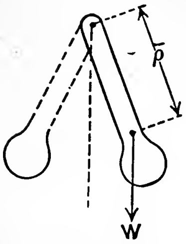

Fig. 39.

T = time of oscillation from one extreme position to the other.

r = radius of gyration.

$$T = \pi\sqrt{\frac{r^2}{\bar{\rho}\cdot g}}.$$

Work, Energy, and Power.

Work (w) is equal to force (F) multiplied by the distance (S) through which it acts.

$$w = F \cdot S.$$

Power (L) is the rate of doing work. $L = \frac{w}{t}$.

Energy is the capacity to do work.

The energy of a moving body, $K.E. = \frac{1}{2}mv^2$.

The kinetic energy of rotation is $K.E. = \frac{1}{2}I \cdot \omega^2$.

Friction.

F = friction.

N = normal force.

f = coefficient of friction.

$F = f \cdot N$.

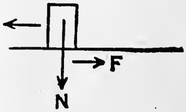

Fig. 40.

Angle of friction, $\phi = \tan^{-1}\frac{F}{N}$.

Average values of f for motion are as follows:

Wood on wood	.25–.50
Metal on wood	.50–.60
Leather on metal	0.56
Leather on metal, lubricated	0.15
Metal on metal, — dry	0.15–.24
Lubricated surfaces:	
Ordinary	0.08
Best	0.03–0.036

For values of f for rest add 40 per cent to above values.

Friction of Belt.

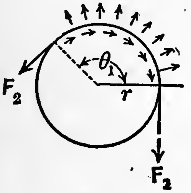

Fig. 41.

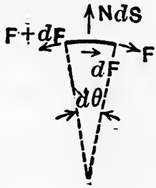

Fig. 42.

$$dF = f \cdot N\,ds = f\,\frac{F}{r}\,ds.$$

$$ds = r\,d\theta, \quad \text{and} \quad f \cdot d\theta = \frac{dF}{F}.$$

$$\therefore \quad f \cdot \theta_1 = \log_e \left[\frac{F_2}{F_1}\right],$$

or $\quad F_1 \cdot e^{f \cdot \theta_1} = F_2$, where θ_1 is in radians.

MECHANICS OF MATERIALS.

Material.	Wt., Lb. per Cu. Ft.	Rings per Inch.	Ultimate Strength, Lb. per Sq. In.				Elas. Limit, Lb. per Sq. In.		Fac. of Safety.			Mod. of Elas. $\times 10^{-6}$.	
			Tension.	Flexure.	Compression.	Shear.	Tension.	Compression.	Steady.	Var.	Shock.	Ten. and Comp.	Shear.
Cast Iron	450	..	20,000	35,000	90,000	18,000	6,000	20,000	6	10	20	15	6
Wrought Iron . .	480	..	50,000	60,000	50,000	40,000	25,000	25,000	4	6	10	28	12
Struct. Steel . . .	490	..	60,000	80,000	60,000	50,000	36,000	36,000	4	6	10	30	13
Strong Steel . . .	490	..	100,000	110,000	120,000	80,000	60,000	60,000	5	8	15	30	......
Brick	125	..	......	500	2,500	1,000	.....	1,000	20	30	40	2	......
Stone	160	..	......	2,000	5,000	1,500	.....	2,000	20	30	40	6	......
Concrete	150	..	300	......	2,500	1,000	.....	1,000	5	10	20	2	......
Timber	35	..	7,000	6,000	*[5,000 600	*[350 1,000	3,000	3,000	†6	10	15	1.5	0.4
White Pine . . .	27	10	7,000	6,000	[4,000 800	[300 1,000	.....	.....	..	..	..	......	......
Longleaf Y.P.. .	40	14	9,000	8,000	[7,000 700	[400 1,500	.....	.....	..	..	..	......	......
Shortleaf Y.P. .	35	10	8,000	7,000	[6,000 700	[400 1,200	.....	.....	..	..	..	......	......
Loblolly Y.P.. .	35	6	7,000	6,000	[5,000 650	[400 1,000	.....	.....	..	..	..	......	......
Douglas Fir . .	30	14	8,000	6,000	[4,500 600	[300 900	.....	.....	..	..	..	......	......
Redwood	25	20	7,000	4,000	[3,500 500	[300 800	.....	.....	..	..	..	......	......
White Oak . . .	48	..	10,000	8,000	[6,000 2,000	[600 4,000	.....	.....	..	..	..	......	......

* Parallel to the grain and across the grain, respectively.

† For compression across the grain and for shear use 3.

NOTATION.

A = area.

b = breadth.

d = depth.

E = modulus of elasticity.

e = total deformation.

F = force.

I = moment of inertia.

I_0 = polar moment of inertia.

J = product of inertia.

l = length.

M = moment.

R = resultant of forces.

r = radius of gyration.

S = unit stress.

s = section modulus.

V = vertical shear.

W = total weight.

w = weight per lineal unit.

Δ = maximum deflection.

ϵ = unit deformation.

Direct Stress.

For an axial tensile or compressive force, or for simple shear,

$$S = \frac{F}{A}.$$

$$\epsilon = \frac{e}{l}.$$

$$E = \frac{S}{\epsilon} = \frac{Fl}{eA}.$$

For tension or compression the deformation is measured along the axis of the member, and for shear it is measured at right angles to the axis of the member.

Eccentric Loads.*

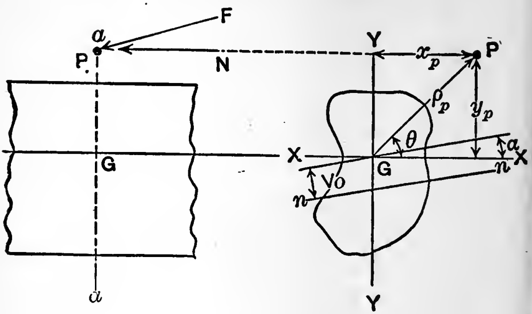

Fig. 43.

Consider a section $a-a$ perpendicular to axis of a bar, and take axes of coördinates through center of gravity.

Let x, y = coördinates of any point of section.

$n-n$ = neutral axis.

v = distance of any point from line through center of gravity and parallel to neutral axis, positive toward P.

v_0 = value of v for neutral axis.

F = force or resultant of forces acting at P.

N = component of F normal to section considered.

S_0 = unit stress at center of gravity.

$S_0 = \frac{N}{A}$.

---

* The method here presented is taken from a paper by L. J. Johnson, M. Am. Soc. C. E., "An Analysis of General Flexure in a Straight Bar of Uniform Cross Section," Trans. Am Soc. C. E., volume LVI, p. 169, 1906.

$$S = S_0 - \frac{S_0}{v_0} \cdot v$$

$$= S_0 - \frac{S_0}{v_0} (y \cos \alpha - x \cdot \sin \alpha)$$

$$= \frac{N}{A} + \frac{N \cdot x_P (y - x \tan \alpha)}{J - I_y \tan \alpha}$$

$$= \frac{N}{A} + \frac{N \cdot y_P (y - x \tan \alpha)}{I_x - J \tan \alpha}$$

$$= \frac{N}{A} + \frac{N \cdot \rho_P (y - x \cdot \tan \alpha) \cos \theta}{J - I_y \cdot \tan \alpha}$$

$$= \frac{N}{A} + \frac{N \cdot \rho_P (y - x \cdot \tan \alpha) \sin \theta}{I_x - J \cdot \tan \alpha}$$

$$= \frac{N}{A} + \frac{N (y_P I_y - x_P J)\, y + N (x_P I_x - y_P J)\, x}{I_x I_y - J^2}$$

$$= \frac{N}{A} + N \cdot \rho_P \times$$

$$\left[\frac{(I_y \sin \theta - J \cdot \cos \theta)\, y + (I_x \cos \theta - J \sin \theta)\, x}{I_x I_y - J^2}\right].$$

In the above equations $\frac{N}{A}$ is the portion of S which is direct stress, and the other term is the portion due to the bending moment, $M = N \cdot \rho_P$. If s represent the section modulus

$$\left(\frac{I_x I_y - J^2}{(I_y \sin \theta - J \cdot \cos \theta)\, y + (I_x \cos \theta - J \cdot \sin \theta)\, x}\right),$$

then

$$S = \frac{N}{A} + \frac{M}{s}.$$

NOTE. — The values of the section modulus given in the handbooks are computed from the formula $s = \frac{I}{y}$, which is the value of

s for $J=0$ and for P located on $Y-Y$. For angles and Z-bars J does not equal zero.

In the above equations,

$$\tan\alpha = \frac{I_x - J \cdot \tan\theta}{J - I_y \cdot \tan\theta}$$

$$= \frac{I_x \cot\theta - J}{J \cot\theta - I_y}$$

$$= \frac{I_x \cos\theta - J \cdot \sin\theta}{J \cos\theta - I_y \sin\theta}.$$

For any bar having a section which is symmetrical about either axis, $J=0$, and the values of S become

$$S = \frac{N}{A} + N \cdot \rho_P \left(\frac{I_y \sin\theta \cdot y + I_x \cos\theta \cdot x}{I_x I_y} \right).$$

If for a symmetrical section, P is on $Y-Y$, then $\sin\theta = 1$ and $\cos\theta = 0$, or

$$S = \frac{N}{A} + \frac{N \cdot \rho_P \cdot y}{I_x}$$

$$= \frac{N}{A} + \frac{M \cdot y}{I_x}.$$

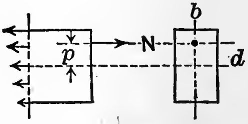

Fig. 44.

For a rectangular section, for which N is applied on $Y-Y$ and p distant from the axis of the bar, the extreme fiber stresses are

$$S = \frac{N}{A}\left(1 \pm 6\frac{p}{d}\right).$$

EQUATION OF NEUTRAL AXIS.

The equation of the neutral axis for an eccentric load is

$$y=\left(\frac{x_P\cdot I_x-y_P\cdot J}{x_P\cdot J-y_P\cdot I_y}\right)x+\frac{I_xI_y-J^2}{A\,(x_P\cdot J-y_P\cdot I_y)}.$$

KERNEL OR CORE-SECTION.

The kernel of a section (sometimes called the core-section) is the area within which P, the point of application of the resultant of the forces, must fall in order that the stress shall be of the same sign throughout the section. It is the area bounded by the locus of the P's corresponding to a series of neutral axes touching the periphery of the section but never crossing the section. For every side of the section there will be an apex of the kernel. If x_a, y_a and x_b, y_b are the coördinates of a and b, which are two consecutive vertices of the section, then the coördinates, x_{ab}, y_{ab}, of the vertex of the kernel corresponding to the side, ab, of the section will be

$$x_{ab}=-\frac{(x_a-x_b)\,J-(y_a-y_b)\,I_y}{A\,(x_ay_b-x_by_a)},$$

$$y_{ab}=-\frac{(x_a-x_b)\,I_x-(y_a-y_b)\,J}{A\,(x_ay_b-x_by_a)}.$$

If ab is parallel to X–X, then

$$x_{ab}=-\frac{J}{A\cdot y_a},\qquad y_{ab}=-\frac{I_x}{A\cdot y_a}.$$

If ab is parallel to Y–Y, then

$$x_{ab}=-\frac{I_y}{A\cdot x_a},\qquad y_{ab}=-\frac{J}{A\cdot x_a}.$$

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The radii vectores of the kernel are lengths which for any θ need only be multiplied by the area of the section (A) to give the section modulus

$$\left(\frac{IxIy-J^2}{(I_y \sin\theta - J\cdot\cos\theta)\, y + (I_y\cdot\cos\theta - J\cdot\sin\theta)\, x}\right),$$

but these lengths must be considered positive if measured on the opposite side of G from P.

SECTION MODULUS POLYGONS.

In the equation $S=\frac{N}{A}+\frac{M}{s}$ (see Eccentric Loads), s is the section modulus. The section modulus polygon is the polygon the lengths of whose radii vectores are the graphical representations of the values of s for extreme fibers for successive values of θ from 0 to 360 degrees. The section modulus polygon is a figure whose sides are parallel to the sides of the kernel of the given section but which lie on opposite sides of the center of gravity from the sides of the kernel.

The most general value of s is

$$\frac{IxIy-J^2}{(I_y \sin\theta - J\cos\theta)\, y + (I_y\cos\theta - J\cdot\sin\theta)\, x}.$$

For any section which is symmetrical about either axis, s becomes

$$s=\frac{IxIy}{I_y \sin\theta\cdot y + I_x\cos\theta\cdot x}.$$

For any symmetrical section for which P lies on Y–Y, $\theta=90°$, hence

$$s=\frac{I_x}{y}.$$

If for any symmetrical section P lies on X–X, $\theta = 0°$, hence

$$s = \frac{I_y}{x}.$$

There will be one vertex of the s-polygon for each side of the polygon bounding the section. If x_a, y_a and x_b, y_b are the coördinates of a and b, two consecutive vertices of the bounding polygon of the section, then the coördinates of the vertex of the s-polygon corresponding to the side ab of the bounding polygon will be

$$x_{ab} = \frac{(x_a - x_b)\,J - (y_a - y_b)\,I_y}{x_a y_b - x_b y_a},$$

$$y_{ab} = \frac{(x_a - x_b)\,I_x - (y_a - y_b)\,J}{x_a y_b - x_b y_a}.$$

If ab is parallel to X–X,

$$x_{ab} = \frac{J}{y_a}, \qquad y_{ab} = \frac{I_x}{y_a}.$$

If ab is parallel to Y–Y,

$$x_{ab} = \frac{I_y}{x_a}, \qquad y_{ab} = \frac{J}{x_a}.$$

For sections symmetrical about either X–X, or Y–Y, $J = 0$, and the values of $\frac{I_x}{y_a}$ and $\frac{I_y}{x_a}$ can be found in the handbooks issued by the steel companies, under the column marked "Section Modulus." The vertices can then be plotted and connected by straight lines to form the s-polygon. From this s-polygon the values of s for any value of θ can be obtained graphically.

The most advantageous plane of loading for any section will be that having the greatest value of s.

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DIAGONAL STRESSES.

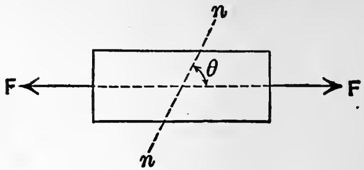

Fig. 45.

F = axial load.

A = area of section normal to axis of bar.

n–n = any diagonal section.

θ = angle which n–n makes with axis.

S = unit axial stress.

S_s = unit shear along plane normal to axis.

S_n = unit tension or compression normal to section n–n.

S_{sn} = unit shear along section n–n.

For combined direct stress and vertical shear,

$$S_n = \frac{S}{2}(1 - \cos 2\theta) + S_s \cdot \sin 2\theta.$$

$$S_{sn} = \frac{S}{2} \cdot \sin 2\theta + S_s \cdot \cos 2\theta.$$

The maximum or minimum value of S_n occurs when $\cot 2\theta = -\frac{S}{2S_s}$, and is

$$\text{max. } S_n = \frac{1}{2}S \pm \left(S_s^2 + \frac{S^2}{4}\right)^{\frac{1}{2}}.$$

The maximum value of S_{sn} occurs when $\tan 2\theta = +\frac{S}{2S_s}$, and is

$$\text{max. } S_{sn} = \left(S_s^2 + \frac{S^2}{4}\right)^{\frac{1}{2}}.$$

For axial load only, $S_s = 0$, hence

$$S_n = \frac{S}{2}(1 - \cos 2\theta) = S \cdot \sin^2\theta = \frac{F}{A} \cdot \sin^2\theta.$$

$$S_{sn} = \frac{S}{2} \cdot \sin 2\theta = \frac{F}{2A} \sin 2\theta.$$

The maximum value of S_n occurs when $\theta = 90°$, and is then the unit axial stress.

The maximum value of S_{sn} occurs when $\theta = 45°$, and is $\frac{S}{2}$ or $\frac{F}{2A}$.

THIN PIPES, CYLINDERS, AND SPHERES.

S = unit stress in metal.

t = thickness of metal.

d = diameter.

p = unit pressure of liquid or gas.

θ = angle which the direction of p makes with X–X.

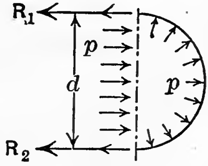

Fig. 46.

For the transverse stress across a longitudinal section of a pipe or cylinder,

$$R_1 = R_2 = \frac{1}{2}\Sigma p \cdot \cos\theta = \frac{1}{2} p \cdot d.$$

$$S = \frac{R_1}{t} = \frac{p \cdot d}{2t}.$$

For the longitudinal stress across a transverse section of a pipe, or for the stress in a thin hollow sphere,

$$S = \frac{p \cdot \frac{1}{4}\pi d^2}{\pi d \cdot t} = \frac{p \cdot d}{4t},$$

which is one-half of the unit transverse stress in a pipe having the same diameter and thickness.

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RIVETED JOINTS.

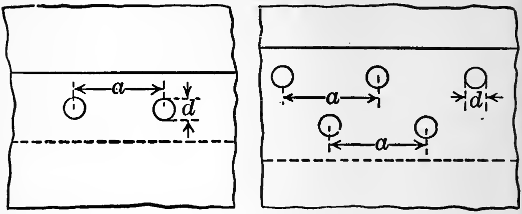

Fig. 47.

a = distance center to center of two consecutive rivets in one row.

d = diameter of rivet or rivet hole.

F = stress in unriveted plate in length a.

t = thickness of plate.

S_t = unit tensile stress.

S_c = unit compressive or bearing stress.

S_s = unit shearing stress.

e_t = efficiency of joint for tension.

e_c = efficiency of joint for compression.

e_s = efficiency of joint for shear.

m = number of shearing sections of rivets in distance a. (Notice that for butt joints each rivet has two shearing areas.)

n = number of bearing areas of rivets in distance a.

$$F = t\,(a-d)\,S_t = m \cdot \frac{1}{4}\pi d^2 \cdot S_s = n \cdot t \cdot d \cdot S_c.$$

$$e_t = \frac{a-d}{a}.$$

$$e_s = \frac{m \cdot \pi \cdot d^2 S_s}{4 \cdot atS_t}.$$

$$e_c = \frac{n \cdot dS_c}{aS_t}.$$

For maximum efficiency, make $e_t = e_s = e_c$, for which

$$d = \frac{4 \cdot n \cdot S_c \cdot t}{m \cdot \pi \cdot S_s},$$

and $$a = \frac{4\,nS_c t}{m\pi S_s}\left(1 + n\,\frac{S_c}{S_t}\right).$$

The allowable value of S_c is usually 2 S_s.

For single riveted lap joints the maximum efficiency is approximately 55 per cent, for double riveted lap joints approximately 70 per cent, for triple riveted lap joints approximately 75 per cent, and for triple and double riveted butt joints approximately 80 per cent.

BEAMS.

Vertical Shear. The vertical shear at any section of a horizontal beam is equal to the sum of the vertical components of the reactions to the left of the section minus the sum of the vertical components of the loads to the left of the section.

For *any* beam the vertical shear upon the right side of the left support of any span is

$$V_1 = \frac{M_2 - M_1}{l} + \frac{1}{2}\,wl + \Sigma F\left(1 - \frac{a}{l}\right),$$

where

M_1 = the moment at the left support,

M_2 = the moment at the right support,

w = the uniform load per lineal unit,

F = any concentrated load,

a = the distance from the left support to F,

l = the length of span.

Shearing Stresses. If V = vertical shear at any section,

$$S_s = \frac{V}{A},$$

where S_s is the average unit shear.

The actual unit vertical shear at any point is equal to the unit horizontal shear at that point, and may be determined by the following equation:

$$S_s = \frac{V}{I \cdot b} \cdot \sum_y^c (y \cdot dA),*$$

where b is the breadth of the section at the given point, y is the distance of the point considered from the neutral axis, and c is the distance from the neutral axis to the extreme fiber on the same side as the point considered.

The maximum value of S_s occurs at the neutral axis, and is

$$\text{max. } S_s = \frac{V}{I \cdot b} \int_0^c y \cdot dA = \frac{V}{I \cdot b} \cdot A_1 y_1,$$

where A_1 is the area of the portion of the section on one side of the neutral axis, and y_1 is the distance from the neutral axis to the center of gravity of the portion of the section on one side of the neutral axis.

For a rectangular section, the maximum unit shear is $\frac{3}{2}$ of the mean unit shear.

For *Diagonal Shear*, see Diagonal Stresses, page 62.

Bending Moment. The bending moment at any point for any beam is

$$M = M_1 + V_1 x - \tfrac{1}{2} wx^2 - \Sigma F(x-a),$$

---

* See "Merriman's Mechanics of Materials," page 269.

where

M_1 = bending moment at the left support,
V_1 = vertical shear upon the right side of the left support,
F = any concentrated load upon the left of the section considered,
x = distance from the left support to the section considered.

For any beam of one span V_1 is equal to the vertical component of the left reaction.

The maximum positive moments occur at those sections for which $\frac{dM}{dx}$ becomes equal to or passes through zero, that is where the shear becomes or passes through zero. The negative moments at the supports may have the largest numerical values, and for these points $\frac{dM}{dx}$ does not equal zero, since the tangents to the moment curve are not horizontal at these points.

Theorem of Three Moments. For any two consecutive spans of a continuous beam, let

M_1 = moment at the left support,
M_2 = moment at the middle support,
M_3 = moment at the right support,
l_1 = length of the first span,
l_2 = length of the second span,
l = length of span for equal spans,
w_1 = uniform load per lineal unit on first span,
w_2 = uniform load per lineal unit on second span,
F_1 = any concentrated load on the first span,
F_2 = any concentrated load on the second span,
a_1 = distance from first support to F_1,
a_2 = distance from middle support to F_2.

Then, for *uniform loads only,*

$$M_1l_1+2\,M_2\,(l_1+l_2)+M_3l_2=-\frac{1}{4}\,w_1l_1^3-\frac{1}{4}\,w_2l_2^3.$$

For *equal spans* with *equal uniform loads,*

$$M_1+4\,M_2+M_3=-\frac{1}{2}\,wl^2.$$

For *concentrated loads only,*

$$M_1l_1+2\,M_2\,(l_1+l_2)+M_3l_2$$
$$=-F_1\left(a_1l_1-\frac{a_1^3}{l_1}\right)-F_2\left(2\,a_2l_2-3\,a_2^2+\frac{a_2^3}{l_2}\right).$$

Flexural Stresses. The *tensile* and *compressive stresses* in a beam, produced by bending, can be determined by placing $\frac{N}{A}=0$ in the formula for S given under Eccentric Loads, which gives

$$S=\frac{M}{s}.$$

For *combined flexure and direct stress,* the tensile and compressive stresses can be computed for prisms by the formulæ given under *Eccentric Loads,* and for long members by the formulæ given for *Eccentrically Loaded Columns.*

Elastic Curves. The curve which is assumed by the neutral surface of a beam under load is called the elastic curve.

The *radius of curvature* of the elastic curve is

$$R=\frac{EI}{M}=\frac{dl^3}{dx\cdot d^2y}=\frac{dx^2}{d^2y},$$

from which the equation of the elastic curve can be obtained, for any particular case, by placing M equal to $EI\frac{d^2y}{dx^2}$, and by making two integrations to obtain an equation in terms of x and y.

The *deflection* of a beam at any given point is obtained by substituting the particular value of x in the equation of the elastic curve and solving for y. The maximum deflection occurs at the section for which

$$\frac{dy}{dx}=0.$$

(For particular cases, see Table of Beams.)

TABLE OF BEAMS.

NOTE. — The equations for elastic curves and the values of Δ apply only to beams of uniform section.

Beams Supported at Both Ends and Uniformly Loaded.

$$R_1=R_2$$

$$=\frac{1}{2}wl=\frac{W}{2}.$$

$$V=R_1-wx.$$

$$M=R_1x-\frac{1}{2}wx^2$$

$$=\frac{1}{2}wlx-\frac{1}{2}wx^2$$

$$=\frac{1}{2}Wx-\frac{1}{2}wx^2.$$

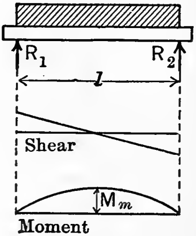

Fig. 48.

$$M_m=\frac{1}{8}wl^2=\frac{1}{8}Wl.$$

$$EI\frac{d^2y}{dx^2}=\frac{1}{2}wlx-\frac{1}{2}wx^2.$$

$$24\,EIy=w\,(-x^4+2\,lx^3-l^3x).$$

$$y=\Delta \text{ when } x=\frac{l}{2}.$$

$$\Delta=\frac{5}{384}\,\frac{wl^4}{EI}=\frac{5}{384}\,\frac{Wl^3}{EI}.$$

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Beam Supported at Both Ends and Loaded with a Concentrated Load at Center of Span.

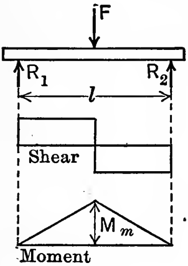

Fig. 49.

$$R_1 = R_2 = \frac{1}{2}F.$$

$$V = R_1, \text{ or } V = R_2.$$

$M = R_1 x$, on the left of F,

$= R_1 x - F\left(x - \frac{l}{2}\right)$, on the right of F.

$$M_m = \frac{1}{4}Fl.$$

$EI\dfrac{d^2y}{dx^2} = \dfrac{1}{2}Fx$, on the left of F.

$48\,EIy = F\,(4\,x^3 - 3\,l^2x)$, on the left of F.

$$\Delta = \frac{1}{48}\,\frac{Fl^3}{EI}.$$

(For both uniform and concentrated loads, combine the results for each.)

Beam Supported at Both Ends and Loaded with a Concentrated Load Distant *a* from the Left Support.

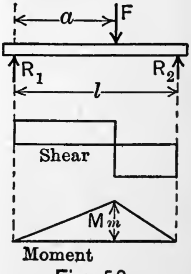

Fig. 50.

$$R_1 = F\left(\frac{l-a}{l}\right).$$

$$R_2 = F - R_1 = F\left(\frac{a}{l}\right).$$

$V = R_1$, on the left of F,

$= R_2$, on the right of F.

$M = R_1 x$, on the left of F,

$= R_1 x - F\,(x - a)$, on the right of F.

$$M_m = Fa\left(1 - \frac{a}{l}\right).$$

$EI\dfrac{d^2y}{dx^2} = R_1 x$, on the left of F,

$= R_1 x - F\,(x - a)$, on the right of F.

For the curve on the left of F,

$$6\,EI_y = F\left(1-\frac{a}{l}\right)x^3 - F\left(2\,al - 3\,a^2 + \frac{a^3}{l}\right)x.$$

The maximum deflection (Δ) occurs at the section for which

$$x = \sqrt{\frac{2\,al - a^2}{3}},$$

and is $$\Delta = \frac{F}{3\,EI}\left(\frac{2\,al - a^2}{3}\right)^{\frac{3}{2}}\left(1-\frac{a}{l}\right).$$

Beam Supported at Both Ends and Loaded with Several Concentrated Loads.

$$R_1 = \frac{\Sigma\, F\,(l-a)}{l}.$$

$$R_2 = \frac{\Sigma\, Fa}{l} = \Sigma\, F - R_1.$$

$$V = R_1 - \sum\nolimits_0^x F.$$

$$M = R_1 x - \sum\nolimits_0^x F(x-a).$$

The maximum moment (M_m) occurs at the section for which $R_1 - \sum_0^x F$ equals or passes through zero.

For a system of movable loads the maximum moment will occur under one of the loads, the loads being in such a position that the center of the span is midway between the center of gravity of all the loads and the section at which the maximum moment occurs.

The maximum deflection of a beam loaded with several loads is the sum of the deflections produced by each load at the section

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at which the maximum deflection for the entire system of loads occurs. The deflections produced by each load can be obtained

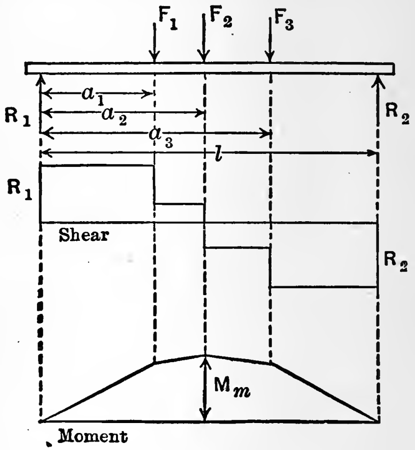

Fig. 51.

by means of the equation of the elastic curve for a single load.

Cantilever Beam with Uniform Load.

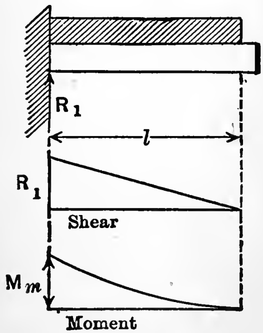

Fig. 52.

$$R_1 = wl = W.$$

$$R_2 = 0.$$

$$V = R_1 - wx.$$

$$M = \frac{1}{2} w \, (l - x)^2,$$

or if x is taken from the free end,

$$M = \frac{1}{2} wx^2.$$

$$M_m = \frac{1}{2} wl^2 = \frac{1}{2} Wl.$$

$$EI\frac{d^2y}{dx^2}=\frac{1}{2}wl^2-wlx+\frac{1}{2}wx^2.$$

$$24\,EIy=wx^4-4\,wlx^3+6\,wl^2x^2.$$

$$\Delta=\frac{1}{8}\frac{wl^4}{EI}=\frac{1}{8}\frac{Wl^3}{EI}.$$

Cantilever Beam with Concentrated Load at the Free End.

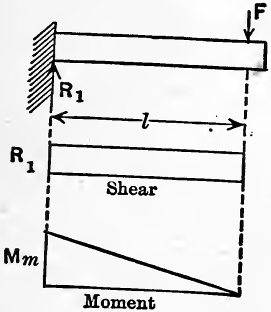

Fig. 53.

$$R_1=F.$$

$$R_2=0.$$

$$V=R_1.$$

$$M=F\ (l-x).$$

$$M_m=Fl.$$

$$EI\frac{d^2y}{dx^2}=F\ (l-x).$$

$$6\,EIy=3\,Flx^2-Fx^3.$$

$$\Delta=\frac{1}{3}\frac{Fl^3}{EI}.$$

Beam Fixed at Both Ends and Uniformly Loaded.

$$R_1=R_2=\frac{1}{2}wl=\frac{1}{2}W.$$

$$V=R_1-wx.$$

$$M = -\frac{1}{12}wl^2 + \frac{1}{2}wlx - \frac{1}{2}wx^2.$$

$$M_c = \frac{1}{24}wl^2 = \frac{1}{24}Wl.$$

$$EI\frac{d^2y}{dx^2} = M_1 + \frac{1}{2}wlx - \frac{1}{2}wx^2.$$

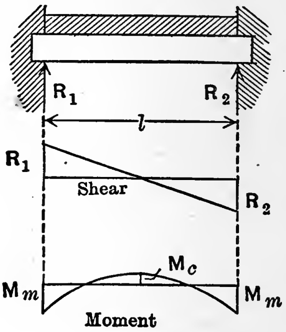

Fig. 54.

By placing $\frac{dy}{dx} = 0$ when $x = 0$ and when $x = l$,

$$M_1 = -\frac{1}{12}wl^2 = -\frac{1}{12}Wl = M_m.$$

$$24\,EIy = w(-l^2x^2 + 2\,lx^3 - x^4).$$

$$\Delta = \frac{1}{384}\frac{wl^4}{EI} = \frac{1}{384}Wl^3.$$

Beam Fixed at Both Ends and Loaded at the Center of the Span with a Concentrated Load.

$$R_1 = R_2 = \frac{1}{2}F.$$

$$V = R_1, \text{ on the left of } F,$$

$$= R_2, \text{ on the right of } F.$$

$$M = -\frac{1}{8}Fl + \frac{1}{2}Fx, \text{ on the left of } F,$$

$$= -\frac{1}{8}Fl + \frac{1}{2}Fx - F\left(x - \frac{l}{2}\right),$$

on the right of F.

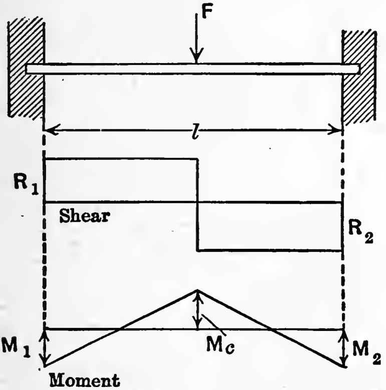

Fig. 55.

$$EI\frac{d^2y}{dx^2} = M_1 + \frac{1}{2}Fx, \text{ on the left of } F,$$

$$= M_1 + \frac{1}{2}Fx - F\left(x - \frac{l}{2}\right),$$

on the right of F.

By placing $\frac{dy}{dx} = 0$ when $x = 0$ and when $x = \frac{l}{2}$:

$M_1 = -\frac{1}{8}Fl$. $M_c = +\frac{1}{8}Fl$. $M_1 = M_c = M_m$.

$48\,EIy = 4\,Fx^3 - 3\,Flx^2$, on the left of F.

$$\Delta = \frac{1}{192}\frac{Fl^3}{EI}.$$

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Beam Fixed at Both Ends and Loaded with a Concentrated Load Distant *a* from the Left Support.

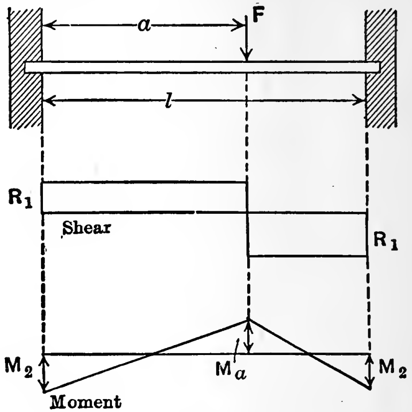

Fig. 56.

$$R_1 = F\left(1 - 3\frac{a^2}{l^2} + 2\frac{a^3}{l^3}\right).$$

$$R_2 = F\frac{a^2}{l^2}\left(3 - 2\frac{a}{l}\right).$$

$V = R_1$, on the left of F,

$= R_2$, on the right of F.

$M = M_1 + R_1x$, on the left of F,

$= M_1 + R_1x - F(x-a)$, on the right of F

$$M_1 = -Fa\left(1 - 2\frac{a}{l} + \frac{a^2}{l^2}\right).$$

$$M_2 = -\frac{Fa^2}{l}\left(1 - \frac{a}{l}\right).$$

$$M_a = +F\frac{a^2}{l}\left(2 - 4\frac{a}{l} + 2\frac{a^2}{l^2}\right).$$

$EI\dfrac{d^2y}{dx^2} = M_1 + R_1x$, on the left of F.

$6\,EIy = 3\,M_1x^2 + R_1x^3$, on the left of F.

The maximum deflection (Δ) occurs at the section for which $x = \frac{2\,al}{l+2\,a}$.

$$\Delta = \frac{2\,M_1 a^2 l^2}{EI\,(l+2\,a)^2} + \frac{4\,R_1 a^3 l^3}{3\,EI\,(l+2\,a)^3}.$$

Continuous Beam with Uniform Loads.

w_1 = load per lineal unit on l_1.

w_2 = load per lineal unit on l_2, etc.

W_1 = total load on l_1.

W_2 = total load on l_2, etc.

$R_1 = V_1$.

$R_2 = V_{2a} + V_{2b}$.

$R_3 = V_{3a} + V_{3b}$.

$R_4 = V_{4a} + V_{4b}$.

$R_n = V_n$.

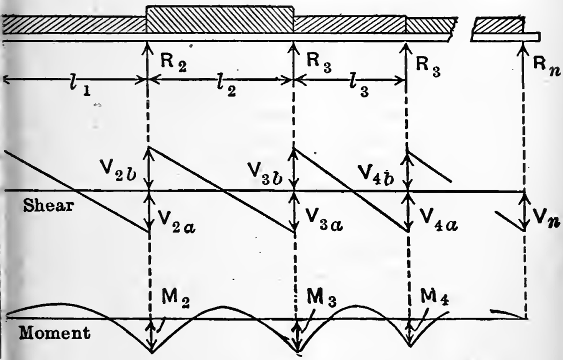

Fig. 57.

For a *continuous beam supported but not fixed at the ends*, use the theorem of three moments, writing the equation for the first and second spans, for the second and third spans, and so on, to the end. Solve the simultaneous equa-

tions, thus obtained for the moments at the supports. Then

$$V_1 = \frac{M_2}{l_1} + \frac{1}{2} w_1 l_1.$$

$$V_{2a} = W_1 - V_1.$$

$$V_{2b} = \frac{M_3 - M_2}{l_2} + \frac{1}{2} w_2 l_2.$$

$$V_{3a} = W_2 - V_{2b}, \text{ etc.}$$

For equal spans with equal uniform load over the entire beam, the ends of the beam resting upon supports, the moment at any support is Kwl^2 or KWl, and the vertical shear is Nwl or NW, where K and N have the values given in the following table. For many practical calculations the moment at a support one span from the end is assumed to be $-\frac{1}{10}Wl$, and for intermediate supports $-\frac{1}{12}Wl$.

For a *continuous beam with fixed ends* consider an imaginary span to be added at each end of the beam, with the free ends resting upon supports. Then write the equation of three moments for each two consecutive spans, making $l = 0$ for the first and last spans, and compute the moments at the supports as shown above.

Continuous Beam with Concentrated Loads.

Determine the moments at the supports in a similar manner to that employed for continuous beam with uniform load, employing the equation of three moments for concentrated loads.

COEFFICIENTS FOR UNIFORMLY LOADED CONTINUOUS BEAMS.*

No. of Spans	Values of K for Moment						Values of N for Shear											
	M_1	M_2	M_3	M_4	M_5	M_6	V_{1a}	V_{1b}	V_{2a}	V_{2b}	V_{3a}	V_{3b}	V_{4a}	V_{4b}	V_{5a}	V_{5b}	V_{6a}	V_{6b}
1	0	0	...	...	...	...	0	$\frac{1}{2}$	$\frac{1}{2}$	0	...	...	...	...	...	...	...	...
2	0	$\frac{1}{8}$	0	...	...	...	0	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	0	...	...	...	...	...	...
3	0	$\frac{1}{10}$	$\frac{1}{10}$	0	...	...	0	$\frac{4}{10}$	$\frac{6}{10}$	$\frac{5}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{4}{10}$	0	...	...	...	...
4	0	$\frac{3}{28}$	$\frac{2}{28}$	$\frac{3}{28}$	0	...	0	$\frac{11}{28}$	$\frac{17}{28}$	$\frac{15}{28}$	$\frac{13}{28}$	$\frac{13}{28}$	$\frac{15}{28}$	$\frac{17}{28}$	$\frac{11}{28}$	0	...	...
5	0	$\frac{4}{38}$	$\frac{3}{38}$	$\frac{3}{38}$	$\frac{4}{38}$	0	0	$\frac{15}{38}$	$\frac{23}{38}$	$\frac{20}{38}$	$\frac{18}{38}$	$\frac{19}{38}$	$\frac{19}{38}$	$\frac{18}{38}$	$\frac{20}{38}$	$\frac{23}{38}$	$\frac{15}{38}$	0

* Taken from Merriman's "Mechanics of Materials."

STRUTS AND COLUMNS.

Euler's Formula.

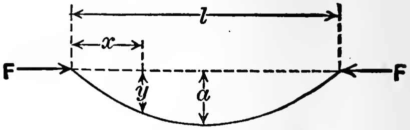

Fig. 58.

$$EI\frac{d^2y}{dx^2} = -Fy.$$

$$dx = \left(\sqrt{\frac{EI}{F}}\right)\left(\frac{dy}{\sqrt{a^2-y^2}}\right).$$

$$x = \sqrt{\frac{EI}{F}}\cdot\sin^{-1}\left(\frac{y}{a}\right), \text{ or}$$

$$y = a\cdot\sin\left(x\sqrt{\frac{F}{EI}}\right).$$

Since $y=a$ when $x=\frac{l}{2}$, $\frac{l}{2}\sqrt{\frac{F}{EI}}$ must equal $\frac{\pi}{2}$, or

$$F = EI\frac{\pi^2}{l^2}.$$

$$\frac{F}{A} = \pi^2E\left(\frac{r}{l}\right)^2, \textit{ for round ends.}$$

For *one end round and the other end fixed*, replace l by $\frac{4}{3}l$ and π by 2π, which gives

$$F = \frac{9}{4}EI\frac{\pi^2}{l^2}.$$

$$\frac{F}{A} = \frac{9}{4}\pi^2E\left(\frac{r}{l}\right)^2.$$

For *both ends fixed*, replace l by $\frac{3}{2}l$ and π by 3π, in the formula for round ends, which gives

$$F = 4EI\frac{\pi^2}{l^2}.$$

$$\frac{F}{A} = 4\pi^2E\left(\frac{r}{l}\right)^2.$$

Rankine's Formula. (Sometimes called Gordon's Formula.)

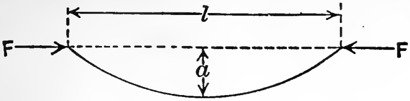

Fig. 59.

From the formula for eccentric loads for a symmetrical section (page 57), the maximum stress will be

$$S=\frac{F}{A}+\frac{My}{I},$$

where y is the distance from the neutral axis to the extreme fiber.

But, $I=Ar^2$, $M=Fa$ and $a=K\frac{l^2}{y}$, where K is some constant depending upon character and condition of the column. Hence

$$S=\frac{F}{A}\left[1+K\left(\frac{l}{r}\right)^2\right], \text{ or}$$

$$\frac{F}{A}=\frac{S}{1+K\left(\frac{l}{r}\right)^2}.$$

The following values of K are recommended in the Civil Engineers' Pocket Book:*

Material.	Both Ends Fixed.	One End Fixed, One End Round.	Both Ends Round.
Timber	1/3000	1.95/3000	3/3000
Cast Iron . . .	1/5000	1.95/5000	4/5000
Wrought Iron .	1/36000	1.95/36000	4/36000
Steel	1/25000	1.95/25000	4/25000

Ritter's Formula. Ritter's formula is the same as Rankine's formula except that the

* American Civil Engineers' Pocket Book, p. 307.

expression $\frac{S_e}{nE}$ is used for K, in which S_e is the elastic limit of the material, and n is equal to π^2 for round ends, $\frac{9}{4}\,\pi^2$ for one end round and one end fixed, and $4\,\pi^2$ for both ends fixed.

The Straight Line Formula. The straight line formula is

$$\frac{F}{A}=S-C\frac{l}{r},$$

where C is a constant depending upon the character and condition of the column.

Merriman gives the value of C in the above equation to be

$$C=\frac{2}{3}S\left(\frac{S}{3\,nE}\right)^{\frac{1}{2}},$$

which is obtained by making the straight line a tangent to the curve for Euler's formula passing through the point S for $\frac{l}{r}=0$, the values of n being those given for Ritter's formula.

Values of constants for the straight line formula, as determined by T. H. Johnson, *for rupture*, are given in the Civil Engineers' Pocket Book* as follows:

Kind of Column.	S.	C.	Limit l/r.
Wrought Iron:			
Flat Ends	42,000	128	218
Hinged Ends . .	42,000	157	178
Round Ends . .	42,000	203	138
Structural Steel:			
Flat Ends	52,500	179	195
Hinged Ends . .	52,500	220	159
Round Ends . .	52,500	284	123
Cast Iron:			
Flat Ends	80,000	438	122
Hinged Ends . .	80,000	537	99
Round Ends . .	80,000	693	77
Oak, Flat Ends . .	5,400	28	128

* American Civil Engineers' Pocket Book, p. 308.

Some of the values of constants commonly used for designing steel columns, by the straight line formula are as follows:

Member.	S.	C.	Limit l/r.	Authority.
R. R. Bridges:				
Chords, L. L. . .	10,000	45	100	Cooper
Chords, D. L. . .	20,000	90	100	Cooper
Posts (Thru), L. L.	8,500	45	100	Cooper
Posts (Thru), D. L.	17,000	90	100	Cooper
Posts (Deck), L. L.	9,000	40	100	Cooper
Posts (Deck), D. L.	18,000	80	100	Cooper
Laterals (Wind) .	13,000	60	120	Cooper
Any Member . .	16,000†	70	*100 / 120	Am. Ry. Eng. and M. of W. Assoc.
Highway Bridges:				
Struts	16,000	70	*125 / 150	Ketchum
Chords, L. L. . .	12,000	55	*125 / 150	Ketchum
Chords, D. L. . .	24,000	110	*125 / 150	Ketchum
Posts (Thru), L. L.	10,000	45	*125 / 150	Ketchum
Posts (Thru), D. L.	20,000	90	*125 / 150	Ketchum
Posts (Deck), L. L.	11,000	40	*125 / 150	Ketchum
Posts (Deck), D. L.	22,000	80	*125 / 150	Ketchum
Laterals, Wind . .	13,000	60	*125 / 150	Ketchum
Girder Stiffeners.	12,000	55		Ketchum
Buildings:				
Columns	16,000	70	*125 / 150	Ketchum
Columns	16,000	70	*120 / 150	Chicago

For cast iron columns, for which $\frac{l}{r}$ does not exceed 70, the Chicago ordinance allows

$$10{,}000-60\,\frac{l}{r}.$$

For timber columns, the formula is changed to

$$\frac{F}{A}=S\left(1-C\,\frac{l}{d}\right),$$

* Main members and laterals, respectively.

† Impact of live loads to be taken into account by adding $I=S\,\frac{300}{L+300}$, in which S = actual live load, and L = length of bridge loaded.

in which S is the allowable compressive stress along the grain, and d is the diameter. The Chicago ordinance (Mr. Benj. E. Winslow's formula) uses $\frac{1}{80}$ for C, for values of $\frac{l}{d}$ not greater than 30. Ketchum's Specifications for Steel Frame Buildings gives $C = \frac{1}{100}$.

Eccentrically Loaded Columns. By adding the bending stress $\frac{My}{I}$ to Rankine's formula, replacing M by Fe, and I by Ar^2, the formula becomes

$$S = \frac{F}{A}\left[1 + K\frac{l^2}{r^2} + \frac{ey}{r^2}\right],$$

in which the constants are those given for Rankine's formula, e is the eccentricity, and y is the distance from the neutral axis to the extreme fiber.

A more general formula for combining direct and bending stresses is

$$S = \frac{F}{A} \pm \frac{My}{I \mp \frac{\alpha F l^2}{\beta E}},^*$$

in which M is the apparent bending moment, y is the distance to the extreme fiber, I is the moment of inertia, E is the modulus of elasticity, and α and β are constants, β/α being 9.6 for a simple beam uniformly loaded and 12 for a simple beam with a load at the center.

The following formula for steel struts, given in Ketchum's Specifications for Steel Frame Buildings, is a special case of the last formula.

$$S = \frac{F}{A} + \frac{My}{I - \frac{Fl^2}{10\,E}}.$$

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* See "Apparent Combined Stresses," Merriman's "Mechanics of Materials."

TORSION.

Circular Sections. .

Twisting moment, $M = Fa.$

Circular Secfions

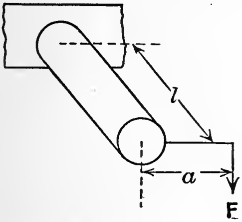

Fig. 60.

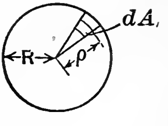

Fig. 61.

Resisting moment, $M_r = \int \frac{\rho^2}{R} S\,dA$, where S is the shearing stress at the extreme fiber.

$$M = M_r, \text{ or}$$

$$M = \frac{SI_0}{R},$$

where I_0 is the polar moment of inertia.

For a *solid round shaft* $\frac{I_0}{R} = \frac{1}{2}\pi R^3$, hence

$$M = \frac{1}{2}\pi R^3 S, \quad \text{or} \quad S = \frac{2\,M}{\pi R^3}.$$

Non-Circular Sections. (Taken from Merriman's "Mechanics of Materials.") For non-circular sections the above formulæ are only approximate.

For an *elliptical section* whose major axis is m and whose minor axis is n the maximum stress is

$$S = \frac{16\,Fa}{\pi mn^2}, \quad \text{or}$$

$$M = \frac{\pi mn^2 S}{16}.$$

For a *rectangular section* whose long side is m and whose short side is n, the maximum stress is

$$S = \frac{9}{2}\frac{Fa}{mn^2}, \quad \text{or}$$

$$M = \frac{2}{9} mn^2 S.$$

Transmission of Power. The horse-power which is transmitted by a shaft is

$$\text{H.P.} = \frac{2\,\pi a \cdot F \cdot \omega}{550 \times 12},$$

where a = moment arm in inches,

ω = number of revolutions per sec.

But $Fa = \frac{SI_0}{R}$, hence

$$\text{H.P.} = \frac{2\,\pi\omega SI_0}{550 \times 12\,R} = 0.000952\,\frac{\omega SI_0}{R}.$$

ELLIPSOID OF STRESS.

For any point within a stressed body, the resultant unit stress upon any plane is proportional to the radius vector of an ellipsoid. The principal axes of the ellipsoid coincide with the principal stresses, which stresses are normal to the planes upon which they act. For a plane not normal to a principal axis the resultant stress is not normal to the plane.

REINFORCED CONCRETE.

Notation.

Let A = area, b = width of beam, b' = width of stem, d = depth to center of steel, E = modulus of elasticity, f = unit stress, M = moment, $n = E_s \div E_c$, P = total load, p = ratio of area of longitudinal steel to area of section of member,

q = ratio of volume of circumferential steel to volume of column, s = spacing, subscript (c) refers to concrete, subscript (s) refers to steel, t = thickness of flange, u = unit bond stress, V = total shear, v = unit shearing stress, Σ_0 = sum of perimeters of bars, and *other values* are as indicated in the figures or as specifically stated.

Columns.

For *columns with longitudinal steel only.*

$$P = f_c A\,[1 + (n-1)\,p].$$
$$f_s = n f_c.$$

For *columns with spiral and longitudinal steel,* the proper form of equation is not well established. The ultimate unit load may be expected to be

$$\frac{P}{A} = f_c\,(1-p) + f_s p + K f_s' q,$$

in which f_s is the yield point of longitudinal steel, f_s' is the yield point of the circumferential steel, and K is a factor the value of which will usually be between 1.0 and 1.5

Beams.

For *beams*, in general,

$$f_s = M \div (A_s jd).$$
$$f_c = \frac{f_s k}{n\,(1-k)}.$$
$$jd = d - z.$$
$$v = V \div (b' jd).$$
$$u = V \div (j\,d\Sigma_0).$$

Per vertical stirrup,

$$P = Vs/jd.$$

Per stirrup at 45°,

$$P = 0.7\,Vs/jd.$$

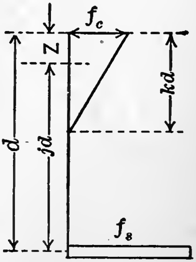

Fig. 62.

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For *rectangular beams,*

$$p = 1 \div \left[2\,\frac{f_s}{f_c}\left(\frac{f_s}{nf_c}+1\right)\right].$$

$$k = \sqrt{2\,pn + (pn)^2} - pn.$$

$$z = kd \div 3.$$

$$f_c = 2\,M/jkbd^2 = 2\,pf_s/k.$$

(For $f_s = 16{,}000$ and $f_c = 650$, $p = 0.0077$.)

$\left(j \text{ is approximately } \frac{7}{8}.\right)$

For *T-beams,*

$$kd = \frac{2\,n\,dA_s + bt^2}{2\,nA_s + 2\,bt}.$$

$$z = \frac{(3\,kd - 2\,t)\,t}{(2\,kd - t)^3}.$$

$\left(\text{For thin flanges } z \text{ is often assumed } \frac{t}{2}.\right)$

For *beams reinforced for compression,*

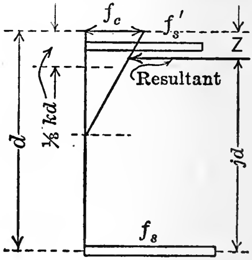

Fig. 63.

$$k = \left[2\,n\left(p + p'\,\frac{d'}{d}\right) + n^2\,(p+p')^2\right]^{\frac{1}{2}} - n\,(p+p').$$

$$z = \frac{\frac{1}{3}\,k^3 d + 2\,p'nd'\left(k - \frac{d'}{d}\right)}{k^2 + 2\,p'n\left(k - \frac{d'}{d}\right)}.$$

For $p' = 0.5\,p$, $d' \div d = 0.10$, and $n = 15$;
$p = 0.0135$ to make f_s 16,000 when f_c is 747, for which $M = 185\,bd^2$; $p = 0.010$ to make f_s 16,000 when f_c is 650, for which $M = 140\,bd^2$.

For $p' = 0.5\,p$, $d' \div d = 0.15$, and $n = 15$;
$p = 0.013$ to make f_s 16,000 when f_c is 747, for which $M = 175\ bd^2$; $p = 0.0094$ to make f_s 16,000 when f_c is 650, for which $M = 130\ bd^2$.

For $p' = p$, $d' \div d = 0.10$, and $n = 15$;
$p = 0.0195$ to make f_s 16,000 when f_c is 747, for which $M = 275\ bd^2$; $p = 0.014$ to make f_s 16,000 when f_c is 650, for which $M = 200\ bd^2$.

For $p' = p$, $d' \div d = 0.15$, and $n = 15$;
$p = 0.018$ to make f_s 16,000 when f_c is 747, for which $M = 250\ bd^2$; $p = 0.0125$ to make f_s 16,000 when f_c is 650, for which $M = 175\ bd^2$.

Flat Slab Floors.

For flat slab floors extending over several panels in each direction, the following requirements are in accordance with the recommendations * of the Joint Committee on Concrete and Reinforced Concrete.

Column Capitals. The minimum edge thickness should be $1\frac{1}{2}$ inches. The slope of the conical surface should not be more than 45 degrees with the vertical. The minimum diameter (or dimension parallel to edge of panel) should be not less than one-fifth of the panel distance (measured center to center of adjacent columns), and it is desirable to use 0.225 times the panel distance.

Dropped Panels. The minimum width should be four-tenths of the panel distance, and the maximum offset should be five-tenths

* For the Final Report of the Joint Committtee on Concrete and Reinforced Concrete see Proceedings of the American Society for Testing Materials, vol. XVII, 1917, pp. 202–262.

of the thickness of the slab outside of the dropped panel.

Slab Thickness. For, $t =$ the total thickness of slab in inches, L = panel distance in feet, and w = the total dead and live load in pounds per square foot; minimum values of t should be $0.024\,L\sqrt{w} + 1\frac{1}{2}$ for slabs without dropped panels, $0.020\,L\sqrt{w} + 1$ for slabs with dropped panels, and $0.03\,L\sqrt{w} + 1\frac{1}{2}$ for the dropped panels themselves. Also, t should not be less than six inches, nor less than one-thirty-second of the panel distance for floors, nor less than one-fortieth of the panel distance for roofs.

Bending Moments in Girdless Slabs. For c = diameter of capital in feet, and panel distances in feet and in accordance with Fig. 64, and for other values as already given, interior panels may be designed upon the assumption that the sum of the positive bending moments for one inner and two outer sections on one line of length L_1 is $\frac{1}{25}\,wL_1(L_2 - \frac{2}{3}\,c)^2$ foot-pounds, of which at least 25 per cent should be resisted by the inner section, while the two outer sections should resist at least 55 per cent of the positive moment in slabs without dropped panels, and at least 60 per cent in slabs with dropped panels. Also, for the slab thickness away from the dropped panels, at least 70 per cent of the positive moment should be resisted by the two outer sections.

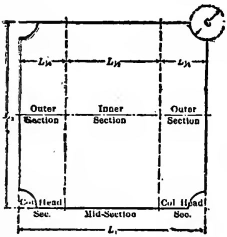

Fig. 64.

For interior panels, assume the sum of the negative moments to be resisted by one mid-

section and two column-head sections along one line of length L_1 to be $\frac{1}{15} wL_1 (L_2 - \frac{2}{3} c)^2$ foot-pounds, of which at least 20 per cent should be resisted by the mid-section; while the two column-head sections $\left(\text{of length } \frac{L_1}{4}, \text{ each}\right)$ should resist at least 65 per cent of the total moment for slabs without dropped panels, and at least 80 per cent for slabs with dropped panels.

Wall Panels. At the first row of columns away from the wall and also at the sections halfway from this row of columns to the wall, increase the moments by 20 per cent of the values as determined for interior panels. If wall girders or cantilever restraint does not exist at the wall, increase the moments of the outer section and the column head section by 20 per cent of the values as determined for interior panels, for designing reinforcement parallel to the wall.

Shear and Diagonal Tension. As a measure of diagonal tension assume $v = \frac{wL}{24\,jd}$ for slabs without dropped panels, and $v = \frac{wL}{20\,jd}$ for slabs with dropped panels. For punching shear at peripheries of capitals and dropped panels, assume a total shear 25 per cent greater than the actual punching shear, computed on the basis of a load which is uniformly distributed.

Bending Moments in Columns should be given special consideration.

HYDRAULICS.

NOTATION.

A = area in sq. ft., a = area in sq. in., D = diameter in ft., E = energy, F = force, f = friction factor, $g = 32.2$, h = head, h_f = friction head, I = moment of inertia, L = length in ft., M = statical moment, P = total pressure, p = unit pressure, q = quantity in cu. ft. per sec., r = hydraulic radius, s = slope, V = theoretical velocity, v = actual velocity, w = density of water.

STATIC PRESSURE.

$p = wh$, in which for water at ordinary temperatures w is 62.4 lb. per cu. ft. The density of water for particular temperatures is shown in Fig. 65.

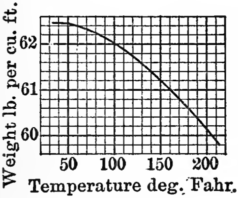

Fig. 65.

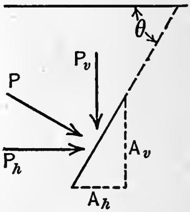

Fig. 66.

For h in feet,

$$p = 62.4\, h \text{ lb. per sq. ft.}$$
$$= 0.433\, h \text{ lb. per sq. in.}$$

or $$h = 2.306\, p, \text{ for } p \text{ in lb. per sq. in.}$$

$$P = pA$$
$$= \int wh\, dA, \text{ for any surface,}$$
$$= whA, \text{ for a horizontal surface,}$$

or $\qquad = \frac{1}{2} whA$, for a rectangular surface with one edge at the surface of the water h being measured to the lower edge.

If p is the average unit pressure,

$$P_v = P \cos\theta = pA_h.$$
$$P_h = P \sin\theta = pA_v.$$

CENTER OF PRESSURE.

$$y_p = I_0 \div M_0$$
$$= Ar_0^2 \div Ay_c$$
$$= y_c + \frac{r_{cg}^2}{y_c},$$

in which r is the radius of gyration.

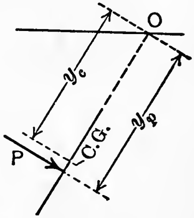

Fig. 67.

WEIRS.

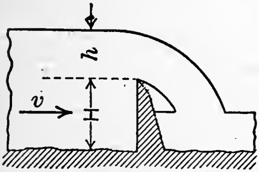

Fig. 68.

q = cu. ft. per sec.
h = observed head in ft.
v = velocity of approach.
Velocity head,

$$h_v = \frac{v^2}{2g}.$$

Rectangular Weirs.

Francis' formula is

$$q = 3.33\,[L - 0.1\,nh]\,[(h + h_v)^{\frac{3}{2}} - h_v^{\frac{3}{2}}],$$

in which n is the number of end contractions.

Fteley and Stearns' formula for suppressed weirs is

$$q = 3.31\,L\,(h + 1.5\,h_v)^{\frac{3}{2}} + 0.007\,L.$$

Bazin's formula for suppressed weirs is $q =$

$$\left(0.405 + \frac{0.00984}{h}\right)\left[1 + 0.55\left(\frac{h}{H+h}\right)^2\right] L\sqrt{2g}\cdot h^{\frac{3}{2}}.$$

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A *general equation* for discharge is

$$q = c\,\tfrac{2}{3}\,\sqrt{2g}\cdot L\,(h + nh_v)^{\frac{3}{2}},$$

for which Hamilton Smith's values of n are **1.4** for contracted weirs and $1\tfrac{1}{3}$ for suppressed weirs.

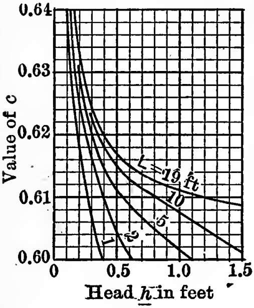

Fig. 69.
Contracted Weirs.

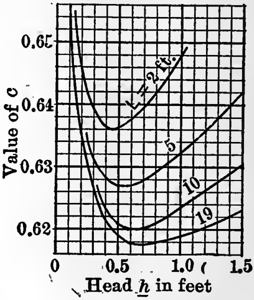

Fig. 70.
No End Contraction.

Smith's values of c for contracted weirs are plotted in Fig. 69 and for suppressed weirs in Fig. 70.

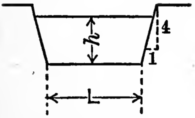

Fig. 71.
Cippoletti Weir.

Trapezoidal Weirs.

Cippoletti's formula is

$$q = 3.367\,Lh^{\frac{3}{2}}.$$

Triangular Weirs.

$$q = c\cdot\frac{8}{15}\tan\theta\,\sqrt{2g}\cdot h^{\frac{5}{2}}.$$

For $\theta = 45°$, and for an average value of c,

$$q = 2.6\,h^{\frac{5}{2}}.$$

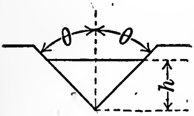

Fig. 72.
Triangular Weir.

Submerged Weirs.

The formula for submerged weirs given in the American Civil Engineers' Pocket Book* is

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* American Civil Engineers' Pocket Book, page 854.

$$q = cL\sqrt{2gh}\cdot\left[h - \frac{1}{3}(h - h')\right],$$

in which c is from 0.58 to 0.63 for a sharp crest.

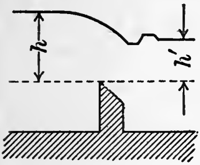

Fig: 73.
Submerged Weir.

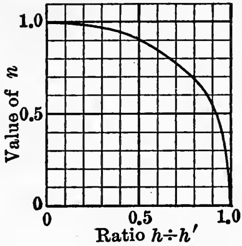

Fig. 74.

*Herschel's formula** is

$$q = 3.33\,L(nh)^{\frac{3}{2}},$$

in which n has values indicated in Fig. 74.

ORIFICES AND JETS.

Discharge.

$$V = \sqrt{2gh} \quad \text{or} \quad h = \frac{V^2}{2g}.$$

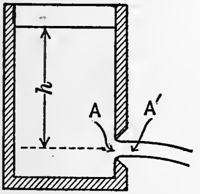

Fig. 75.
Standard Orifice.

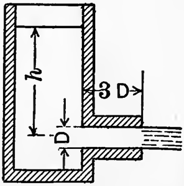

Fig. 76.
Standard Tube.

For a *standard orifice*,

$$v = \text{from } 0.97\,V \text{ to } 0.99\,V.$$
$$A' = \text{from } 0.57\,A \text{ to } 0.62\,A.$$
$$q = cA\sqrt{2gh},$$

in which an average value of c is 0.61.

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* Trans. Am. Soc. C. E., 1885, vol. XIV, p. 194.

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For a *standard tube*,

$$v = 0.82\,V = 0.82\sqrt{2\,gh},$$

$$q = 0.82\,A\sqrt{2\,gh}.$$

An *inward projecting tube* may reduce the discharge to $0.5\,A\sqrt{2\,gh}$, and a diverging or compound tube will increase the discharge.

Force and Energy.

The *energy of a jet* discharging W lb. of water is $W\dfrac{v^2}{2\,g}$.

The *force of a jet* discharging W lb. of water per second, and impinging at right angles to a fixed plate, is $W\dfrac{v}{g}$.

The *impulse* exerted by a jet is equal to the *reaction*. For a jet deflected by a fixed plate,

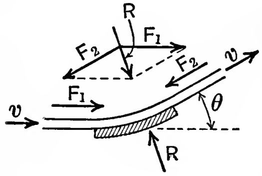


Fig. 77.

$$F_1 = F_2 = W\frac{v}{g},$$

$$R = 2\,F_1 \sin\frac{\theta}{2}$$

$$= 2\,W\frac{v}{g}\sin\frac{\theta}{2}.$$

The total component of force parallel to F_1 (Fig. 77) is

$$F_1 - F_2\cos\theta = W\frac{v}{g}(1 - \cos\theta).$$

For *moving plates* the force upon the plate is that which would be exerted upon a fixed plate by a jet having a velocity equal to the relative velocity of the given jet to the plate, and the work done can be computed from the forces (impulse and reaction) and the velocity of the plate. The velocity of the plate determines the distance through which the force acts.

Also, the energy given up by the jet may be computed by the formula $\frac{W}{2g}(v_1^2 - v_2^2)$, in which v_1 is the absolute velocity with which the jet strikes the plate and v_2 is the absolute velocity of the jet leaving the plate.

FLOW IN PIPES.

Long Pipes.

Fanning's formula is

$$h_f = 4f\frac{L}{D}\frac{v^2}{2g}, \quad \text{or} \quad v = \sqrt{\frac{2gDh_f}{4fL}},$$

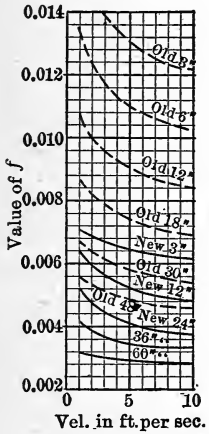


Fig. 78.

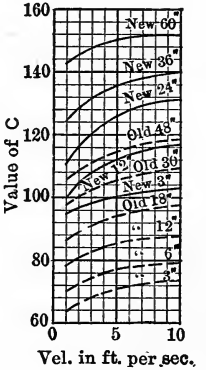


Fig. 79.

whence $$q = 3.15\left[\frac{h_f D^5}{fL}\right]^{\frac{1}{2}}$$

or $$D = 0.632\left[\frac{fLq^2}{h_f}\right]^{\frac{1}{5}}.$$

Values of f for cast iron pipe are indicated in Fig. 78.* (The formula for h_f is frequently

---

* Plotted for average values given in American Civil Engineers' Pocket Book, pp. 845, 846,

used in the form $h_f = f\frac{L}{D}\frac{v^2}{2g}$, in which case the value of f is four times the value here used.)

Chezy's formula is

$$v = C\sqrt{rs},$$

in which r is the hydraulic radius, which is equal to $D/4$ for a pipe, s is the slope of the hydraulic grade line or the friction head divided by the length, and C is a coefficient, for which values for cast iron pipes are indicated in Fig. 79.

The Chezy formula is used for flow in open channels as well as for flow in pipes.

Flamant's formulæ are

$$v = 86.38\, D^{\frac{5}{7}} s^{\frac{4}{7}} \text{ for new cast iron pipes,}$$

$$v = 76.28\, D^{\frac{5}{7}} s^{\frac{4}{7}} \text{ for old cast iron pipes,}$$

in which s is the same as for the Chezy formula.

Various Losses of Head.

The *loss at entrance* for a pipe is

$$h_f = \left(\frac{1}{c_v^2} - 1\right)\frac{v^2}{2g},$$

in which c_v is the coefficient of velocity.

For a square edge at the entrance, the loss may be taken as $0.5\frac{v^2}{2g}$, or for an inward projecting pipe it may be considered to be $\frac{v^2}{2g}$.

The *loss due to expansion* at a point of sudden enlargement is

$$h_f = \frac{(v_1 - v_2)^2}{2g},$$

in which v_1 is the velocity in the smaller section and v_2 is the velocity in the larger section.

Other losses of head occur at elbows, valves, and sudden contractions. These are ordinarily stated in the form $K\frac{v^2}{2g}$, in which K is a coeffi-

cient for the particular case. For practical problems the equivalent length of pipe may often be used.

Equivalent Pipe Length.

A convenient method of taking account of losses of head at entrance, elbows, curves, and fittings, and the head remaining as velocity head $\left(\frac{v^2}{2\,g}\right)$ is to add to the actual length of pipe a length in which the friction loss would be equivalent to the particular loss, using the total equivalent length of pipe in computing size or flow. The equivalent length of pipe required to produce any loss, $K\,\frac{v^2}{2\,g}$, is $\frac{K}{4\,f}$ times the diameter, in which f is the friction factor for Fanning's formula. For ordinary computations for iron pipe the following equivalent lengths may be used:

For loss at entrance, 25 diameters,
For loss at an elbow, 10 diameters,
For loss at end, $\left(\frac{v^2}{2\,g}\right)$, 50 diameters.

Bernoulli's Theorem.

Neglecting friction, $P/w+v^2/2\,g+z$ is a constant for all points along a given pipe, z being the elevation of the point above a given plane of reference.

FLOW IN CHANNELS.

Chezy's formula is

$$v=C\sqrt{rs},$$

in which s is the slope, r is the hydraulic radius, which is equal to the area of the cross-section of the water divided by the length of the wetted perimeter, C is a coefficient which depends upon the roughness of the channel, and v is the mean velocity.

Kutter's formula for the value of C for use in the Chezy formula is

$$C = \frac{\frac{1.811}{n} + 41.65 + \frac{0.00281}{s}}{1 + \frac{n}{\sqrt{r}}\left(41.65 + \frac{0.00281}{s}\right)},$$

in which r and s are the same as given for the Chezy formula, and n is the coefficient of roughness for which some of the values are as follows:

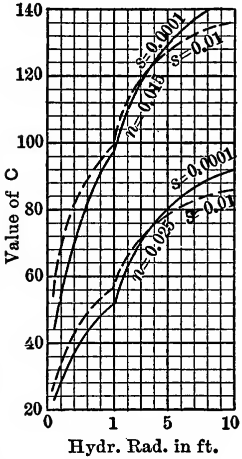


Fig. 80.
Kutter's Coefficient.

$n = 0.010$ for neat cement,

$n = 0.013$ for clean brick and sewers,

$n = 0.015$ for unclean sewers,

$n = 0.020$ for new canals,

$n = 0.025$ for ordinary canals,

$n = 0.035$ for canals in bad condition.

Values of C for $n = 0.015$ and $n = 0.025$ are indicated in Fig. 80, for $s = 0.01$ and for $s = 0.0001$.

Bazin's formula for the value of C for use in the Chezy formula is

$$C = \frac{157.6}{1 + \frac{1.811\, m}{\sqrt{r}}}$$

or

$$C = \frac{87}{0.552 + \frac{m}{\sqrt{r}}},$$

in which r is the hydraulic radius and m is a coefficient of roughness for which some of the values are:

$m=0.16$ for planks or bricks,
$m=1.30$ for ordinary canals,
$m=1.75$ for canals in bad condition.

The *ratio of the mean velocity in a channel to the maximum surface velocity* is subject to a considerable variation, its approximate value being 0.8.

The *ratio of the mean velocity for any vertical section to the velocity at the mid-depth* is approximately 0.98.

For any vertical section, the velocity at 0.6 of the depth from the surface will be approximately the mean velocity for the section.

For any vertical section the mean velocity is approximately 0.9 of the surface velocity.

HYDRAULIC GRADE LINE

The hydraulic grade line is the line connecting the points to which water would rise in a piezometer tube, if the tube were applied to consecutive points throughout the length of a pipe or conduit. The distance from the pipe to the hydraulic grade line at any point is the pressure head at the given point. The slope of the hydraulic grade line is the hydraulic gradient. The difference in elevation between any two points on the hydraulic grade line is the loss of head which exists between the two corresponding points of the conduit.

HEAT ENG.
ELEC. ENG.
TABLES

HEAT ENGINEERING.

COMPILED BY G. A. GOODENOUGH
Professor of Thermodynamics, University of Illinois

ELEMENTS OF THERMODYNAMICS.

Notation.

M = weight of substance, in lb.
p = absolute pressure, in lb. per sq. ft.
t = temperature, deg. F.
$T = t + 459.6$ = absolute temperature.
V, v = volume, in cu. ft.
U, u = internal energy, in B.t.u.
I, i = thermal head, in B.t.u.
S, s = entropy.
Q, q = heat absorbed in B.t.u.
$J = 777.6$ = mechanical equivalent of heat; *i.e.*, 777.6 ft. lb. = 1 B.t.u.
$A = \frac{1}{J}$ = reciprocal of mechanical equivalent.
W = external work done during a change of state.
c_v = specific heat at constant volume.
c_p = specific heat at constant pressure.

The small letters, v, u, i, s refer to 1 lb. of the substance, the capital letters V, U, I, S refer to M lb. Thus $V = Mv$, $S = Ms$, etc.

Fundamental Equations: Definitions.

The state of a substance initially given by p_1, v_1, T_1 changes to a second state given by p_2, v_2, T_2. The work done by the substance in expanding is $W_{12} = \int_{v_1}^{v_2} p\,dV$; and if Q_{12} denotes the heat absorbed during the process, the first law is expressed by the energy equation

$$Q_{12} = U_2 - U_1 + A \int_{V_1}^{V_2} p\,dV.$$

Or in differential form,

$$dQ = du + Ap\,dV.$$

The thermal head I is defined by the equation

$$I = U + ApV,$$

whence $$dQ = dI - AV\,dp.$$

For a change of state at constant pressure, $dQ = dI$, or

$$Q_{12} = I_2 - I_1.$$

Similarly, for a change of state at constant volume

$$Q_{12} = U_2 - U_1.$$

The entropy S may be defined by the relation

$$dS = \frac{dQ}{T} + \frac{dH}{T},$$

where H denotes, not the heat absorbed by the substance from the surroundings, but the heat generated within the substance due to friction, wire drawing, etc. If the change of state is adiabatic (no heat absorbed or rejected), then $dQ = 0$ and $ds = \frac{dH}{T}$. If the change is also frictionless, $dH = 0$, and $dS = 0$, or S is constant. In many changes H is negligible, whence

$$ds = \frac{dQ}{T}, \quad \text{or} \quad dQ = T\,dS.$$

$$Q_{12} = \int_{s_1}^{s_2} T\,dS.$$

It follows that if the change of state is represented graphically on a plane with T and S as the axes, the area between the curve and the S-axis represents the heat absorbed.

PERFECT GASES.

The characteristic equation of a perfect gas is

$$pv = BT, \quad \text{or} \quad pV = MBT,$$

in which B is the so-called gas constant. The equation may be given the homogeneous form

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} = \frac{pV}{T} = B.$$

For a perfect gas

$$c_p - c_v = AB = \frac{1}{777.6} B$$

and $$c_p/c_v = k.$$

VALUES OF B, c_p, c_v, AND k FOR GASES.

Gas.	B	c_p	c_v	k
Air	53.34	0.240	0.171	1.40
Hydrogen	765.86	3.42	2.44	1.40
Nitrogen	54.99	0.247	0.176	1.40
Oxygen	48.25	0.217	0.155	1.40
Carbon monoxide	55.14	0.243	0.172	1.41

For a change of a gas from an initial state p_1, V_1, T_1, to a final state p_2, V_2, T_2,

$$U_2 - U_1 = Mc_v (T_2 - T_1) = \frac{A}{k-1}(p_2V_2 - p_1V_1),$$

$$I_2 - I_1 = Mc_p (T_2 - T_1) = \frac{Ak}{k-1}(p_2V_2 - p_1V_1),$$

$$S_2 - S_1 = M\left[c_p \log_e \frac{V_2}{V_1} + c_v \log_e \frac{p_2}{p_1}\right].$$

Special Changes of State.

1. Constant Volume. $\frac{p_2}{p_1} = \frac{T_2}{T_1}$. $W_{12} = 0$.

$$Q_{12} = U_2 - U_1 = Mc_v (t_2 - t_1).$$

$$S_2 - S_1 = Mc_v \log_e \frac{T_2}{T_1}.$$

2. Constant Pressure. $\frac{V_2}{V_1} = \frac{T_2}{T_1}$.

$$W_{12} = p(V_2 - V_1) = MB(t_2 - t_1).$$

$$Q_{12} = Mc_p (t_2 - t_1) = \frac{Ak}{k-1} W_{12}.$$

$$S_2 - S_1 = Mc_p \log_e \frac{T_2}{T_1}.$$

3. Constant Temperature (Isothermal).

$$p_1V_1 = p_2V_2. \quad W_{12} = p_1V_1 \log_e \frac{V_2}{V_1}.$$

$$U_2 - U_1 = 0.$$

$$Q_{12} = AW_{12} = AMBT \log_e \frac{V_2}{V_1}.$$

$$S_2 - S_1 = \frac{Q_{12}}{T} = AMB \log_e \frac{V_2}{V_1}.$$

4. Adiabatic. $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}.$

$$W_{12} = J\,(U_1 - U_2) = \frac{1}{k-1}\,(p_1V_1 - p_2V_2).$$

$$Q_{12} = 0. \quad S_2 - S_1 = 0.$$

Also $W_{12} = \frac{p_1V_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right].$

5. pv^n = const. The expansion and compression of gases in motors, compressors, etc., may be represented by curves having the equation $pv^n = C$, where n is a constant. The specific heat associated with such a process is $c_n = c_v \frac{n-k}{n-1}$, whence for $1 < n < k$, c_n is negative.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}.$$

$$W_{12} = \frac{1}{1-n}\,(p_2V_2 - p_1V_1)$$

$$= \frac{p_1V_1}{n-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right].$$

$$Q_{12} = Mc_n\,(t_2 - t_1). \quad S_2 - S_1 = Mc_n \log_e \frac{T_2}{T_1}.$$

$$AW_{12} : U_2 - U_1 : Q_{12} = k - 1 : 1 - n : k - n.$$

ELEC. ENG.

TABLES

SATURATED AND SUPERHEATED STEAM.

Notation.

The symbols v, u, i, and s have the same significance as in the general notation; however, these symbols with a prime (v', s', etc.) refer to 1 lb. of water at the boiling temperature, and with a double prime (v'', u'', etc.) they refer to saturated steam. In addition, let

r = latent heat, *i.e.*, heat required to vaporize 1 lb. of liquid at given constant pressure and temperature.

$\psi = Ap\,(v'' - v')$ = heat equivalent of external work required in vaporization.

ρ = increase of energy during vaporization.

x = quality of mixture, *i.e.*, ratio of weight of steam present to total weight of mixture.

c' = specific heat of water.

Fundamental Relations.

$$i'' = i' + r. \quad u'' = u' + \rho. \quad r = \rho + \psi.$$

$$s' = \int_{491.6}^{T} c' \frac{dT}{T}. \quad s'' = s' + \frac{r}{T}.$$

$$\frac{r}{T} = A\,(v'' - v')\frac{dp}{dt}. \quad \text{(Clapeyron's relation.)}$$

For a mixture of steam and water having a quality x,

$$i = i' + xr = i'' - (1 - x)\,r.$$

$$u = u'' - (1 - x)\,\rho.$$

$$s = s' + x\frac{r}{T} = s'' - (1 - x)\frac{r}{T}.$$

$$v = v' + x\,(v'' - v') = xv'' \text{ approx.}$$

Equations for Superheated Steam.

[In the following equations, take p in lb. per sq. *inch.*]

$$v = \frac{BT}{p} - \left(1 + 3\,ap^{\frac{1}{2}}\right)\frac{m}{T^4} + 0.018.$$

$$c_p = 0.32 + 0.000126\,T + \frac{23583}{T^2} + p\left(1 + 2\,ap^{\frac{1}{2}}\right)\frac{C'}{T}\cdot$$

$$i = 0.32\,T + 0.000063\,T^2 - \frac{23583}{T^5} - p\left(1 + 2\,ap^{\frac{1}{2}}\right)\frac{C''}{T^4} + 948.54.$$

$$s = 0.73683\log T + 0.000126\,T - \frac{11792}{T^2} - 0.254\log p - \left(1 + 2\,ap^{\frac{1}{2}}\right)\frac{C'''}{T^5} - 0.0807.$$

$$u = i - 0.1852\,pv.$$

Constants in the preceding formulas:

$\log B = \bar{1}.77448.$	$\log C' = 11.39361.$
$\log 3\,a = \bar{2}.71000.$	$\log C'' = 10.79155.$
$\log 2\,a = \bar{2}.53391.$	$\log C''' = 10.69464.$
$\log m = 10.82500.$	

Tables of the Properties of Steam.

Two tables of the properties of steam are included among the tables of this book. The first gives the important properties of saturated steam, while the second gives properties of superheated steam and also of mixtures of steam and water within certain limits. In this second table values of the entropy from 1.50 to 1.85 inclusive appear at the top of the page, and values of the pressures are given in the first column. By following a column the variation

of the volume v and the thermal head i during an adiabatic change of state is observed. The column designated by x gives the temperature of the superheated steam above the heavy dividing line and the quality of the mixture below this line.

Changes of State in Steam and Water Mixtures.

1. Isothermal or Constant Pressure. $t =$ const. $p =$ const.

$$W_{12} = p\,(V_2 - V_1) = Mp\,(v'' - v')\,(x_2 - x_1).$$
$$U_2 - U_1 = M\rho\,(x_2 - x_1). \quad Q_{12} = Mr\,(x_2 - x_1).$$

2. Adiabatic. $s =$ const.

$$s_1' + \frac{x_1 r_1}{T_1} = s_2' + x_2 \frac{r_2}{T_2}. \quad Q_{12} = 0.$$

$$W_{12} = (U_1 - U_2)\,J = JM\,[(i_1' + x_1\rho_1) - (i_2' + x_2\rho_2)].$$

3. Constant Volume. $v =$ const.

$$x_1 v_1'' = x_2 v_2'', \quad \text{or} \quad x_2 = x_1 \frac{v_1''}{v_2''}.$$
$$W_{12} = 0. \quad Q_{12} = U_2 - U_1$$
$$= M\,[(i_2' + x_2\rho_2) - (i_1' + x_1\rho_1)].$$

FLOW OF COMPRESSIBLE FLUIDS.

Fundamental Equations.

Let A denote the cross-section of the pipe or tube through which the fluid is flowing, w the mean velocity of the fluid across the section, and M the weight in lb. flowing through the section per second. If the flow is adiabatic, as may usually be assumed, the following equations apply.

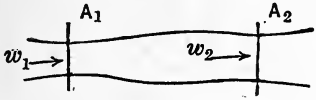
1. Equation of continuity.

$$Aw = Mv, \quad \text{or} \quad \frac{A_1 w_1}{v_1} = \frac{A_2 w_2}{v_2}.$$

2. Equation of energy.

$$Ji + \frac{w^2}{2g} = \text{const.}, \quad \text{or} \quad Ji + \frac{w_1^2}{2g} = Ji_2 + \frac{w_2^2}{2g}.$$

The second equation may be expressed by the statement: the sum of the thermal head and the velocity head is a constant.



The two equations hold good for flow with friction. The effect of frictional resistances is to increase the thermal head Ji and decrease the velocity head $\frac{w^2}{2g}$ by an equal amount.

In the case of flow from a reservoir, as a steam boiler, the initial section A_1 may be considered inside the reservoir and the velocity w_1 may be neglected in comparison with the exit velocity w_2. In this case the second equation becomes

$$\frac{w_2^2}{2g} = J(i_1 - i_2)$$

or $\quad w = \sqrt{2gJ}\sqrt{(i_1 - i_2)} = 223.7\sqrt{(i_1 - i_2)}.$

For air, or other gases of similar nature,

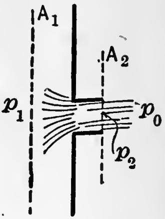
$$J(i_1 - i_2) = \frac{k}{k-1}(p_1 v_1 - p_2 v_2).$$

For steam, values of i are given in the steam tables.

Discharge Through Orifices.

Let p_1 denote the pressure in the reservoir, p_0 the pressure in the region into which the fluid discharges, and p_2 the pressure in the plane of the orifice, that is, at section A_2. If p_0 is less than a certain critical value mp_1, then p_2 takes

the value mp_1, and the discharge is constant for all values of p_0. If, however, p_0 is greater than mp_1, $p_2 = p_0$, and the discharge decreases as p_0 approaches p_1. The value of m depends upon the properties of the fluid. For saturated or slightly wet steam $m = 0.58$; for superheated steam $m = 0.55$; and for air and similar gases $m = 0.53$.



Case 1. $p_0 \leqq mp_1$. Discharge is independent of p_0. $p_2 = mp_1$. Take the flow as frictionless and find i_1 and i_2 corresponding to p_1 and p_2 from the second of the steam tables. Then

$$w_2 = 223.7 \sqrt{(i_1 - i_2)} \quad \text{and} \quad M = \frac{Aw_2}{v_2}.$$

For example, steam at 190 lb. pressure superheated to 450° flows through an orifice $\frac{3}{8}$ inch in diameter into a region in which the pressure is 60 lb. $p_2 = 0.55\ p_1 = 104.5$ lb., and from the steam table $i_1 = 1241$, $i_2 = 1189$ B.t.u. and $v_2 = 4.26$ cu. ft. Also $A = 0.1104$ sq. in. $=$ 0.000767 sq. ft.

$$w_2 = 223.7 \sqrt{1241 - 1189} = 1613 \text{ ft. per sec.}$$

$$M = \frac{0.000767 \times 1613}{4.26} = 0.29 \text{ lb. per sec.,}$$

$$\text{or } 17.4 \text{ lb. per min.}$$

For saturated steam, the discharge may be calculated approximately by one of the following empirical formulas:

1. Napier's rule, $M = \dfrac{pA}{70}$.
2. Grashof's formula, $M = 0.0165\ Ap^{0.97}$.
3. Rateau's formula,

$$M = \frac{pA}{1000}(16.367 - 0.96 \log p).$$

In these formulas p should be taken in lb. per sq. *inch* and A in square *inches*. Then M will give weight discharged per second.

For the discharge of air with $p_0 < 0.53\,p_1$, Fliegner's formula,

$$M = 0.53\ \frac{pA}{\sqrt{T}},$$

may be used.

Case 2. $p_0 > mp_1$. $p_2 = p_0$. The discharge depends upon p_0 and p_1.

For steam, determine i_1 and i_0, also v_0, then

$$w_2 = 223.7\ \sqrt{(i_1 - i_0)},\quad M = \frac{Aw_2}{v_0}.$$

For air the discharge in this case is given by the formula

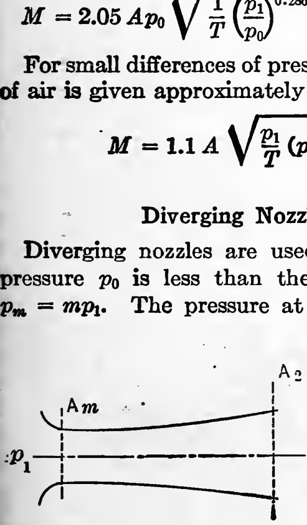
$$M = 2.05\,Ap_0 \sqrt{\frac{1}{T}\left(\frac{p_1}{p_0}\right)^{0.286}} \sqrt{\left(\frac{p_1}{p_0}\right)^{0.286} - 1}.$$

For small differences of pressure the discharge of air is given approximately by the formula

$$M = 1.1\,A \sqrt{\frac{p_1}{T}(p_1 - p_0)}.$$

Diverging Nozzles.

Diverging nozzles are used when the back pressure p_0 is less than the critical pressure $p_m = mp_1$. The pressure at the smallest section, or *throat*, takes the value p_m, and if the nozzle is properly proportioned, the pressure at the end section A_2 is p_0, the back pressure; *i.e.*, $p_2 = p_0$. Taking $w_1 = 0$ in the reservoir, the second general equation gives



$$Ji_1 = Ji_m + \frac{w_m^2}{2g} = Ji_0 + \frac{w_0^2}{2g}.$$

If the flow is adiabatic and frictionless, the entropy remains constant and the three thermal heads i_1, i_m, and i_0 are found in the second steam table. The effective drop of head through the nozzle is $i_1 - i_0$. The effect of frictional resistances is to decrease this drop by $y\,(i_1 - i_0)$, where y is a coefficient that may vary from 0.08 to 0.20 depending upon the size and smoothness of the nozzle.

	Thermal Head at End Section.	Quality at End Section.	Volume at End Section.
Without friction.	i_0	x_0	v_0
With friction.	$i_0' = i_0 + y(i_1 - i_0)$	$x_0' = x_0 + \frac{y(i_1 - i_0)}{r_0}$	$v_0' = x_0' v_0''$

As an example, consider the design of a nozzle to discharge 0.7 lb. per second. The steam is initially at a pressure of 200 lb. per sq. in. and is superheated to 548° F.; and the back pressure is 40 lb. per sq. in. The coefficient y is taken as 0.14. From the second steam table, in the column $s = 1.65$, the following values are found: $i_1 = 1295$, i_m (for $p_m = 110$ lb.) $= 1235$, $i_0 = 1150$, $v_m = 4.58$, $v_0 = 10.28$. The loss of jet energy due to friction is $0.14\,(1295 - 1150) = 20.3$ B.t.u. Without friction x_0 at section A_2 is from the table 0.978, with friction it is $0.978 + \frac{20.3}{935.5} = 1.00$; hence the specific volume at section A_2 is 10.51. Using the fundamental equations, the following results are obtained:

	i_1 1295	i_m 1235	$i_1 - i_m$ 60	w_m 1733
	i_1	i_0	$i_1 - i_0$	w_0
Without friction...	1295	1150	145	2694
With friction......		1170.3	124.7	2498
	v_m 4.58	A_m (sq. ft.) 0.00185		d_m (inch) 0.582
	x_0	v_0	A_0 (sq.ft)	d_0 (inch)
Without friction...	0.978	10.28	0.00267	0.700
With friction......	1.00	10.51	0.00295	0.735

Flow of Gases and Vapors in Mains.

The general equation of flow in pipes of circular cross-section, assuming that there is no transmission of heat is

$$v\,dp + \frac{cw^2}{d}\,dL = 0,$$

in which d denotes the diameter and L the length of the pipe, and c is the coefficient of resistance.

If the drop of pressure is small, as is the case in short mains, this equation gives the approximate relation

$$p' = p_1 - p_2 = c\,\frac{w^2L}{vd}. \qquad (a)$$

When, on the other hand, the drop of pressure is considerable, integration of the general equation gives the relation

$$p_1^2 - p_2^2 = \frac{32}{T_1^2}\,c\,\frac{M^2BTL}{d^5}, \qquad (b)$$

in which M denotes the weight of air flowing per second.

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1. Flow of Steam. Since the drop of pressure is small formula (a) is used. The coefficient c is not constant but varies with the diameter of the pipe. Taking the diameter in inches, and the length L in feet formula (a) reduces finally to

$$p' = 0.000131\left(1 + \frac{3.6}{d}\right)\frac{M^2 vL}{d^5},$$

or
$$M = 87.5\left[\frac{p'd^5}{vL\left(1 + \frac{3.6}{d}\right)}\right]^{\frac{1}{2}}.$$

In these formulas M denotes the weight of steam flowing in pounds per minute, and v the volume of a pound of steam at the mean pressure p' in lb. per sq. *inch.*

2. Flow of Compressed Air. Let V denote the volume in cubic feet of free air at 70° F. and a pressure of 14.7 lb. per sq. in. flowing per minute. Since in the flow of compressed air, the drop of pressure is relatively large, formula (b) is used. By proper transformations it may be given the form

$$V = 3.061\left[\frac{d^5\,(p_1^2 - p_2^2)}{cL}\right]^{\frac{1}{2}},$$

with
$$c = 0.003\left(1 + \frac{3.6}{d}\right).$$

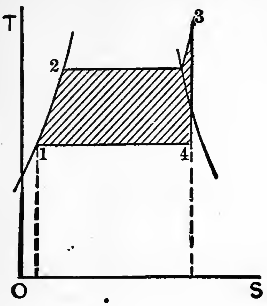
Here again d is to be taken in inches, L in feet, and p_1, p_2 in pounds per square inch.

THE STEAM ENGINE.

Ideal Rankine Cycle.

Representing the changes of state on the *TS*-plane (see Fig.), the medium receives heat in the boiler and superheater during the processes 1–2–3; the line 3–4 represents adiabatic expansion in the cylinder; and the line 4–1 represents rejection of heat to the condenser.

Heat absorbed $= q_1 = i_3 - i_1$.
Heat rejected $= q_2 = i_4 - i_1$.
Heat available for work $= q_1 - q_2 = i_3 - i_4$.



Thermodynamic efficiency of cycle

$$= \eta_R = \frac{i_3 - i_4}{i_3 - i_1}.$$

Steam required per H.P. hr. $= N_R = \dfrac{2546}{i_3 - i_4}$.

B.t.u. required to give 1 H.P. hr.

$$= N_R (i_3 - i_1) = \frac{2546}{\eta_R}.$$

Values of i_1, i_3, and i_4 are obtained directly from the steam tables. For example, steam is furnished at a pressure of 190 lb. per sq. in. superheated to 450° F., and the condenser pressure is 3 in. of mercury. Then

$i_1 = 83$ B.t.u. $i_3 = 1241$ B.t.u. $i_4 = 913$ B.t.u.

Heat available for work $= 1241 - 913 = 328$ B.t.u.; steam consumption per H.P. hr. $= \dfrac{2546}{328}$ $= 7.76$ lb.; thermodynamic efficiency of cycle $= \dfrac{328}{1241 - 83} = 0.283$; B.t.u. required to produce 1 H.P. hr. $= 7.76\,(1241 - 83) = 8986$ B.t.u.

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Efficiency of the Actual Engine.

Under the same conditions of operation, the actual engine transforms a smaller amount of heat into work per pound of steam supplied than the ideal Rankine engine. Let q_R and q_a denote the heat transformed by the Rankine engine and the actual engine, respectively. The efficiency of the actual engine is defined by the relation

$$\eta = \frac{q_a}{q_R}.$$

This efficiency ranges from 0.50 to 0.80 in steam engines and steam turbines.

Let N_a denote the actual steam consumption per H.P. hr. Then

$$N_a = \frac{2546}{q_a} = \frac{N_R}{\eta}; \quad \text{or} \quad \eta = \frac{N_R}{N_a};$$

and the heat required to give 1 H.P. hr. is

$$N_a (i_3 - i_1) = \frac{2546}{\eta \eta_R}.$$

In the example preceding let the efficiency of the actual engine based on the ideal Rankine engine be 0.70; then the steam consumption is $7.76 \div 0.70 = 11.1$ lb. per H.P. hr., and the heat required per H.P. hr. is $8986 \div 0.70 =$ 12,837 B.t.u.

STEAM BOILERS.

Let i_1 = thermal heat (heat of liquid) of water fed to boiler.

i_2 = thermal heat (total heat) of steam formed.

M = weight of water evaporated per hour.

M_e = equivalent weight of water evaporated per hour from and at 212° F.

f = factor of evaporation.

H = rated horsepower of boiler.

By definition a boiler horsepower is equivalent to the evaporation of 34.5 lb. of water per hour from and at 212° F.

$$f = \frac{M_e}{M} = \frac{i_1 - i_2}{971.7}.$$

$$H = \frac{M_e}{34.5} = \frac{M\,(i_1 - i_2)}{33{,}520}.$$

$$M_e = fM.$$

CONDENSERS.

Steam enters the condenser at a known pressure p_1 and a quality x_1, which is frequently assumed as 1. The thermal head of the entering steam is $i_1 = i_1' + x_1 r_1$; that of the condensed steam leaving the condenser at the temperature t_2 is i_2'. If M lb. of condensing water is required and the temperature at entering and leaving are t_3 and t_4, respectively, then

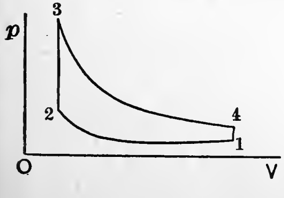
$$M = \frac{i_1' + x_1 r_1 - i_2'}{t_4 - t_3}.$$

INTERNAL COMBUSTION ENGINES.

The ideal cycles employed for internal combustion motors are the following:

1. Explosive, Otto.
2. Slow burning, non-explosive, Joule or Brayton, Diesel.

Otto Cycle.



Compression 1–2 and expansion 3–4 are assumed to be adiabatic. The line 2–3 represents the rapid heating at constant volume.

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{p_3}{p_4}\right)^{\frac{k-1}{k}} = \left(\frac{v_1}{v_2}\right)^{k-1}.$$

Heat absorbed $= Q_1 = Mc_v \, (T_3 - T_2)$.

If Q_1 is the heating value of the fuel and M the weight of the charge of fuel and air, the final temperature T_3 is

$$T_3 = T_2 + \frac{Q_1}{Mc_v}.$$

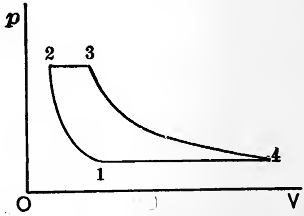
The efficiency of the cycle is

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{v_2}{v_1}\right)^{k-1} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}.$$

Work of cycle $= W = \eta Q_1$
$= JMc_v \, (T_3 - T_4 - T_2 + T_1)$.

Joule or Brayton Cycle.

The absorption of heat in the process 2–3 is at constant pressure.



$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{v_4}{v_3}\right)^{k-1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}.$$

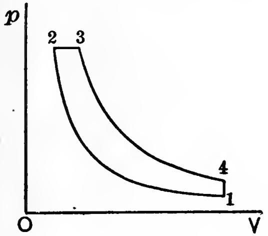
$$Q_1 = Mc_p \, (T_3 - T_2); \quad T_3 = T_2 + \frac{Q_1}{Mc_p}.$$

$$\text{Efficiency} = \eta = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}.$$

Work of cycle $= \eta Q_1 = JMc_p \, (T_3 - T_2 - T_4 + T_1)$.

Diesel Cycle.

Air is compressed to a pressure of 500 lb. per sq. in. or more, and the fuel injected into this air burns at nearly constant pressure.



$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}. \qquad T_3 = T_2 + \frac{Q_1}{MC_p}.$$

$$\frac{v_3}{v_2} = \frac{T_3}{T_2}. \qquad \frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}.$$

$$\text{Efficiency} = \eta = 1 - \frac{T_4 - T_1}{k\,(T_3 - T_2)}.$$

Work of cycle =

$$\eta Q_1 = JM\,[c_p\,(T_3 - T_2) - c_v\,(T_4 - T_1)].$$

AIR COMPRESSION.

Let V_1 = volume of free air entering compressor cylinder per stroke at pressure p_1 (atmospheric, or slightly lower).

V_2 = volume of the same air when compressed to the higher pressure p_2.

W = work required per strike.

H = net horsepower required to drive the compressor.

N = r.p.m. of double acting compressor.

The compression is assumed to follow the law pv^n = const. The value of n lies between

1.2 and 14 depending upon the effectiveness of the water jacket. An average value is 1.3.

$$V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}}.$$

$$W = \frac{n}{n-1}(p_2V_2 - p_1V_1)$$

$$= \frac{n}{n-1}p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right].$$

(p_2 and p_1 in lb. per sq. *foot.*)

$$H = \frac{2\,NW}{33{,}000}.$$

If the compressor has no clearance the volume V_0 swept through by the piston is equal to V_1. If there is clearance, the air caught in the clearance space expands from p_2 to p_1 and as a result $V_1 < V_0$. Let m = ratio of clearance volume to V_0; then

$$V_0 = \frac{V_1}{1 + m - m\left(\frac{p_2}{p_1}\right)^n}.$$

Compound Compression.

If the air is compressed in two stages (1) from p_1 to an intermediate pressure p' (2) from p' to p_2, then for minimum work of compression

$$p' = \sqrt{p_1p_2}$$

and $$W = \frac{2\,n}{n-1}p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{2n}} - 1\right].$$

For compression in three stages with cooling between the stages, the proper intermediate pressures are

$$p' = \sqrt[3]{p_1^2p_2}, \qquad p'' = \sqrt[3]{p_1p_2^2},$$

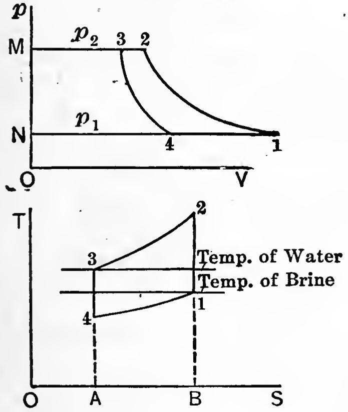
and the work of compression per stroke is

$$W = \frac{3\,n}{n-1}p_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{3n}} - 1\right].$$

REFRIGERATION.

Air as the Medium.

Air is compressed adiabatically, as shown by 1–2, cooled at constant pressure (2–3), expanded adiabatically (3–4) in a separate expansion cylinder, and then passing through the brine absorbs heat from it, as represented by 4–1.



Let Q = heat absorbed from brine or cold room per minute.

Q' = heat rejected to cooling water per minute.

M = weight of air circulated per minute.

H = horsepower (net) required.

p_1, p_2 = lower and higher pressures, respectively.

c_p = specific heat of air at constant pressure.

The temperatures T_1 and T_3 are fixed by the brine and cooling water; the temperatures T_4 and T_2 are obtained from the relation

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}.$$

$$Q = Mc_p\,(T_1 - T_4).$$
$$Q' = Mc_p\,(T_2 - T_3).$$

$$\text{Work per minute} = J\,(Q' - Q)$$
$$= JMc_p\,[(T_2 - T_3) - (T_1 - T_4)]$$
$$= JQ\,\frac{T_2 - T_1}{T_1}.$$

$$H = \frac{Q}{42.43}\,\frac{T_2 - T_1}{T_1}.$$

If N is the number of working strokes per minute, the required volume of the compressor cylinder (neglecting clearance) is

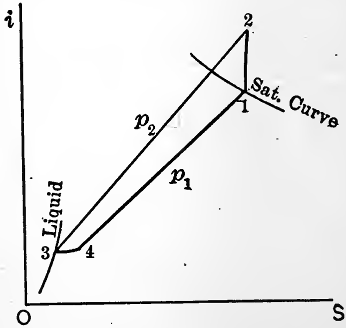
$$V_c = \frac{MBT_1}{Np_1};$$

that of the expansion cylinder is

$$V_e = \frac{MBT_4}{Np_1}.$$

Vapor as the Medium.

Adiabatic compression (1–2) is followed by rejection of heat (2–3) to the cooling water



until the medium is a liquid (at point 3). Liquid passes through expansion valve dropping

in pressure from p_2 to p_1 and attains state represented by point 4, with $i_3 = i_4$. Line 4–1 represents absorption of heat from brine at constant pressure p_1. Using same notation as in preceding section,

$$Q = M(i_1 - i_4) = M(i_1 - i_3),$$
$$Q' = M(i_2 - i_3).$$

Work required per minute $= JM(i_2 - i_1)$.

$$H = \frac{M(i_2 - i_1)}{42.43}.$$

Let v'' denote the volume of 1 lb. of the saturated vapor at the lower pressure p_1; and N the number of working strokes per minute; then the volume of the compressor cylinder is (neglecting clearance)

$$V_c = \frac{Mv''}{N}.$$

If the cooling water enters at the temperature t_1 and leaves at temperature t_2, the weight G required per minute is

$$G = \frac{M(i_2 - i_3)}{t_2 - t_1}.$$

Values of i_1, i_2, and i_3 are obtained from the tables of saturated and superheated ammonia or carbon dioxide.

ELECTRICAL ENGINEERING FORMULÆ.

COMPILED BY H. H. HIGBIE.

Professor of Electrical Engineering, University of Michigan.

NOTATION.

A = area, square centimeters.

A_m = cross-section area magnetic circuit, sq. cm.

B = flux density, maxwells per sq. cm., gausses.

B_m = cyclic maximum flux density, gausses.

b = susceptance, mhos.

C = capacitance, farads.

C_0 = capacitance to neutral, per mile of transmission line, farads.

d = distance, centimeters.

= diameter, mils.

E = e.m.f., volts, effective or square-root-mean-square value.

= unvarying voltage in d-c. circuit.

E_{av} = average value of varying e.m.f., volts.

E_m = maximum instantaneous value of varying e.m.f.

E_0 = volts to neutral, r.m.s. value.

E_r = volts consumed in overcoming resistance.

E_g = e.m.f. generated, volts.

E_t = e.m.f. between terminals, volts.

e = e.m.f. at any instant, volts.

F = force, dynes.

f = frequency, cycles per second.

$\mathfrak{F}$ = magnetomotive force (m.m.f.), gilberts.

g = conductance, mhos.

H = magnetizing force, field intensity, gausses in air, dynes force on unit pole.

I = current, amperes, effective or r.m.s. value.

= unvarying current in d-c. circuit.

I_t = current from terminals, amperes.

I_f = current in (shunt) field, amperes.

I_a = total current through armature, amperes.

i = current, amperes, at any instant.

i_c = charging current at any instant, amperes.

k = specific inductive capacity or dielectric constant.

K = constant.

L = self-inductance of electric circuit, henrys.

l = length.

l_m = length magnetic circuit, centimeters.

l_w = length of wire, centimeters.

M = mutual inductance, henrys, of two electrical circuits magnetically interlinked.

m = power factor.

= strength of magnet, in unit poles.

n = reactive factor.

= angular velocity, revolutions per second.

N = turns in coil or electrical circuit.

p = instantaneous power, watts.

= number of field poles.

P = average power, watts.

P_H = power lost due to hysteresis, watts.

P_E = power lost due to eddy-currents, watts.

P_r = power, watts, transformed into heat in overcoming resistance.

Q, q = quantity of electricity, coulombs, amperes × seconds.
R = resistance, ohms.
$\mathcal{R}$ = reluctance, oersteds.
r = radius.
s = number of parallel paths between armature terminals.
T = torque, pound-feet.
t = thickness, thousandths of inch, mils.
= temperature, degrees Centigrade.
= time elapsed, seconds.
v = velocity, centimeters per second.
V = volume, cubic inches.
w = weight, pounds.
W_m = energy of magnetic field, watt-sec. or joules.
W_c = energy stored in condenser, watt-seconds.
X = reactance, ohms.
y = admittance, mhos.
Z = impedance, ohms.
= number useful conductors on armature.
α_0 = temp. coeff. of resistance, based on 0° C.
ϵ = base of Naperian logarithms = 2.7183.
η = efficiency, ratio.
θ = angle.
= time-phase difference expressed in electrical degrees.
μ = permeability, ratio.
ρ_0 = resistivity at 0° C., ohms per centimeter cube.
Φ = magnetic flux, maxwells.
ω = angular velocity, radians per second.

MAGNETIC FORCES AND FIELDS.

(a) *Field due to a pole at a point.*

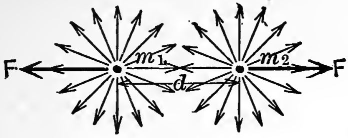


Fig. 1.

H_2 (at m_1 due to m_2) $= \frac{F}{m_1}$ = force (dynes), exerted on unit north pole.

$$F = \frac{m_1 m_2}{d^2} = m_1 H_2 = m_2 H_1.$$

(b) *Field due to current in straight conductor.*

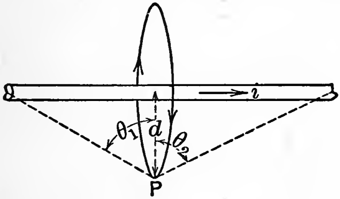


Fig. 2.

$$H \text{ (at } P \text{ due to } i) = \frac{i}{10 \cdot d} (\sin \theta_1 + \sin \theta_2).$$

Field (dynes force on unit north pole) at P is downward into paper if current flows toward right, and upward if current flows toward left. Field is circular and concentric with axis of conductor.

(c) *Force on conductor due to current and field.*

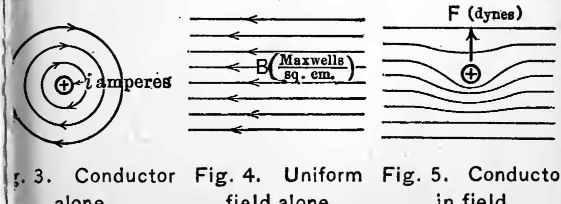


Fig. 3. Conductor alone. Fig. 4. Uniform field alone. Fig. 5. Conductor in field.

TABLES

F (dynes on each centimeter length of wire) $= B\dfrac{i}{10}$, whence

Pounds force on wire

$$= 22.5 \times 10^{-8} \cdot Bl_w i$$

$$= \frac{57.1}{10^8} Bi \times \text{(length of wire, inches).}$$

This formula presumes that i is in direction at right angles to B. If the directions of i and B form an angle θ, the preceding expression for force must be multiplied by $\sin\theta$. This force is perpendicular to both i and B; it is in direction away from the side of the conductor where the field has been made more dense, and toward the side where the field has been made less dense (Fig. 5).

(*d*) *Law of the magnetic circuit.*

$$\Phi = \frac{\mathfrak{F}}{\mathfrak{R}} = \frac{0.4\,\pi N i}{l_m/\mu A_m}.$$

$$B = \frac{\Phi}{A_m} = \mu H,$$

whence

$$H = 0.4\,\pi \frac{Ni}{l_m}.$$

and

Amp.-turns per inch length of magnetic circuit

$$= 0.3132 \times \left(\frac{\text{maxwells per square inch}}{\mu}\right).$$

See page 150 for magnetization curves.

MAGNETICALLY INDUCED ELECTROMOTIVE FORCE.

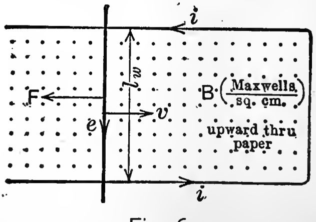


Fig. 6.

$$e\text{ (induced)} = \frac{Bvl_w}{10^8} = \frac{1}{10^8}\frac{d\Phi}{dt},$$

where Φ is the total maxwells linking the single turn of circuit shown. Direction of e is always such that force produced on current in same direction as e, by the field, would be in direction opposite to the velocity which produces e.

In general, if N turns are linked by a varying flux Φ maxwells, then

$$e\text{ (induced)} = \frac{1}{10^8}\frac{d}{dt}(\Phi N) = \frac{N}{10^8}\cdot\frac{d\Phi}{dt}.$$

If a current i amperes flows, the conductor must move against a force $\left(\frac{Bil_w}{10}\right)$ dynes, whence

$$ei = \frac{Bvl_w}{10^8} \times \frac{10 \cdot F}{Bl_w} = \frac{Fv\text{ dyne-cm. per sec}}{10^7}, \text{ or}$$

volts $\times$ amperes = watts

$$= \frac{\text{ergs per sec}}{10^7} = 746 \times \text{horse power.}$$

INDUCTANCE OF AN ELECTRIC CIRCUIT.

(*a*) *General.* An electric circuit has 1 henry inductance if 1 volt is induced in it when the current changes at rate of 1 ampere per second. A non-inductive circuit is one which builds no magnetic field when current flows. The induced e.m.f. must always *oppose the change* of current.

$$e\text{ (induced)} = -L\frac{di}{dt} = -\frac{1}{10^8}\frac{d}{dt}(\Phi N),$$

$$L = \frac{1}{10^8}\frac{d}{di}(\Phi N),$$

$$e\text{ (average)} = -\frac{1}{10^8} \times \frac{\Phi N}{t},$$

$$L\text{ (average)} = \frac{1}{10^8} \times \frac{\Phi N}{i},$$

where Φ is the flux produced by i, which links all of the turns N, and L is the average inductance within the current limits 0 to i, or flux limits 0 to Φ. If not all the flux links all the turns, but Φ_1 maxwells link N_1 turns, Φ_2 maxwells link N_2 turns, etc., we have

$$L \text{ (average)} = \frac{1}{10^8} \times \frac{\Phi_1 N_1 + \Phi_2 N_2 + \cdots}{i}.$$

(*b*) *Self-inductance of a transmission line* in air, henrys per mile length of each single wire, is given by the equation

$$L_w = \left(0.08047 + 0.74113 \log_{10} \frac{d}{r}\right) \times 10^{-3},$$

where d is distance between centers of outgoing and return wires, and r is radius of wire, both in terms of same unit of length. Tables of inductance and reactance for transmission lines, found in Handbooks, are calculated from this formula; it applies also to each mile of each wire of a three-wire line if wires are all equidistant.

(*c*) *Mutual inductance of two electric circuits.*

$$e_1 = -M \frac{di_2}{dt}, \quad \text{and} \quad e_2 = -M \frac{di_1}{dt},$$

$$M_{(av)} = \frac{1}{10^8} \cdot \frac{\Phi_2 N_1}{i_2} = \frac{1}{10^8} \cdot \frac{\Phi_1 N_2}{i},$$

where i_1 amperes in one circuit cause Φ_1 maxwells to link with the N_2 turns of the other circuit, or i_2 amperes in the second circuit cause Φ_2 maxwells to link with the N_1 turns of the first circuit. If all of the flux produced by either of the two circuits links with all the turns of both circuits, we have:

$$L_1 L_2 = M^2.$$

ENERGY STORED IN MAGNETIC FIELD.

$$W_m = \tfrac{1}{2} Li^2 = \int_0^i Li\, di$$

gives the watt-seconds or joules of energy stored in magnetic field due to current i amperes in circuit having constant inductance L henrys.

$$W_B = \frac{B^2}{8\pi\mu} = \frac{\mu H^2}{8\pi}$$

gives the ergs per cubic centimeter in a magnetic field of density B maxwells per square centimeter in a medium having constant permeability μ.

POWER DISSIPATED IN MAGNETIC CIRCUIT.

$$P_H = K_1 f B_m^{1.6}\, V = K_2 f B_m^{1.6}\, w.$$
$$P_E = K_3 f^2 B_m^2 t^2 V = K_4 f^2 B_m^2 t^2 w.$$

Values of K, and of P_H or P_E for any assigned values of f, B_m, V, w and t may be calculated from data given on pages 150, 151. It is assumed that the flux varies harmonically with respect to time, and that it is uniformly distributed throughout the iron.

CONDENSERS AND ELECTROSTATICS.

(*a*) *General.*

$$C = \frac{q}{e}, \text{ or } Q = Ce.$$

C is in farads when q is in coulombs or ampere-seconds, and e is in volts.

$$i_c = \frac{dq}{dt} = C\frac{de}{dt}, \text{ or } e = \frac{1}{C}\int i_c\, dt.$$

(*b*) For several condensers in parallel, the equivalent total capacitance is

$$C_{eq} = C_1 + C_2 + C_3 + \cdot \cdot \cdot .$$

(*c*) For several condensers in series, the equivalent total capacitance is given by the equation

TAB-LES

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots.$$

(*d*) *Capacitance of a parallel-plate condenser.*

$$C = 0.08842\, k \frac{A}{d} \times 10^{-12} \text{ farads}$$

$$= 0.08842\, k \frac{A}{d} \times 10^{-6} \text{ microfarads,}$$

where d is the uniform distance, in centimeters, between oppositely charged surfaces each of A square centimeters area, and k is the specific inductive capacity of the dielectric between. It is assumed that d is small in comparison with dimensions of plates. Values of k for common insulating materials are given on page 151.

(*e*) *Capacitance of single-conductor cable with grounded metal sheath.*

$$C = \frac{0.03882\, k}{\log_{10}(r_i/r_0)} \times 10^{-6} \text{ farads per mile,}$$

where r_0 is the external radius of the inner cylindrical conductor and r_i is the internal radius of the outer sheath, both in terms of same units. Total capacitance is directly proportional to length, since capacitances of successive miles are all in parallel.

(*f*) *Capacitance to Neutral of each wire of a transmission line in air.*

$$C_0 = \frac{0.03882}{\log_{10}(d/r)} \times 10^{-6} \text{ farads per mile,}$$

where r is the radius of the wire and d is the distance between centers, both in terms of same units. It is assumed that d is large compared with r, and both small compared with distance to surrounding objects. For a two-wire line, capacitance between wires (per mile distance) is one-half the value given above, as the condensers from each wire to neutral

are in series. For three wires spaced equidistant (at vertices of equilateral triangle) as for three-phase line, the same formula gives capacitance to neutral per mile.

(*g*) *Charging Current per mile of a transmission line in air.*

$$I_c = 2\pi f C_0 E_0,$$

where E_0 equals 0.50 times r.m.s. value of volts between line wires for a single-phase two-wire line, and 0.577 times r.m.s. volts between wires for a three-phase three-wire line. Harmonic e.m.f. and balanced voltages are assumed. I_c is r.m.s. amperes if E_0 is r.m.s. volts. Tables of charging current and line capacitance found in Handbooks are in accord with these formulæ.

(*h*) *Energy stored in a condenser or in its dielectric.*

$$W_c = \int Ce\,de = \frac{1}{2}Ce^2 = \frac{1}{2}\cdot\frac{Q^2}{C} = \frac{1}{2}Qe,$$

where W_c is the watt-seconds or joules required to raise condenser of C farads to a potential difference of e volts between terminals or plates, the charge being Q coulombs or ampere-seconds.

CIRCUITS CARRYING DIRECT CURRENT (UNVARYING).

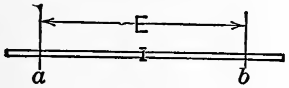


Fig. 7.

When a conductor carries an unvarying current, the e.m.f. between any two points a and b is directly and exactly proportional to the current. That is,

$$\frac{E}{I} = R = \text{a constant}$$

$$= \text{resistance of conductor } ab.$$

If E is in volts and I in amperes, R is expressed as "ohms resistance." It is assumed that no e.m.f. is generated (as by battery or dynamo) between a and b.

(b) *Resistance of a conductor.* R is a constant for any given temperature, material and dimensions of conductor; it varies with each of these factors as indicated in the following equations.

$R = R_0 (1 + \alpha_0 t)$ when dimensions and material of conductor remain unchanged.

$R_0 = \rho_0 \frac{l}{A}$ when temperature is constant at 0° C.

R_0 is ohms resistance at 0° C., and R is ohms for same conductor at t° C. α_0, the temperature coefficient for resistance, equals 0.00427 for standard annealed copper and has practically the same value for most pure metals (including aluminum, and soft steel) although it varies greatly among alloys, non-metals and solutions. See an Electrical Engineering Handbook. ρ is the resistivity, varying greatly with the nature and treatment of the conductor material; see page 152. l is the length of conductor and A its cross-section area in plane normal to direction of current flow, in same units used to determine ρ_0.

$$R_0 = 6.0153 \rho_0 10^6 \left(\frac{\text{length in feet}}{\text{section area in circular-mils}}\right) \text{ ohms.}$$

One circular-mil is area of circle 0.001 inch diameter.

For *round copper wires*, at 20° C. or 68° F., the following relations form the basis for tables in Roebling's book "Wire in Electrical Construction":

$$\text{Ohms per 1000 feet} = \frac{10371.2}{d^2}.$$

$$\text{Pounds per 1000 feet} = 0.003027\, d^2.$$

d^2 = section area in circ. mils
= (diam. in inches $\times$ 1000)^2.

Values of d for standard (Brown & Sharpe) gauge numbers are given on page 153.

(c) *Total resistance of a series circuit.*

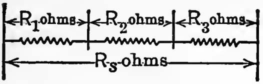


Fig. 8.

$$R_s = R_1 + R_2 + R_3.$$

(d) *Equivalent Resistance of a Parallel Circuit.*

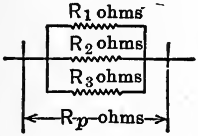


Fig. 9.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3},$$

assuming that none of the paths contains any source of e.m.f.

(e) *Power lost in a conductor.*

$$P_r = E_r I = IR \times I = I^2 R = \frac{E_r^2}{R},$$

where P_r is the watts transformed into heat in a conductor of R ohms resistance carrying I amperes. $E_r = IR$ is the volts consumed in overcoming resistance.

(f) *Series circuits carrying direct current.* Relations of current, e.m.f., and power.

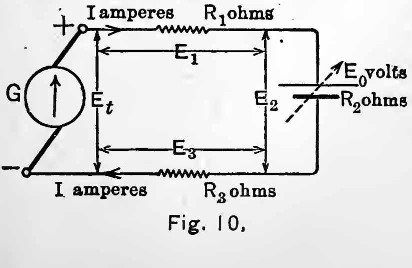


Fig. 10.

Consider a generator G impressing an unvarying e.m.f. E_t upon a series circuit consisting of $R_1 + R_3$ ohms, and a battery (or a motor) having internal resistance R_2 ohms and a generated "back e.m.f." E_0 volts directed opposite to E_t (as indicated by dotted arrow). Then, if I represent the amperes flowing,

$$E_t - E_0 = I(R_1 + R_2 + R_3).$$
$$E_t = E_1 + E_2 + E_3$$
$$= IR_1 + (E_0 + IR_2) + IR_3.$$

P_G = power output of generator = E_tI watts.

P_r = power transformed into heat in entire external circuit.

$P_r = I^2(R_1 + R_2 + R_3)$ watts.

P_T = power transformed chemically in battery (or mechanically in motor) generating E_0.

$P_T = E_0I$ watts.

P_2 = power *input* to battery (or motor).

$P_2 = (E_0 + IR_2)I = E_0I + I^2R_2.$

If the connections are changed so that E_0 acts in same direction as E_t, then the sign of E_0 is reversed in the above equations:

$$E_t + E_0 = I(R_1 + R_2 + R_3).$$
$$P_2 = -(E_0 - IR_2)I = (E_0I - I^2R_2)$$

watts *output* from generator of E_0.

(*g*) *Parallel circuits carrying direct current.* Relations of current, e.m.f., and power.

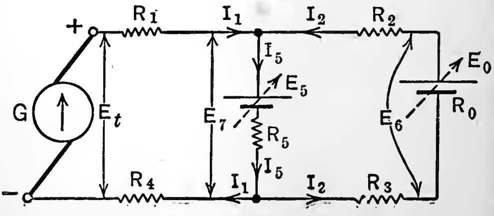


Fig 11.

Consider a generator G impressing an unvarying e.m.f. E_t upon a load having resistance R_5 ohms and internal e.m.f. E_5 volts, over line wires having resistances R_1 and R_4 ohms; let a battery whose generated e.m.f. (on open-circuit) is E_0 volts and internal resistance R_0 ohms be connected in parallel with G to this same load R_5. Directions of E_0 and E_5 are indicated by dotted arrows.

Mark, as indicated, the directions in which currents I_1, I_2, I_5 in various parts of the circuit may flow; if the wrong direction happens to be chosen for any current, the algebraic solution of the following simultaneous equations will give negative value for that current. We may now write:

$$E_t - E_5 = I_1R_1 + I_5R_5 + I_1R_4$$

or $$E_7 = E_5 + I_5R_5 = E_t - I_1(R_1 + R_4),$$

and $$E_0 - E_5 = I_2R_2 + I_2R_0 + I_2R_3 + I_5R_5$$

or $$E_7 = (E_0 - I_2R_0) - I_2(R_2 + R_3)$$
$$= E_6 - I_2(R_2 + R_3)$$

and $I_1 + I_2 = I_5$.

Numerical values having been assigned to E_t, E_0 and E_5 in volts, and to all the resistances in ohms, we should be able to find corresponding values for I_1, I_2, I_3 by solving these equations.

(*h*) *Solution of Networks.* As indicated in the preceding examples, the solution of any series-parallel arrangement of circuits, or network, depends on the application of two principles, commonly known as *Kirchoff's Laws:*

(*a*) In any closed circuit the algebraic sum of the products of the current and resistance in each of the conductors in the circuit is equal to the electromotive force in the circuit. In applying this, account must be taken of the

relative direction of the e.m.f.'s and the currents in various parts of the circuit.

(*b*) The algebraic sum of the currents which meet at any point is zero; or, the sum of currents *toward* a juncture must be equal to the sum of currents *away* from that juncture.

DIRECT-CURRENT MACHINES.

(*a*) *Electromotive force generated* in the armature between terminals is

$$E_g = \frac{p\Phi Zn}{10^8\, s} \text{ volts,}$$

where the armature has altogether Z conductors on its outer surface arranged in s parallel circuits, and revolves at n revolutions per second in a field of p poles from each of which Φ maxwells enter the armature. If the dynamo operates as generator I is in same direction as E_g; if it operates as motor, E_g is in opposition to I. Brushes are assumed to be on neutral points.

(*b*) *Terminal voltage* of a d-c. dynamo is

$$E_t = E_g \pm (R_a I_a + R_{se} I_{se} + R_{cp} I_{cp}),$$

where R_a, R_{se}, R_c are the resistances of armature, series field and commutating-poles, respectively, in ohms; and I_a, I_{se}, I_{cp} are the currents in the corresponding parts, amperes. The + sign is used if the dynamo operates as motor, the − sign if it operates as generator. For shunt-wound dynamo the $R_{se}I_{se}$ term is omitted, and if it has no commutating-poles omit the term $R_{cp}I_{cp}$.

(*c*) *Torque* of a dynamo is

$$T = \frac{0.1174\, p\Phi Z I_a}{10^8\, s} \text{ pound-feet,}$$

where T is the total torque magnetically developed on an armature with Z surface conduc-

tors arranged in s parallel paths, due to a total current I_a amperes, when Φ maxwells enter the armature from each of p poles. The torque at pulley must be slightly greater than this in a generator, or slightly less in a motor, on account of friction (and magnetic losses if the dynamo is rotating).

(*d*) *Speed of a motor* is

$$n = \frac{10^8 E_g s}{p\Phi Z} = \frac{10^8 s\,(E_t - RI)}{p\Phi Z} \text{ rev. per sec.,}$$

where RI is the total resistance drop in the armature circuit across which E_t volts is impressed, including series field and commutating-pole winding if the motor has such.

(*e*) *Efficiency and Losses* in a d-c. dynamo.

For a *Generator:*

$$\eta = \frac{\text{watts output}}{\text{watts input}} = \frac{E_t I_t}{E_t I_t + P_f + P_{se} + P_{cp} + P_s}.$$

For a *Motor:*

$$\eta = \frac{E_t I_t - P_f - P_{se} - P_{cp} - P_s}{E_t I_t}.$$

$P_f = I_f^2 R_f$ = heat loss in shunt field coils and rheostat.

$P_{se} = I_{se}^2 R_{se}$ = heat loss in series field coils and regulating shunt.

$P_{cp} = I_{cp}^2 R_{cp}$ = heat loss in commutating-pole winding.

P_s = stray power, including hysteresis and eddy-current losses in armature core and in pole faces, and friction losses in bearings, brushes and windage.

GROWTH AND DECAY OF CURRENT IN INDUCTIVE CIRCUIT.

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right),$$

where i is the amperes flowing in a circuit having resistance R ohms and self-inductance L henrys

TAB-LES

arranged in series, at an instant t seconds after an unvarying e.m.f. E volts has been applied. Current assumed to start from zero.

If the impressed e.m.f. E is removed from a circuit of resistance R ohms and self-inductance L henrys, when it is carrying a steady current $I = \frac{E}{R}$, and the circuit is closed through an additional resistance R_1 ohms, the current becomes i amperes at an instant t seconds after, where

$$i = I\epsilon^{-\frac{R+R_1}{L}t} = \frac{E}{R}\epsilon^{-\frac{R+R_1}{L}t}.$$

The amount of e.m.f. generated in the coil at this instant is

$$e = L\frac{di}{dt} = i(R+R_1) = E\frac{R+R_1}{R}\epsilon^{-\frac{R+R_1}{L}t}.$$

General Equation for electric circuit having resistance R ohms, self-inductance L henrys, and capacitance C farads, all in series is

$$e = Ri + L\frac{di}{dt} + \frac{\int i\,dt}{C},$$

where e volts applied produces a current i amperes which is changing at the rate $\frac{di}{dt}$ amperes per second. This relation holds at every instant, for any mode of variation of e.m.f. or current.

HARMONIC ALTERNATING CURRENT.

A simple *harmonic e.m.f.* which completes f cycles per second has a value e volts at an instant t seconds after it has attained its maximum positive value E_m volts, where

$$e = E_m \cos 2\pi ft = E_m \cos \omega t.$$

This e.m.f. will produce a simple *harmonic current* (i amperes) in any circuit having resistance R ohms, self-inductance L henrys, and capacitance C farads, where

$$i = I_m \cos(\omega t - \theta),$$

$$I_m = \frac{E_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}},$$

$$= \frac{E_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{E_m}{Z},$$

$$\theta = \text{arc tan}\,\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} = \text{arc cos}\left(\frac{R}{Z}\right).$$

Effective or square-root-mean-square *value* of this e.m.f. is

$$E = \frac{E_m}{\sqrt{2}} = 0.707\, E_m,$$

and of this current is

$$I = 0.707\, I_m = \frac{E}{Z}.$$

Average value of this e.m.f. (during one half-cycle) is

$$E_{av} = \frac{2}{\pi} E_m = 0.636\, E_m.$$

Form-factor $= \dfrac{E_{effective}}{E_{average}} = \dfrac{0.707}{0.636}$

$= 1.11$ for harmonic e.m.f. or current.

Impedance $= Z = \dfrac{E}{I} = \sqrt{R^2 + X^2}.$

Reactance $= X = \left(2\,\pi f L - \dfrac{1}{2\,\pi f C}\right).$

Power at any *instant* in a circuit where E (r.m.s.) volts produces I (r.m.s.) amperes, lagging (or leading) θ electrical degrees $\left(\text{or } \frac{\theta}{360} \text{ of } \frac{1}{f} \text{ seconds}\right)$ with respect to E, is

TAB-LES

$$p\ (\text{watts}) = ie = EI\cos\theta + EI\cos(4\pi ft - \theta)$$
$$= E_m \cos\omega t \times Im\cos(\omega t - \theta).$$

Average power in this circuit is

$$P = EI\cos\theta = \text{average of } p \text{ for complete cycle.}$$

$$\textit{Power-factor} = \frac{P}{EI} = \frac{\text{power}}{\text{apparent power}},$$
$$= \cos\theta \text{ (when } e \text{ and } i \text{ are harmonic)}$$
$$= \frac{R}{Z} = \frac{I^2R}{IZ \cdot I}.$$

Series Circuits carrying simple harmonic *Alternating Current.*

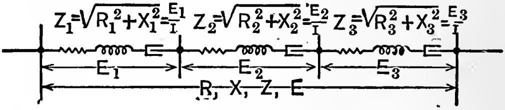


Fig. 12.

$$R = R_1 + R_2 + R_3.$$
$$X = X_1 + X_2 + X_3.$$
$$Z = \sqrt{R^2 + X^2}.$$
$$I = \frac{E}{Z}.$$

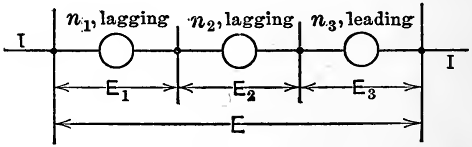


Fig. 13.

Three units in series have e.m.f.'s E_1, E_2, E_3 (r.m.s. volts) respectively, power factors m_1, m_2, m_3, and reactive factors n_1, n_2, n_3 respectively, where

$$m = \cos\theta$$
$$n = \sin\theta = \sqrt{1 - m^2}.$$

All carry the same current, I (r.m.s. amperes).

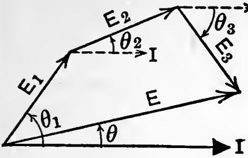


Fig. 14.

Component of E in phase with I $= m_1E_1 + m_2E_2 + m_3E_3 = E_R$.

Component of E at 90° to I $= n_1E_1 + n_2E_2 + n_3E_3 = E_X$.

If I lags, nE is positive; if I leads E, then nE is negative.

Total voltage $= E = \sqrt{E_R^2 + E_X^2}$.

Total power factor $= m = \cos\theta = \dfrac{E_R}{E}$.

Total reactive factor $= n = \sin\theta = \dfrac{E_X}{E}$.

Total power $= P = P_1 + P_2 + P_3 = m_1E_1I + m_2E_2I + m_3E_3I = mEI$.

Total reactive volt-amperes $= n_1E_1I + n_2E_2I + n_3E_3I = nEI$.

Total apparent power $= \sqrt{(mEI)^2 + (nEI)^2}$
$= EI$ (volt-amperes).

Parallel Circuits carrying simple harmonic *Alternating Current.*

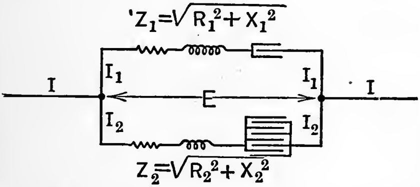


Fig 15

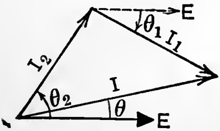


Fig. 16.

Component of I_1 in phase with E equals

$$I_1 \times \frac{R_1}{Z_1} = \frac{R_1}{Z_1^2} \times E = g_1 E.$$

Component of I_2 in phase with E equals

$$I_2 \times \frac{R_2}{Z_2} = \frac{R_2}{Z_2^2} \times E = g_2 E.$$

Component of I in phase with E equals

$$(g_1 + g_2)\, E = gE.$$

Component of I_1 at 90° to E equals

$$I_1 \times \frac{X_1}{Z_1} = \frac{X_1}{Z_1^2} \times E = b_1 E.$$

Component of I_2 at 90° to E equals

$$I_2 \times \frac{X_2}{Z_2} = \frac{X_2}{Z_2^2} \times E = b_2 E.$$

Component of I at 90° to E equals

$$(b_1 + b_2)\, E = bE.$$

When the current leads, b is considered as positive, and when the current lags b is negative,

$$I = \sqrt{(gE)^2 + (bE)^2} = E\sqrt{g^2 + b^2} = yE.$$

Equivalent impedance

$$= Z = \frac{E}{I} = \frac{1}{y} = \frac{1}{\sqrt{g^2 + b^2}}.$$

Equivalent resistance

$$= Z \cdot \frac{g}{y} = \frac{g}{g^2 + b^2} = R_{\text{eq.}}$$

Equivalent reactance

$$= Z \cdot \frac{b}{y} = \frac{b}{g^2 + b^2} = X_{\text{eq.}}$$

Instead of a simple combination of R, L and C, path No. 1 may be an induction motor taking I_1 amperes at E volts with power factor $m_1 = \frac{R_1}{Z_1}$, reactive factor $n_1 = \frac{X_1}{Z_1}$ (lagging); while path No. 2 may be an over-excited synchronous motor taking I_2 amperes at E volts with power

factor $m_2 = \frac{R_2}{Z_2}$, reactive factor $n_2 = \frac{X_2}{Z_2}$ (leading). In this case b_1 and b_2 would have opposite signs but inasmuch as both paths take in power, g will have the same sign in both cases.

$$\begin{aligned}\text{Total power} &= P = P_1 + P_2 = I_1^2R_1 + I_2^2R^2 = I^2Req.\\ &= m_1EI_1 + m_2EI_2 = g_1E^2 + g_2E^2\\ &= mEI = gE^2.\end{aligned}$$

Total reactive volt-amperes

$$\begin{aligned}&= I_1^2X_1 + I_2^2X_2 = I^2Xeq.\\ &= n_1EI_1 + n_2EI_2 = b_1E^2 + b_2E^2\\ &= nEI = bE^2.\end{aligned}$$

$$\begin{aligned}\text{Total apparent power} &= \sqrt{(mEI)^2 + (nEI)^2}\\ &= E^2\sqrt{g^2 + b^2}\\ &= EI \text{ (volt-amperes)}.\end{aligned}$$

$$\text{Total power factor} = m = \frac{m_1I_1 + m_2I_2}{I},$$

$$m = \frac{gE}{I} = \frac{g}{\sqrt{g^2 + b^2}}.$$

Conductance, g, mhos.
Susceptance, b, mhos.
Admittance, y, mhos.

The significance and use of these three quantities, and the relation of each to R and X in either series or parallel circuits, should be evident from the preceding examples.

THREE-PHASE CIRCUITS.

(*a*) *Star or Wye Connection.*

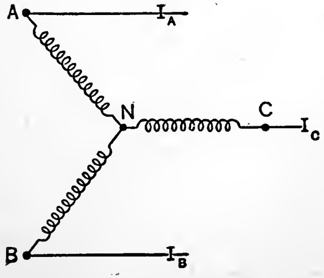


Fig. 17.

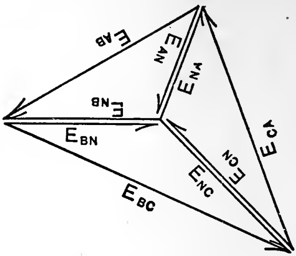


Fig. 18.

For balanced or unbalanced condition:

NOTE. — The dots indicate that vectors, not arithmetic values, are added.

$$\dot{E}_{AB} = \dot{E}_{AN} + \dot{E}_{NB} = -\dot{E}_{NA} + \dot{E}_{NB}.$$
$$\dot{E}_{BC} = \dot{E}_{BN} + \dot{E}_{NC} = -\dot{E}_{NB} + \dot{E}_{NC}.$$
$$\dot{E}_{CA} = \dot{E}_{CN} + \dot{E}_{NA} = -\dot{E}_{NC} + \dot{E}_{NA}.$$
$$I_A = I_{NA}, \qquad I_B = I_{NB}, \qquad I_C = I_{NC}.$$
$$\dot{I}_{NA} + \dot{I}_{NB} + \dot{I}_{NC} = 0.$$

(if no current flows in a neutral connection).

$$\dot{E}_{AB} + \dot{E}_{BC} + \dot{E}_{CA} = 0.$$

$$\text{Total power} = E_{NA}I_{NA}\cos\theta_{NA} + E_{NB}I_{NB}\cos\theta_{NB} + E_{NC}I_{NC}\cos\theta_{NC}.$$

The three phases are "balanced" when

$$I_A = I_B = I_C.$$
$$E_{AB} = E_{BC} = E_{CA} = \sqrt{3}\,E_{NA}.$$
$$E_{NA} = E_{NB} = E_{NC}.$$
$$\theta_{NA} = \theta_{NB} = \theta_{NC}.$$

Phase angle between line voltage and phase voltage is 30° when phases are balanced.

Delta or Mesh Connection.

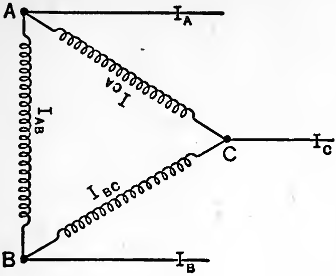


Fig. 19.

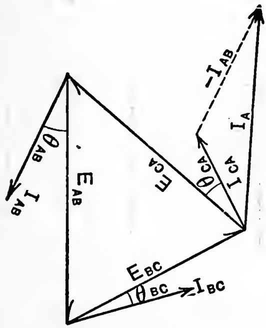


Fig. 20.

For balanced or unbalanced condition:

$$I_A = I_{CA} + I_{BA} = I_{CA} - I_{AB}.$$
$$I_B = I_{AB} + I_{CB} = I_{AB} - I_{BC}.$$
$$I_C = I_{BC} + I_{AC} = I_{BC} - I_{CA}.$$

(For simplicity, only the first of these equations is illustrated in Fig. 20.)

$$E_{AB} + E_{BC} + E_{CA} = 0.$$

$$\text{Total power} = E_{AB} I_{AB} \cos\theta_{AB} + E_{BC} I_{BC} \cos\theta_{BC} + E_{CA} I_{CA} \cos\theta_{CA}$$

When the three phases are balanced, we have also:

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$$I_A = I_B = I_C = \sqrt{3}\, I_{AB}.$$
$$I_{AB} = I_{BC} = I_{CA}.$$
$$\theta_{AB} = \theta_{BC} = \theta_{CA}.$$

Phase angle between line current and phase current is 30° when phases are balanced.

(*c*) *Power in Three-phase Systems.*

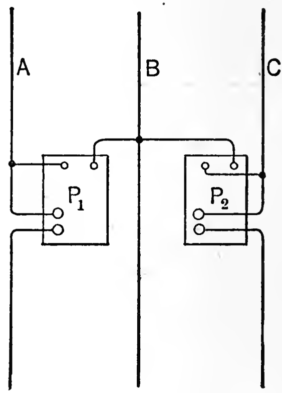

Fig. 21.

Let E_P, I_P and $\cos \theta_P$ be the e.m.f., current and power factor within each of the three phases, which may be connected either in wye or in delta; let E_L be the e.m.f. between line wires, I_L the current in each line wire, and $\cos \theta_L$ the power factor of the entire system. Then, for balanced system,

$$\text{Total power} = 3\, E_P I_P \cos\theta_P = \sqrt{3}\, E_L I_L \cos \theta_L$$

$$\cos \theta_L = \frac{\text{total watts}}{\text{total volt-amperes}} = \cos \theta_P.$$

In the balanced system, the total power is unvarying — is the same at every instant.

Fig. 21 shows how to connect two identically similar wattmeters (P_1 and P_2) so that the alge-

braic sum of their indications equals the total power being transmitted over any three-wire system *ABC* (which may be three-phase). This is correct for any power factor, and for either balanced or unbalanced loads. For balanced loads the values P_1 and P_2 are equal at power factor 1.00; one of them becomes zero at power factor 0.50, and becomes negative for power factors lower than 0.50.

Power factor may be calculated from the wattmeter readings if load is balanced, as follows:

$$\cos\theta = \frac{P_1 + P_2}{2\sqrt{P_1^2 - P_1P_2 + P_2^2}}.$$

TRANSFORMERS: VOLTAGE AND CURRENT RATIOS

If practically all flux links both primary and secondary coils, as is usually the case in "constant voltage transformers," the ratio of primary turns in series to secondary turns in series is equal to the ratio of e.m.f. between primary terminals to e.m.f. between secondary terminals at zero load, or is equal to the inverse of the ratio of primary load current to secondary load current.

TRANSFORMERS: VOLTAGE REGULATION

With low-tension coils short-circuited, measure the "impedance volts" ($E_Z = ZI$) necessary to impress upon high-tension winding to produce full-load current I_H in high-tension circuit, and measure also the (total) "impedance watts" P_{SC} then being supplied to the transformer. Then

$$(XI) = \sqrt{E_Z^2 - (RI)^2}, \qquad \text{and} \qquad (RI) = \frac{P_{SC}}{I_H}$$

TAB-LES

from which we draw the following diagram as for a simple series circuit:

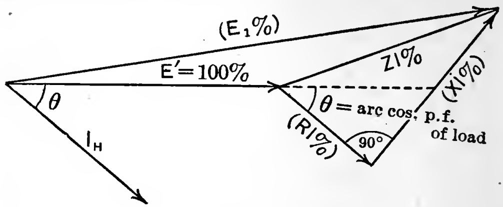

Fig. 22.

wherein E' (= 100% of itself) represents rated high-tension voltage, cos θ is the power factor of load between secondary terminals, ($RI\%$) and ($XI\%$) designate the e.m.f.'s RI and XI referred to above but expressed now as percentages of the rated high-tension voltage. From this diagram it follows that $(E_1\%)^2 = (100 \cos\theta + RI\%)^2 + (100\sqrt{1-(\cos\theta)^2} + XI\%)^2$

Per cent voltage regulation of transformer = $E_1\% - 100$, wherein ($E_1\%$) represents e.m.f. necessary to impress upon high-tension coils, as per cent of rated $h-t$ e.m.f.

Transformers in parallel are treated as impedances in parallel, since E' must be the same for all that are paralleled, as also E_1 and ZI.

DISTRIBUTING LINES AND SHORT TRANSMISSIONS: REGULATION

The same diagram and equations given above for transformers should serve also for calculating voltage regulation of short transmissions (where the distribution of capacitance, inductance and leakance need not be considered). In this case E' represents the voltage required to be delivered at load end, I_H is total current in line to load, ($RI\%$) is the resistance drop as percentage of load voltage E', ($XI\%$) is the reactance drop

as per cent, $\cos \theta$ is the power factor of the load, $(E_1\%)$ is the e.m.f. necessary to impress at input end of line as per cent of load voltage. At zero load E' changes so as to equal E_1, therefore the change of load voltage is the algebraic difference between E' and E_1.

Wire Table for Round Wires.

Gauge Number, Brown & Sharpe.	Diameter in Mils.	Guage Number, Brown & Sharpe.	Diameter In Mils.
0000	460.0	19	36.0
000	410.0	20	32.0
00	365.0	21	28.5
0	325.0	22	25.3
1	289.0	23	22.6
2	258.0	24	20.1
3	229.0	25	17.9
4	204.0	26	15.9
5	182.0	27	14.2
6	162.0	28	12.6
7	144.0	29	11.3
8	128.0	30	10.0
9	114.0	31	8.9
10	102.0	32	8.0
11	91.0	33	7.1
12	81.0	34	6.3
13	72.0	35	5.6
14	64.0	36	5.0
15	57.0	37	4.5
16	51.0	38	4.0
17	45.0	39	3.5
18	40.0	40	3.1

1 mil = 0.001 inch.

Magnetization Curves for Electrical Steels.

Kilo-Maxwells per Square Inch.	Ampere-turns per Inch Length of Magnetic Circuit.	
	Sheets.	Castings.
10	1.32	....
20	1.66	....
30	2.03	8.82
40	2.48	11.4
50	3.07	14.5
60	3.97	18.5
70	5.50	24.3
80	8.30	36.0
90	15.30	56.7
100	40.00	97.0
110	135.00	182.00
120	336.00	370.00
130	1050.00	1100.00

Data from Pender's Handbook for Electrical Engineers.

Hysteresis Loss, Watts per Pound, at 60 Cycles when B_m = 10,000 Gausses.

Metal.	Range of Values.	
	From	To
Silicon Steel Annealed Sheets..................	0.55	1.36
Ordinary Electrical Sheets, Annealed...............	0.84	3.5
Soft Cast Steel...........	2.7	11.00
Cast Iron................	10.00	14.00
Forged Steel..............	13.00	22.00

Specific gravity:

Ordinary Electrical Sheets = 7.7.

Silicon Steel = 7.5.

Data from Pender's Handbook for Electrical Engineers.

Eddy-current Loss, Watts per Pound, for Sheets 0.0141 Inch Thick, at 60 Cycles when B_m = 10,000 Gausses.

Kind of Sheets.	Range of Values.		
	From	To	Average.
Silicon Steel.........	0.12	0.27	0.18
Ordinary Electrical..	0.34	0.70	0.608

Data from Pender's Handbook for Electrical Engineers.

Dielectric Constants.

Substance.	Value of k.
Air...........................	1.00
Glass.........................	5.5 to 10.0
Rubber........................	2.0 to 4.0
Gutta Percha..................	2.9
Mica..........................	2.5 to 5.9
Paper.........................	1.7 to 4.0
Oil...........................	2.0 to 2.5
Paraffin......................	1.9 to 2.3
Shellac.......................	2.7 to 3.8

Data from Standard Handbook for Electrical Engineers.

Resistivity (ρ_0) at 0° Cent., in Ohms Between Opposite Faces of a Centimeter Cube.

Conductors.		Insulators.	
Copper, annealed	1.589×10^{-6}	Rubber	$10^{14} \times (1 \text{ to } 40)$
Copper, hard drawn	1.60×10^{-6}	Porcelain	6×10^{9}
Aluminum, wire	2.607×10^{-6}	Mica	$10^{13} \times (5 \text{ to } 10)$
Soft steel	$10^{-6} \times (11.8 \text{ to } 15.9)$	Petroleum	2×10^{16}
Hard steel	45.6×10^{-6}	Paraffin	$10^{16} \times (3 \text{ to } 300)$
Nichrome	$10^{-6} \times (98.7 \text{ to } 109.2)$	Fiber	$10^{7} \times (2 \text{ to } 10)$
Tungsten	$10^{-6} \times (4.37 \text{ to } 5.42)$	Slate	$10^{8} \times (2 \text{ to } 4)$
German silver, 18% Ni	33.1×10^{-6}	Varnish	2×10^{12}
Brass	$10^{-6} \times (3.6 \text{ to } 6.3)$	Wood (dry)	$10^{8} \times (5 \text{ to } 10)$
Climax	87.1×10^{-6}	Glass	9×10^{13}
Carbon	$10^{-6} \times (400 \text{ to } 4000)$	Asbestos	2×10^{11}

10^{-6} ohm = 1 microhm. 10^{6} ohm = 1 megohm.

TABLES

N	0	1	2	3	4	D
100	00000	00043	00087	00130	00173	**43**
1	0432	0475	0518	0561	0604	**43**
2	0860	0903	0945	0988	1030	**42**
3	1284	1326	1368	1410	1452	**42**
4	1703	1745	1787	1828	1870	**42**
5	02119	02160	02202	02243	02284	**41**
6	2531	2572	2612	2653	2694	**41**
7	2938	2979	3019	3060	3100	**40**
8	3342	3383	3423	3463	3503	**40**
9	3743	3782	3822	3862	3902	**40**
110	04139	04179	04218	04258	04297	**39**
1	4532	4571	4610	4650	4689	**39**
2	4922	4961	4999	5038	5077	**39**
3	5308	5346	5385	5423	5461	**38**
4	5690	5729	5767	5805	5843	**38**
5	06070	06108	06145	06183	06221	**38**
6	6446	6483	6521	6558	6595	**37**
7	6819	6856	6893	6930	6967	**37**
8	7188	7225	7262	7298	7335	**37**
9	7555	7591	7628	7664	7700	**36**
120	07918	07954	07990	08027	08063	**36**
1	8279	8314	8350	8386	8422	**36**
2	8636	8672	8707	8743	8778	**35**
3	8991	9026	9061	9096	9132	**35**
4	9342	9377	9412	9447	9482	**35**
5	09691	09726	09760	09795	09830	**35**
6	10037	10072	10106	10140	10175	**34**
7	0380	0415	0449	0483	0517	**34**
8	0721	0755	0789	0823	0857	**34**
9	1059	1093	1126	1160	1193	**33**
130	11394	11428	11461	11494	11528	**33**
1	1727	1760	1793	1826	1860	**33**
2	2057	2090	2123	2156	2189	**33**
3	2385	2418	2450	2483	2516	**32**
4	2710	2743	2775	2808	2840	**32**
5	13033	13066	13098	13130	13162	**32**
6	3354	3386	3418	3450	3481	**32**
7	3672	3704	3735	3767	3799	**32**
8	3988	4019	4051	4082	4114	**31**
9	4301	4333	4364	4395	4426	**31**
140	14613	14644	14675	14706	14737	**31**
1	4922	4953	4983	5014	5045	**31**
2	5229	5259	5290	5320	5351	**30**
3	5534	5564	5594	5625	5655	**30**
4	5836	5866	5897	5927	5957	**30**
5	16137	16167	16197	16227	16256	**30**
6	6435	6465	6495	6524	6554	**30**
7	6732	6761	6791	6820	6850	**29**
8	7026	7056	7085	7114	7143	**29**
9	7319	7348	7377	7406	7435	**29**
150	17609	17638	17667	17696	17725	**29**
1	7898	7926	7955	7984	8013	**29**
2	8184	8213	8241	8270	8298	**28**
3	8469	8498	8526	8554	8583	**28**
4	8752	8780	8808	8837	8865	**28**
5	19033	19061	19089	19117	19145	**28**
6	9312	9340	9368	9396	9424	**28**
7	9590	9618	9645	9673	9700	**28**
8	9866	9893	9921	9948	9976	**27**
9	20140	20167	20194	20222	20249	**27**

N	5	6	7	8	9	D
100	00217	00260	00303	00346	00389	43
1	0647	0689	0732	0775	0817	43
2	1072	1115	1157	1199	1242	42
3	1494	1536	1578	1620	1662	42
4	1912	1953	1995	2036	2078	42
5	02325	02366	02407	02449	02490	41
6	2735	2776	2816	2857	2898	41
7	3141	3181	3222	3262	3302	40
8	3543	3583	3623	3663	3703	40
9	3941	3981	4021	4060	4100	40
110	04336	04376	04415	04454	04493	39
1	4727	4766	4805	4844	4883	39
2	5115	5154	5192	5231	5269	39
3	5500	5538	5576	5614	5652	38
4	5881	5918	5956	5994	6032	38
5	06258	06296	06333	06371	06408	38
6	6633	6670	6707	6744	6781	37
7	7004	7041	7078	7115	7151	37
8	7372	7408	7445	7482	7518	37
9	7737	7773	7809	7846	7882	36
120	08099	08135	08171	08207	08243	36
1	8458	8493	8529	8565	8600	36
2	8814	8849	8884	8920	8955	35
3	9167	9202	9237	9272	9307	35
4	9517	9552	9587	9621	9656	35
5	09864	09899	09934	09968	10003	35
6	10209	10243	10278	10312	0346	34
7	0551	0585	0619	0653	0687	34
8	0890	0924	0958	0992	1025	34
9	1227	1261	1294	1327	1361	33
130	11561	11594	11628	11661	11694	33
1	1893	1926	1959	1992	2024	33
2	2222	2254	2287	2320	2352	33
3	2548	2581	2613	2646	2678	32
4	2872	2905	2937	2969	3001	32
5	13194	13226	13258	13290	13322	32
6	3513	3545	3577	3609	3640	32
7	3830	3862	3893	3925	3956	32
8	4145	4176	4208	4239	4270	31
9	4457	4489	4520	4551	4582	31
140	14768	14799	14829	14860	14891	31
1	5076	5106	5137	5168	5198	31
2	5381	5412	5442	5473	5503	30
3	5685	5715	5746	5776	5806	30
4	5987	6017	6047	6077	6107	30
5	16286	16316	16346	16376	16406	30
6	6584	6613	6643	6673	6702	30
7	6879	6909	6938	6967	6997	29
8	7173	7202	7231	7260	7289	29
9	7464	7493	7522	7551	7580	29
150	17754	17782	17811	17840	17869	29
1	8041	8070	8099	8127	8156	29
2	8327	8355	8384	8412	8441	28
3	8611	8639	8667	8696	8724	28
4	8893	8921	8949	8977	9005	28
5	19173	19201	19229	19257	19285	28
6	9451	9479	9507	9535	9562	28
7	9728	9756	9783	9811	9838	28
8	20003	20030	20058	20085	20112	27
9	0276	0303	0330	0358	0385	27

N	0	1	2	3	4	D
160	20412	20439	20466	20493	20520	27
1	0683	0710	0737	0763	0790	27
2	0952	0978	1005	1032	1059	27
3	1219	1245	1272	1299	1325	26
4	1484	1511	1537	1564	1590	26
5	21748	21775	21801	21827	21854	26
6	2011	2037	2063	2089	2115	26
7	2272	2298	2324	2350	2376	26
8	2531	2557	2583	2608	2634	26
9	2789	2814	2840	2866	2891	26
170	23045	23070	23096	23121	23147	25
1	3300	3325	3350	3376	3401	25
2	3553	3578	3603	3629	3654	25
3	3805	3830	3855	3880	3905	25
4	4055	4080	4105	4130	4155	25
5	24304	24329	24353	24378	24403	25
6	4551	4576	4601	4625	4650	25
7	4797	4822	4846	4871	4895	24
8	5042	5066	5091	5115	5139	24
9	5285	5310	5334	5358	5382	24
180	25527	25551	25575	25600	25624	24
1	5768	5792	5816	5840	5864	24
2	6007	6031	6055	6079	6102	24
3	6245	6269	6293	6316	6340	24
4	6482	6505	6529	6553	6576	24
5	26717	26741	26764	26788	26811	23
6	6951	6975	6998	7021	7045	23
7	7184	7207	7231	7254	7277	23
8	7416	7439	7462	7485	7508	23
9	7646	7669	7692	7715	7738	23
190	27875	27898	27921	27944	27967	23
1	8103	8126	8149	8171	8194	23
2	8330	8353	8375	8398	8421	23
3	8556	8578	8601	8623	8646	22
4	8780	8803	8825	8847	8870	22
5	29003	29026	29048	29070	29092	22
6	9226	9248	9270	9292	9314	22
7	9447	9469	9491	9513	9535	22
8	9667	9688	9710	.9732	9754	22
9	9885	9907	9929	9951	9973	22
200	30103	30125	30146	30168	30190	22
1	0320	0341	0363	0384	0406	22
2	0535	0557	0578	0600	0621	21
3	0750	0771	0792	0814	0835	21
4	0963	0984	1006	1027	1048	21
5	31175	31197	31218	31239	31260	21
6	1387	1408	1429	1450	1471	21
7	1597	1618	1639	1660	1681	21
8	1806	1827	1848	1869	1890	21
9	2015	2035	2056	2077	2098	21
210	32222	32243	32263	32284	32305	21
1	2428	2449	2469	2490	2510	21
2	2634	2654	2675	2695	2715	20
3	2838	2858	2879	2899	2919	20
4	3041	3062	3082	3102	3122	20
5	33244	33264	33284	33304	33325	20
6	3445	3465	3486	3506	3526	20
7	3646	3666	3686	3706	3726	20
8	3846	3866	3885	3905	3925	20
9	4044	4064	4084	4104	4124	20

N	5	6	7	8	9	D
160	20548	20575	20602	20629	20656	27
1	0817	0844	0871	0898	0925	27
2	1085	1112	1139	1165	1192	27
3	1352	1378	1405	1431	1458	26
4	1617	1643	1669	1696	1722	26
5	21880	21906	21932	21958	21985	26
6	2141	2167	2194	2220	2246	26
7	2401	2427	2453	2479	2505	26
8	2660	2686	2712	2737	2763	26
9	2917	2943	2968	2994	3019	26
170	23172	23198	23223	23249	23274	25
1	3426	3452	3477	3502	3528	25
2	3679	3704	3729	3754	3779	25
3	3930	3955	3980	4005	4030	25
4	4180	4204	4229	4254	4279	25
5	24428	24452	24477	24502	24527	25
6	4674	4699	4724	4748	4773	25
7	4920	4944	4969	4993	5018	24
8	5164	5188	5212	5237	5261	24
9	5406	5431	5455	5479	5503	24
180	25648	25672	25696	25720	25744	24
1	5888	5912	5935	5959	5983	24
2	6126	6150	6174	6198	6221	24
3	6364	6387	6411	6435	6458	24
4	6600	6623	6647	6670	6694	24
5	26834	26858	26881	26905	26928	23
6	7068	7091	7114	7138	7161	23
7	7300	7323	7346	7370	7393	23
8	7531	7554	7577	7600	7623	23
9	7761	7784	7807	7830	7852	23
190	27989	28012	28035	28058	28081	23
1	8217	8240	8262	8285	8307	23
2	8443	8466	8488	8511	8533	23
3	8668	8691	8713	8735	8758	22
4	8892	8914	8937	8959	8981	22
5	29115	29137	29159	29181	29203	22
6	9336	9358	9380	9403	9425	22
7	9557	9579	9601	9623	9645	22
8	9776	9798	9820	9842	9863	22
9	9994	30016	30038	30060	30081	22
200	30211	30233	30255	30276	30298	22
1	0428	0449	0471	0492	0514	22
2	0643	0664	0685	0707	0728	21
3	0856	0878	0899	0920	0942	21
4	1069	1091	1112	1133	1154	21
5	31281	31302	31323	31345	31366	21
6	1492	1513	1534	1555	1576	21
7	1702	1723	1744	1765	1785	21
8	1911	1931	1952	1973	1994	21
9	2118	2139	2160	2181	2201	21
210	32325	32346	32366	32387	32408	21
1	2531	2552	2572	2593	2613	21
2	2736	2756	2777	2797	2818	20
3	2940	2960	2980	3001	3021	20
4	3143	3163	3183	3203	3224	20
5	33345	33365	33385	33405	33425	20
6	3546	3566	3586	3606	3626	20
7	3746	3766	3786	3806	3826	20
8	3945	3965	3985	4005	4025	20
9	4143	4163	4183	4203	4223	20

N	0	1	2	3	4	D
220	34242	34262	34282	34301	34321	20
1	4439	4459	4479	4498	4518	20
2	4635	4655	4674	4694	4713	20
3	4830	4850	4869	4889	4908	19
4	5025	5044	5064	5083	5102	19
5	35218	35238	35257	35276	35295	19
6	5411	5430	5449	5468	5488	19
7	5603	5622	5641	5660	5679	19
8	5793	5813	5832	5851	5870	19
9	5984	6003	6021	6040	6059	19
230	36173	36192	36211	36229	36248	19
1	6361	6380	6399	6418	6436	19
2	6549	6568	6586	6605	6624	19
3	6736	6754	6773	6791	6810	19
4	6922	6940	6959	6977	6996	18
5	37107	37125	37144	37162	37181	18
6	7291	7310	7328	7346	7365	18
7	7475	7493	7511	7530	7548	18
8	7658	7676	7694	7712	7731	18
9	7840	7858	7876	7894	7912	18
240	38021	38039	38057	38075	38093	18
1	8202	8220	8238	8256	8274	18
2	8382	8399	8417	8435	8453	18
3	8561	8578	8596	8614	8632	18
4	8739	8757	8775	8792	8810	18
5	38917	38934	38952	38970	38987	18
6	9094	9111	9129	9146	9164	18
7	9270	9287	9305	9322	9340	18
8	9445	9463	9480	9498	9515	17
9	9620	9637	9655	9672	9690	17
250	39794	39811	39829	39846	39863	17
1	9967	9985	40002	40019	40037	17
2	40140	40157	0175	0192	0209	17
3	0312	0329	0346	0364	0381	17
4	0483	0500	0518	0535	0552	17
5	40654	40671	40688	40705	40722	17
6	0824	0841	0858	0875	0892	17
7	0993	1010	1027	1044	1061	17
8	1162	1179	1196	1212	1229	17
9	1330	1347	1363	1380	1397	17
260	41497	41514	41531	41547	41564	17
1	1664	1681	1697	1714	1731	17
2	1830	1847	1863	1880	1896	17
3	1996	2012	2029	2045	2062	16
4	2160	2177	2193	2210	2226	16
5	42325	42341	42357	42374	42390	16
6	2488	2504	2521	2537	2553	16
7	2651	2667	2684	2700	2716	16
8	2813	2830	2846	2862	2878	16
9	2975	2991	3008	3024	3040	16
270	43136	43152	43169	43185	43201	16
1	3297	3313	3329	3345	3361	16
2	3457	3473	3489	3505	3521	16
3	3616	3632	3648	3664	3680	16
4	3775	3791	3807	3823	3838	16
5	43933	43949	43965	43981	43996	16
6	4091	4107	4122	4138	4154	16
7	4248	4264	4279	4295	4311	16
8	4404	4420	4436	4451	4467	16
9	4560	4576	4592	4607	4623	16

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220	34341	34361	34380	34400	34420	20
1	4537	4557	4577	4596	4616	20
2	4733	4753	4772	4792	4811	20
3	4928	4947	4967	4986	5005	19
4	5122	5141	5160	5180	5199	19
5	35315	35334	35353	35372	35392	19
6	5507	5526	5545	5564	5583	19
7	5698	5717	5736	5755	5774	19
8	5889	5908	5927	5946	5965	19
9	6078	6097	6116	6135	6154	19
230	36267	36286	36305	36324	36342	19
1	6455	6474	6493	6511	6530	19
2	6642	6661	6680	6698	6717	19
3	6829	6847	6866	6884	6903	19
4	7014	7033	7051	7070	7088	18
5	37199	37218	37236	37254	37273	18
6	7383	7401	7420	7438	7457	18
7	7566	7585	7603	7621	7639	18
8	7749	7767	7785	7803	7822	18
9	7931	7949	7967	7985	8003	18
240	38112	38130	38148	38166	38184	18
1	8292	8310	8328	8346	8364	18
2	8471	8489	8507	8525	8543	18
3	8650	8668	8686	8703	8721	18
4	8828	8846	8863	8881	8899	18
5	39005	39023	39041	39058	39076	18
6	9182	9199	9217	9235	9252	18
7	9358	9375	9393	9410	9428	18
8	9533	9550	9568	9585	9602	17
9	9707	9724	9742	9759	9777	17
250	39881	39898	39915	39933	39950	17
1	40054	40071	40088	40106	40123	17
2	0226	0243	0261	0278	0295	17
3	0398	0415	0432	0449	0466	17
4	0569	0586	0603	0620	0637	17
5	40739	40756	40773	40790	40807	17
6	0909	0926	0943	0960	0976	17
7	1078	1095	1111	1128	1145	17
8	1246	1263	1280	1296	1313	17
9	1414	1430	1447	1464	1481	17
260	41581	41597	41614	41631	41647	17
1	1747	1764	1780	1797	1814	17
2	1913	1929	1946	1963	1979	17
3	2078	2095	2111	2127	2144	16
4	2243	2259	2275	2292	2308	16
5	42406	42423	42439	42455	42472	16
6	2570	2586	2602	2619	2635	16
7	2732	2749	2765	2781	2797	16
8	2894	2911	2927	2943	2959	16
9	3056	3072	3088	3104	3120	16
270	43217	43233	43249	43265	43281	16
1	3377	3393	3409	3425	3441	16
2	3537	3553	3569	3584	3600	16
3	3696	3712	3727	3743	3759	16
4	3854	3870	3886	3902	3917	16
5	44012	44028	44044	44059	44075	16
6	4170	4185	4201	4217	4232	16
7	4326	4342	4358	4373	4389	16
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280	44716	44731	44747	44762	44778	15
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2	5025	5040	5056	5071	5086	15
3	5179	5194	5209	5225	5240	15
4	5332	5347	5362	5378	5393	15
5	45484	45500	45515	45530	45545	15
6	5637	5652	5667	5682	5697	15
7	5788	5803	5818	5834	5849	15
8	5939	5954	5969	5984	6000	15
9	6090	6105	6120	6135	6150	15
290	46240	46255	46270	46285	46300	15
1	6389	6404	6419	6434	6449	15
2	6538	6553	6568	6583	6598	15
3	6687	6702	6716	6731	6746	15
4	6835	6850	6864	6879	6894	15
5	46982	46997	47012	47026	47041	15
6	7129	7144	7159	7173	7188	15
7	7276	7290	7305	7319	7334	15
8	7422	7436	7451	7465	7480	15
9	7567	7582	7596	7611	7625	14
300	47712	47727	47741	47756	47770	14
1	7857	7871	7885	7900	7914	14
2	8001	8015	8029	8044	8058	14
3	8144	8159	8173	8187	8202	14
4	8287	8302	8316	8330	8344	14
5	48430	48444	48458	48473	48487	14
6	8572	8586	8601	8615	8629	14
7	8714	8728	8742	8756	8770	14
8	8855	8869	8883	8897	8911	14
9	8996	9010	9024	9038	9052	14
310	49136	49150	49164	49178	49192	14
1	9276	9290	9304	9318	9332	14
2	9415	9429	9443	9457	9471	14
3	9554	9568	9582	9596	9610	14
4	9693	9707	9721	9734	9748	14
5	49831	49845	49859	49872	49886	14
6	9969	9982	9996	50010	50024	14
7	50106	50120	50133	0147	0161	14
8	0243	0256	0270	0284	0297	14
9	0379	0393	0406	0420	0433	14
320	50515	50529	50542	50556	50569	14
1	0651	0664	0678	0691	0705	13
2	0786	0799	0813	0826	0840	13
3	0920	0934	0947	0961	0974	13
4	1055	1068	1081	1095	1108	13
5	51188	51202	51215	51228	51242	13
6	1322	1335	1348	1362	1375	13
7	1455	1468	1481	1495	1508	13
8	1587	1601	1614	1627	1640	13
9	1720	1733	1746	1759	1772	13
330	51851	51865	51878	51891	51904	13
1	1983	1996	2009	2022	2035	13
2	2114	2127	2140	2153	2166	13
3	2244	2257	2270	2284	2297	13
4	2375	2388	2401	2414	2427	13
5	52504	52517	52530	52543	52556	13
6	2634	2647	2660	2673	2686	13
7	2763	2776	2789	2802	2815	13
8	2892	2905	2917	2930	2943	13
9	3020	3033	3046	3058	3071	13

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280	44793	44809	44824	44840	44855	15
1	4948	4963	4979	4994	5010	15
2	5102	5117	5133	5148	5163	15
3	5255	5271	5286	5301	5317	15
4	5408	5423	5439	5454	5469	15
5	45561	45576	45591	45606	45621	15
6	5712	5728	5743	5758	5773	15
7	5864	5879	5894	5909	5924	15
8	6015	6030	6045	6060	6075	15
9	6165	6180	6195	6210	6225	15
290	46315	46330	46345	46359	46374	15
1	6464	6479	6494	6509	6523	15
2	6613	6627	6642	6657	6672	15
3	6761	6776	6790	6805	6820	15
4	6909	6923	6938	6953	6967	15
5	47056	47070	47085	47100	47114	15
6	7202	7217	7232	7246	7261	15
7	7349	7363	7378	7392	7407	15
8	7494	7509	7524	7538	7553	15
9	7640	7654	7669	7683	7698	14
300	47784	47799	47813	47828	47842	14
1	7929	7943	7958	7972	7986	14
2	8073	8087	8101	8116	8130	14
3	8216	8230	8244	8259	8273	14
4	8359	8373	8387	8401	8416	14
5	48501	48515	48530	48544	48558	14
6	8643	8657	8671	8686	8700	14
7	8785	8799	8813	8827	8841	14
8	8926	8940	8954	8968	8982	14
9	9066	9080	9094	9108	9122	14
310	49206	49220	49234	49248	49262	14
1	9346	9360	9374	9388	9402	14
2	9485	9499	9513	9527	9541	14
3	9624	9638	9651	9665	9679	14
4	9762	9776	9790	9803	9817	14
5	49900	49914	49927	49941	49955	14
6	50037	50051	50065	50079	50092	14
7	0174	0188	0202	0215	0229	14
8	0311	0325	0338	0352	0365	14
9	0447	0461	0474	0488	0501	14
320	50583	50596	50610	50623	50637	14
1	0718	0732	0745	0759	0772	13
2	0853	0866	0880	0893	0907	13
3	0987	1001	1014	1028	1041	13
4	1121	1135	1148	1162	1175	13
5	51255	51268	51282	51295	51308	13
6	1388	1402	1415	1428	1441	13
7	1521	1534	1548	1561	1574	13
8	1654	1667	1680	1693	1706	13
9	1786	1799	1812	1825	1838	13
330	51917	51930	51943	51957	51970	13
1	2048	2061	2075	2088	2101	13
2	2179	2192	2205	2218	2231	13
3	2310	2323	2336	2349	2362	13
4	2440	2453	2466	2479	2492	13
5	52569	52582	52595	52608	52621	13
6	2699	2711	2724	2737	2750	13
7	2827	2840	2853	2866	2879	13
8	2956	2969	2982	2994	3007	13
9	3084	3097	3110	3122	3135	13

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340	53148	53161	53173	53186	53199	13
1	3275	3288	3301	3314	3326	13
2	3403	3415	3428	3441	3453	13
3	3529	3542	3555	3567	3580	13
4	3656	3668	3681	3694	3706	13
5	53782	53794	53807	53820	53832	13
6	3908	3920	3933	3945	3958	13
7	4033	4045	4058	4070	4083	12
8	4158	4170	4183	4195	4208	12
9	4283	4295	4307	4320	4332	12
350	54407	54419	54432	54444	54456	12
1	4531	4543	4555	4568	4580	12
2	4654	4667	4679	4691	4704	12
3	4777	4790	4802	4814	4827	12
4	4900	4913	4925	4937	4949	12
5	55023	55035	55047	55060	55072	12
6	5145	5157	5169	5182	5194	12
7	5267	5279	5291	5303	5315	12
8	5388	5400	5413	5425	5437	12
9	5509	5522	5534	5546	5558	12
360	55630	55642	55654	55666	55678	12
1	5751	5763	5775	5787	5799	12
2	5871	5883	5895	5907	5919	12
3	5991	6003	6015	6027	6038	12
4	6110	6122	6134	6146	6158	12
5	56229	56241	56253	56265	56277	12
6	6348	6360	6372	6384	6396	12
7	6467	6478	6490	6502	6514	12
8	6585	6597	6608	6620	6632	12
9	6703	6714	6726	6738	6750	12
370	56820	56832	56844	56855	56867	12
1	6937	6949	6961	6972	6984	12
2	7054	7066	7078	7089	7101	12
3	7171	7183	7194	7206	7217	12
4	7287	7299	7310	7322	7334	12
5	57403	57415	57426	57438	57449	12
6	7519	7530	7542	7553	7565	12
7	7634	7646	7657	7669	7680	11
8	7749	7761	7772	7784	7795	11
9	7864	7875	7887	7898	7910	11
380	57978	57990	58001	58013	58024	11
1	8092	8104	8115	8127	8138	11
2	8206	8218	8229	8240	8252	11
3	8320	8331	8343	8354	8365	11
4	8433	8444	8456	8467	8478	11
5	58546	58557	58569	58580	58591	11
6	8659	8670	8681	8692	8704	11
7	8771	8782	8794	8805	8816	11
8	8883	8894	8906	8917	8928	11
9	8995	9006	9017	9028	9040	11
390	59106	59118	59129	59140	59151	11
1	9218	9229	9240	9251	9262	11
2	9329	9340	9351	9362	9373	11
3	9439	9450	9461	9472	9483	11
4	9550	9561	9572	9583	9594	11
5	59660	59671	59682	59693	59704	11
6	9770	9780	9791	9802	9813	11
7	9879	9890	9901	9912	9923	11
8	9988	9999	60010	60021	60032	11
9	60097	60108	60119	60130	60141	11

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340	53212	53224	53237	53250	53263	13
1	3339	3352	3364	3377	3390	13
2	3466	3479	3491	3504	3517	13
3	3593	3605	3618	3631	3643	13
4	3719	3732	3744	3757	3769	13
5	53845	53857	53870	53882	53895	13
6	3970	3983	3995	4008	4020	13
7	4095	4108	4120	4133	4145	12
8	4220	4233	4245	4258	4270	12
9	4345	4357	4370	4382	4394	12
350	54469	54481	54494	54506	54518	12
1	4593	4605	4617	4630	4642	12
2	4716	4728	4741	4753	4765	12
3	4839	4851	4864	4876	4888	12
4	4962	4974	4986	4998	5011	12
5	55084	55096	55108	55121	55133	12
6	5206	5218	5230	5242	5255	12
7	5328	5340	5352	5364	5376	12
8	5449	5461	5473	5485	5497	12
9	5570	5582	5594	5606	5618	12
360	55691	55703	55715	55727	55739	12
1	5811	5823	5835	5847	5859	12
2	5931	5943	5955	5967	5979	12
3	6050	6062	6074	6086	6098	12
4	6170	6182	6194	6205	6217	12
5	56289	56301	56312	56324	56336	12
6	6407	6419	6431	6443	6455	12
7	6526	6538	6549	6561	6573	12
8	6644	6656	6667	6679	6691	12
9	6761	6773	6785	6797	6808	12
370	56879	56891	56902	56914	56926	12
1	6996	7008	7019	7031	7043	12
2	7113	7124	7136	7148	7159	12
3	7229	7241	7252	7264	7276	12
4	7345	7357	7368	7380	7392	12
5	57461	57473	57484	57496	57507	12
6	7576	7588	7600	7611	7623	12
7	7692	7703	7715	7726	7738	11
8	7807	7818	7830	7841	7852	11
9	7921	7933	7944	7955	7967	11
380	58035	58047	58058	58070	58081	11
1	8149	8161	8172	8184	8195	11
2	8263	8274	8286	8297	8309	11
3	8377	8388	8399	8410	8422	11
4	8490	8501	8512	8524	8535	11
5	58602	58614	58625	58636	58647	11
6	8715	8726	8737	8749	8760	11
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8	8939	8950	8961	8973	8984	11
9	9051	9062	9073	9084	9095	11
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1	9273	9284	9295	9306	9318	11
2	9384	9395	9406	9417	9428	11
3	9494	9506	9517	9528	9539	11
4	9605	9616	9627	9638	9649	11
5	59715	59726	59737	59748	59759	11
6	9824	9835	9846	9857	9868	11
7	9934	9945	9956	9966	9977	11
8	60043	60054	60065	60076	60086	11
9	60152	60163	60173	60184	60195	11

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400	60206	60217	60228	60239	60249	11
1	0314	0325	0336	0347	0358	11
2	0423	0433	0444	0455	0466	11
3	0531	0541	0552	0563	0574	11
4	0638	0649	0660	0670	0681	11
5	60746	60756	60767	60778	60788	11
6	0853	0863	0874	0885	0895	11
7	0959	0970	0981	0991	1002	11
8	1066	1077	1087	1098	1109	11
9	1172	1183	1194	1204	1215	11
410	61278	61289	61300	61310	61321	11
1	1384	1395	1405	1416	1426	11
2	1490	1500	1511	1521	1532	11
3	1595	1606	1616	1627	1637	10
4	1700	1711	1721	1731	1742	10
5	61805	61815	61826	61836	61847	10
6	1909	1920	1930	1941	1951	10
7	2014	2024	2034	2045	2055	10
8	2118	2128	2138	2149	2159	10
9	2221	2232	2242	2252	2263	10
420	62325	62335	62346	62356	62366	10
1	2428	2439	2449	2459	2469	10
2	2531	2542	2552	2562	2572	10
3	2634	2644	2655	2665	2675	10
4	2737	2747	2757	2767	2778	10
5	62839	62849	62859	62870	62880	10
6	2941	2951	2961	2972	2982	10
7	3043	3053	3063	3073	3083	10
8	3144	3155	3165	3175	3185	10
9	3246	3256	3266	3276	3286	10
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6	3949	3959	3969	3979	3988	10
7	4048	4058	4068	4078	4088	10
8	4147	4157	4167	4177	4187	10
9	4246	4256	4266	4276	4286	10
440	64345	64355	64365	64375	64385	10
1	4444	4454	4464	4473	4483	10
2	4542	4552	4562	4572	4582	10
3	4640	4650	4660	4670	4680	10
4	4738	4748	4758	4768	4777	10
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6	4933	4943	4953	4963	4972	10
7	5031	5040	5050	5060	5070	10
8	5128	5137	5147	5157	5167	10
9	5225	5234	5244	5254	5263	10
450	65321	65331	65341	65350	65360	10
1	5418	5427	5437	5447	5456	10
2	5514	5523	5533	5543	5552	10
3	5610	5619	5629	5639	5648	10
4	5706	5715	5725	5734	5744	10
5	65801	65811	65820	65830	65839	10
6	5896	5906	5916	5925	5935	10
7	5992	6001	6011	6020	6030	9
8	6087	6096	6106	6115	6124	9
9	6181	6191	6200	6210	6219	9

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400	60260	60271	60282	60293	60304	11
1	0369	0379	0390	0401	0412	11
2	0477	0487	0498	0509	0520	11
3	0584	0595	0606	0617	0627	11
4	0692	0703	0713	0724	0735	11
5	60799	60810	60821	60831	60842	11
6	0906	0917	0927	0938	0949	11
7	1013	1023	1034	1045	1055	11
8	1119	1130	1140	1151	1162	11
9	1225	1236	1247	1257	1268	11
410	61331	61342	61352	61363	61374	11
1	1437	1448	1458	1469	1479	11
2	1542	1553	1563	1574	1584	11
3	1648	1658	1669	1679	1690	10
4	1752	1763	1773	1784	1794	10
5	61857	61868	61878	61888	61899	10
6	1962	1972	1982	1993	2003	10
7	2066	2076	2086	2097	2107	10
8	2170	2180	2190	2201	2211	10
9	2273	2284	2294	2304	2315	10
420	62377	62387	62397	62408	62418	10
1	2480	2490	2500	2511	2521	10
2	2583	2593	2603	2613	2624	10
3	2685	2696	2706	2716	2726	10
4	2788	2798	2808	2818	2829	10
5	62890	62900	62910	62921	62931	10
6	2992	3002	3012	3022	3033	10
7	3094	3104	3114	3124	3134	10
8	3195	3205	3215	3225	3236	10
9	3296	3306	3317	3327	3337	10
430	63397	63407	63417	63428	63438	10
1	3498	3508	3518	3528	3538	10
2	3599	3609	3619	3629	3639	10
3	3699	3709	3719	3729	3739	10
4	3799	3809	3819	3829	3839	10
5	63899	63909	63919	63929	63939	10
6	3998	4008	4018	4028	4038	10
7	4098	4108	4118	4128	4137	10
8	4197	4207	4217	4227	4237	10
9	4296	4306	4316	4326	4335	10
440	64395	64404	64414	64424	64434	10
1	4493	4503	4513	4523	4532	10
2	4591	4601	4611	4621	4631	10
3	4689	4699	4709	4719	4729	10
4	4787	4797	4807	4816	4826	10
5	64885	64895	64904	64914	64924	10
6	4982	4992	5002	5011	5021	10
7	5079	5089	5099	5108	5118	10
8	5176	5186	5196	5205	5215	10
9	5273	5283	5292	5302	5312	10
450	65369	65379	65389	65398	65408	10
1	5466	5475	5485	5495	5504	10
2	5562	5571	5581	5591	5600	10
3	5658	5667	5677	5686	5696	10
4	5753	5763	5772	5782	5792	10
5	65849	65858	65868	65877	65887	10
6	5944	5954	5963	5973	5982	10
7	6039	6049	6058	6068	6077	9
8	6134	6143	6153	6162	6172	9
9	6229	6238	6247	6257	6266	9

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460	66276	66285	66295	66304	66314	9
1	6370	6380	6389	6398	6408	9
2	6464	6474	6483	6492	6502	9
3	6558	6567	6577	6586	6596	9
4	6652	6661	6671	6680	6689	9
5	66745	66755	66764	66773	66783	9
6	6839	6848	6857	6867	6876	9
7	6932	6941	6950	6960	6969	9
8	7025	7034	7043	7052	7062	9
9	7117	7127	7136	7145	7154	9
470	67210	67219	67228	67237	67247	9
1	7302	7311	7321	7330	7339	9
2	7394	7403	7413	7422	7431	9
3	7486	7495	7504	7514	7523	9
4	7578	7587	7596	7605	7614	9
5	67669	67679	67688	67697	67706	9
6	7761	7770	7779	7788	7797	9
7	7852	7861	7870	7879	7888	9
8	7943	7952	7961	7970	7979	9
9	8034	8043	8052	8061	8070	9
480	68124	68133	68142	68151	68160	9
1	8215	8224	8233	8242	8251	9
2	8305	8314	8323	8332	8341	9
3	8395	8404	8413	8422	8431	9
4	8485	8494	8502	8511	8520	9
5	68574	68583	68592	68601	68610	9
6	8664	8673	8681	8690	8699	9
7	8753	8762	8771	8780	8789	9
8	8842	8851	8860	8869	8878	9
9	8931	8940	8949	8958	8966	9
490	69020	69028	69037	69046	69055	9
1	9108	9117	9126	9135	9144	9
2	9197	9205	9214	9223	9232	9
3	9285	9294	9302	9311	9320	9
4	9373	9381	9390	9399	9408	9
5	69461	69469	69478	69487	69496	9
6	9548	9557	9566	9574	9583	9
7	9636	9644	9653	9662	9671	9
8	9723	9732	9740	9749	9758	9
9	9810	9819	9827	9836	9845	9
500	69897	69906	69914	69923	69932	9
1	9984	9992	70001	70010	70018	9
2	70070	70079	0088	0096	0105	9
3	0157	0165	0174	0183	0191	9
4	0243	0252	0260	0269	0278	9
5	70329	70338	70346	70355	70364	9
6	0415	0424	0432	0441	0449	9
7	0501	0509	0518	0526	0535	9
8	0586	0595	0603	0612	0621	9
9	0672	0680	0689	0697	0706	9
510	70757	70766	70774	70783	70791	8
1	0842	0851	0859	0868	0876	8
2	0927	0935	0944	0952	0961	8
3	1012	1020	1029	1037	1046	8
4	1096	1105	1113	1122	1130	8
5	71181	71189	71198	71206	71214	8
6	1265	1273	1282	1290	1299	8
7	1349	1357	1366	1374	1383	8
8	1433	1441	1450	1458	1466	8
9	1517	1525	1533	1542	1550	8

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460	66323	66332	66342	66351	66361	9
1	6417	6427	6436	6445	6455	9
2	6511	6521	6530	6539	6549	9
3	6605	6614	6624	6633	6642	9
4	6699	6708	6717	6727	6736	9
5	66792	66801	66811	66820	66829	9
6	6885	6894	6904	6913	6922	9
7	6978	6987	6997	7006	7015	9
8	7071	7080	7089	7099	7108	9
9	7164	7173	7182	7191	7201	9
470	67256	67265	67274	67284	67293	9
1	7348	7357	7367	7376	7385	9
2	7440	7449	7459	7468	7477	9
3	7532	7541	7550	7560	7569	9
4	7624	7633	7642	7651	7660	9
5	67715	67724	67733	67742	67752	9
6	7806	7815	7825	7834	7843	9
7	7897	7906	7916	7925	7934	9
8	7988	7997	8006	8015	8024	9
9	8079	8088	8097	8106	8115	9
480	68169	68178	68187	68196	68205	9
1	8260	8269	8278	8287	8296	9
2	8350	8359	8368	8377	8386	9
3	8440	8449	8458	8467	8476	9
4	8529	8538	8547	8556	8565	9
5	68619	68628	68637	68646	68655	9
6	8708	8717	8726	8735	8744	9
7	8797	8806	8815	8824	8833	9
8	8886	8895	8904	8913	8922	9
9	8975	8984	8993	9002	9011	9
490	69064	69073	69082	69090	69099	9
1	9152	9161	9170	9179	9188	9
2	9241	9249	9258	9267	9276	9
3	9329	9338	9346	9355	9364	9
4	9417	9425	9434	9443	9452	9
5	69504	69513	69522	69531	69539	9
6	9592	9601	9609	9618	9627	9
7	9679	9688	9697	9705	9714	9
8	9767	9775	9784	9793	9801	9
9	9854	9862	9871	9880	9888	9
500	69940	69949	69958	69966	69975	9
1	70027	70036	70044	70053	70062	9
2	0114	0122	0131	0140	0148	9
3	0200	0209	0217	0226	0234	9
4	0286	0295	0303	0312	0321	9
5	70372	70381	70389	70398	70406	9
6	0458	0467	0475	0484	0492	9
7	0544	0552	0561	0569	0578	9
8	0629	0638	0646	0655	0663	9
9	0714	0723	0731	0740	0749	9
510	70800	70808	70817	70825	70834	8
1	0885	0893	0902	0910	0919	8
2	0969	0978	0986	0995	1003	8
3	1054	1063	1071	1079	1088	8
4	1139	1147	1155	1164	1172	8
5	71223	71231	71240	71248	71257	8
6	1307	1315	1324	1332	1341	8
7	1391	1399	1408	1416	1425	8
8	1475	1483	1492	1500	1508	8
9	1559	1567	1575	1584	1592	8

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520	71600	71609	71617	71625	71634	8
1	1684	1692	1700	1709	1717	8
2	1767	1775	1784	1792	1800	8
3	1850	1858	1867	1875	1883	8
4	1933	1941	1950	1958	1966	8
5	72016	72024	72032	72041	72049	8
6	2099	2107	2115	2123	2132	8
7	2181	2189	2198	2206	2214	8
8	2263	2272	2280	2288	2296	8
9	2346	2354	2362	2370	2378	8
530	72428	72436	72444	72452	72460	8
1	2509	2518	2526	2534	2542	8
2	2591	2599	2607	2616	2624	8
3	2673	2681	2689	2697	2705	8
4	2754	2762	2770	2779	2787	8
5	72835	72843	72852	72860	72868	8
6	2916	2925	2933	2941	2949	8
7	2997	3006	3014	3022	3030	8
8	3078	3086	3094	3102	3111	8
9	3159	3167	3175	3183	3191	8
540	73239	73247	73255	73263	73272	8
1	3320	3328	3336	3344	3352	8
2	3400	3408	3416	3424	3432	8
3	3480	3488	3496	3504	3512	8
4	3560	3568	3576	3584	3592	8
5	73640	73648	73656	73664	73672	8
6	3719	3727	3735	3743	3751	8
7	3799	3807	3815	3823	3830	8
8	3878	3886	3894	3902	3910	8
9	3957	3965	3973	3981	3989	8
550	74036	74044	74052	74060	74068	8
1	4115	4123	4131	4139	4147	8
2	4194	4202	4210	4218	4225	8
3	4273	4280	4288	4296	4304	8
4	4351	4359	4367	4374	4382	8
5	74429	74437	74445	74453	74461	8
6	4507	4515	4523	4531	4539	8
7	4586	4593	4601	4609	4617	8
8	4663	4671	4679	4687	4695	8
9	4741	4749	4757	4764	4772	8
560	74819	74827	74834	74842	74850	8
1	4896	4904	4912	4920	4927	8
2	4974	4981	4989	4997	5005	8
3	5051	5059	5066	5074	5082	8
4	5128	5136	5143	5151	5159	8
5	75205	75213	75220	75228	75236	8
6	5282	5289	5297	5305	5312	8
7	5358	5366	5374	5381	5389	8
8	5435	5442	5450	5458	5465	8
9	5511	5519	5526	5534	5542	8
570	75587	75595	75603	75610	75618	8
1	5664	5671	5679	5686	5694	8
2	5740	5747	5755	5762	5770	8
3	5815	5823	5831	5838	5846	8
4	5891	5899	5906	5914	5921	8
5	75967	75974	75982	75989	75997	8
6	6042	6050	6057	6065	6072	8
7	6118	6125	6133	6140	6148	8
8	6193	6200	6208	6215	6223	7
9	6268	6275	6283	6290	6298	7

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520	71642	71650	71659	71667	71675	8
1	1725	1734	1742	1750	1759	8
2	1809	1817	1825	1834	1842	8
3	1892	1900	1908	1917	1925	8
4	1975	1983	1991	1999	2008	8
5	72057	72066	72074	72082	72090	8
6	2140	2148	2156	2165	2173	8
7	2222	2230	2239	2247	2255	8
8	2304	2313	2321	2329	2337	8
9	2387	2395	2403	2411	2419	8
530	72469	72477	72485	72493	72501	8
1	2550	2558	2567	2575	2583	8
2	2632	2640	2648	2656	2665	8
3	2713	2722	2730	2738	2746	8
4	2795	2803	2811	2819	2827	8
5	72876	72884	72892	72900	72908	8
6	2957	2965	2973	2981	2989	8
7	3038	3046	3054	3062	3070	8
8	3119	3127	3135	3143	3151	8
9	3199	3207	3215	3223	3231	8
540	73280	73288	73296	73304	73312	8
1	3360	3368	3376	3384	3392	8
2	3440	3448	3456	3464	3472	8
3	3520	3528	3536	3544	3552	8
4	3600	3608	3616	3624	3632	8
5	73679	73687	73695	73703	73711	8
6	3759	3767	3775	3783	3791	8
7	3838	3846	3854	3862	3870	8
8	3918	3926	3933	3941	3949	8
9	3997	4005	4013	4020	4028	8
550	74076	74084	74092	74099	74107	8
1	4155	4162	4170	4178	4186	8
2	4233	4241	4249	4257	4265	8
3	4312	4320	4327	4335	4343	8
4	4390	4398	4406	4414	4421	8
5	74468	74476	74484	74492	74500	8
6	4547	4554	4562	4570	4578	8
7	4624	4632	4640	4648	4656	8
8	4702	4710	4718	4726	4733	8
9	4780	4788	4796	4803	4811	8
560	74858	74865	74873	74881	74889	8
1	4935	4943	4950	4958	4966	8
2	5012	5020	5028	5035	5043	8
3	5089	5097	5105	5113	5120	8
4	5166	5174	5182	5189	5197	8
5	75243	75251	75259	75266	75274	8
6	5320	5328	5335	5343	5351	8
7	5397	5404	5412	5420	5427	8
8	5473	5481	5488	5496	5504	8
9	5549	5557	5565	5572	5580	8
570	75626	75633	75641	75648	75656	8
1	5702	5709	5717	5724	5732	8
2	5778	5785	5793	5800	5808	8
3	5853	5861	5868	5876	5884	8
4	5929	5937	5944	5952	5959	8
5	76005	76012	76020	76027	76035	8
6	6080	6087	6095	6103	6110	8
7	6155	6163	6170	6178	6185	8
8	6230	6238	6245	6253	6260	7
9	6305	6313	6320	6328	6335	7

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580	76343	76350	76358	76365	76373	7
1	6418	6425	6433	6440	6448	7
2	6492	6500	6507	6515	6522	7
3	6567	6574	6582	6589	6597	7
4	6641	6649	6656	6664	6671	7
5	76716	76723	76730	76738	76745	7
6	6790	6797	6805	6812	6819	7
7	6864	6871	6879	6886	6893	7
8	6938	6945	6953	6960	6967	7
9	7012	7019	7026	7034	7041	7
590	77085	77093	77100	77107	77115	7
1	7159	7166	7173	7181	7188	7
2	7232	7240	7247	7254	7262	7
3	7305	7313	7320	7327	7335	7
4	7379	7386	7393	7401	7408	7
5	77452	77459	77466	77474	77481	7
6	7525	7532	7539	7546	7554	7
7	7597	7605	7612	7619	7627	7
8	7670	7677	7685	7692	7699	7
9	7743	7750	7757	7764	7772	7
600	77815	77822	77830	77837	77844	7
1	7887	7895	7902	7909	7916	7
2	7960	7967	7974	7981	7988	7
3	8032	8039	8046	8053	8061	7
4	8104	8111	8118	8125	8132	7
5	78176	78183	78190	78197	78204	7
6	8247	8254	8262	8269	8276	7
7	8319	8326	8333	8340	8347	7
8	8390	8398	8405	8412	8419	7
9	8462	8469	8476	8483	8490	7
610	78533	78540	78547	78554	78561	7
1	8604	8611	8618	8625	8633	7
2	8675	8682	8689	8696	8704	7
3	8746	8753	8760	8767	8774	7
4	8817	8824	8831	8838	8845	7
5	78888	78895	78902	78909	78916	7
6	8958	8965	8972	8979	8986	7
7	9029	9036	9043	9050	9057	7
8	9099	9106	9113	9120	9127	7
9	9169	9176	9183	9190	9197	7
620	79239	79246	79253	79260	79267	7
1	9309	9316	9323	9330	9337	7
2	9379	9386	9393	9400	9407	7
3	9449	9456	9463	9470	9477	7
4	9518	9525	9532	9539	9546	7
5	79588	79595	79602	79609	79616	7
6	9657	9664	9671	9678	9685	7
7	9727	9734	9741	9748	9754	7
8	9796	9803	9810	9817	9824	7
9	9865	9872	9879	9886	9893	7
630	79934	79941	79948	79955	79962	7
1	80003	80010	80017	80024	80030	7
2	0072	0079	0085	0092	0099	7
3	0140	0147	0154	0161	0168	7
4	0209	0216	0223	0229	0236	7
5	80277	80284	80291	80298	80305	7
6	0346	0353	0359	0366	0373	7
7	0414	0421	0428	0434	0441	7
8	0482	0489	0496	0502	0509	7
9	0550	0557	0564	0570	0577	7

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580	76380	76388	76395	76403	76410	7
1	6455	6462	6470	6477	6485	7
2	6530	6537	6545	6552	6559	7
3	6604	6612	6619	6626	6634	7
4	6678	6686	6693	6701	6708	7
5	76753	76760	76768	76775	76782	7
6	6827	6834	6842	6849	6856	7
7	6901	6908	6916	6923	6930	7
8	6975	6982	6989	6997	7004	7
9	7048	7056	7063	7070	7078	7
590	77122	77129	77137	77144	77151	7
1	7195	7203	7210	7217	7225	7
2	7269	7276	7283	7291	7298	7
3	7342	7349	7357	7364	7371	7
4	7415	7422	7430	7437	7444	7
5	77488	77495	77503	77510	77517	7
6	7561	7568	7576	7583	7590	7
7	7634	7641	7648	7656	7663	7
8	7706	7714	7721	7728	7735	7
9	7779	7786	7793	7801	7808	7
600	77851	77859	77866	77873	77880	7
1	7924	7931	7938	7945	7952	7
2	7996	8003	8010	8017	8025	7
3	8068	8075	8082	8089	8097	7
4	8140	8147	8154	8161	8168	7
5	78211	78219	78226	78233	78240	7
6	8283	8290	8297	8305	8312	7
7	8355	8362	8369	8376	8383	7
8	8426	8433	8440	8447	8455	7
9	8497	8504	8512	8519	8526	7
610	78569	78576	78583	78590	78597	7
1	8640	8647	8654	8661	8668	7
2	8711	8718	8725	8732	8739	7
3	8781	8789	8796	8803	8810	7
4	8852	8859	8866	8873	8880	7
5	78923	78930	78937	78944	78951	7
6	8993	9000	9007	9014	9021	7
7	9064	9071	9078	9085	9092	7
8	9134	9141	9148	9155	9162	7
9	9204	9211	9218	9225	9232	7
620	79274	79281	79288	79295	79302	7
1	9344	9351	9358	9365	9372	7
2	9414	9421	9428	9435	9442	7
3	9484	9491	9498	9505	9511	7
4	9553	9560	9567	9574	9581	7
5	79623	79630	79637	79644	79650	7
6	9692	9699	9706	9713	9720	7
7	9761	9768	9775	9782	9789	7
8	9831	9837	9844	9851	9858	7
9	9900	9906	9913	9920	9927	7
630	79969	79975	79982	79989	79996	7
1	80037	80044	80051	80058	80065	7
2	0106	0113	0120	0127	0134	7
3	0175	0182	0188	0195	0202	7
4	0243	0250	0257	0264	0271	7
5	80312	80318	80325	80332	80339	7
6	0380	0387	0393	0400	0407	7
7	0448	0455	0462	0468	0475	7
8	0516	0523	0530	0536	0543	7
9	0584	0591	0598	0604	0611	7

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640	80618	80625	80632	80638	80645	7
1	0686	0693	0699	0706	0713	7
2	0754	0760	0767	0774	0781	7
3	0821	0828	0835	0841	0848	7
4	0889	0895	0902	0909	0916	7
5	80956	80963	80969	80976	80983	7
6	1023	1030	1037	1043	1050	7
7	1090	1097	1104	1111	1117	7
8	1158	1164	1171	1178	1184	7
9	1224	1231	1238	1245	1251	7
650	81291	81298	81305	81311	81318	7
1	1358	1365	1371	1378	1385	7
2	1425	1431	1438	1445	1451	7
3	1491	1498	1505	1511	1518	7
4	1558	1564	1571	1578	1584	7
5	81624	81631	81637	81644	81651	7
6	1690	1697	1704	1710	1717	7
7	1757	1763	1770	1776	1783	7
8	1823	1829	1836	1842	1849	7
9	1889	1895	1902	1908	1915	7
660	81954	81961	81968	81974	81981	7
1	2020	2027	2033	2040	2046	7
2	2086	2092	2099	2105	2112	7
3	2151	2158	2164	2171	2178	7
4	2217	2223	2230	2236	2243	7
5	82282	82289	82295	82302	82308	7
6	2347	2354	2360	2367	2373	7
7	2413	2419	2426	2432	2439	6
8	2478	2484	2491	2497	2504	6
9	2543	2549	2556	2562	2569	6
670	82607	82614	82620	82627	82633	6
1	2672	2679	2685	2692	2698	6
2	2737	2743	2750	2756	2763	6
3	2802	2808	2814	2821	2827	6
4	2866	2872	2879	2885	2892	6
5	82930	82937	82943	82950	82956	6
6	2995	3001	3008	3014	3020	6
7	3059	3065	3072	3078	3085	6
8	3123	3129	3136	3142	3149	6
9	3187	3193	3200	3206	3213	6
680	83251	83257	83264	83270	83276	6
1	3315	3321	3327	3334	3340	6
2	3378	3385	3391	3398	3404	6
3	3442	3448	3455	3461	3467	6
4	3506	3512	3518	3525	3531	6
5	83569	83575	83582	83588	83594	6
6	3632	3639	3645	3651	3658	6
7	3696	3702	3708	3715	3721	6
8	3759	3765	3771	3778	3784	6
9	3822	3828	3835	3841	3847	6
690	83885	83891	83897	83904	83910	6
1	3948	3954	3960	3967	3973	6
2	4011	4017	4023	4029	4036	6
3	4073	4080	4086	4092	4098	6
4	4136	4142	4148	4155	4161	6
5	84198	84205	84211	84217	84223	6
6	4261	4267	4273	4280	4286	6
7	4323	4330	4336	4342	4348	6
8	4386	4392	4398	4404	4410	6
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2	0787	0794	0801	0808	0814	**7**
3	0855	0862	0868	0875	0882	**7**
4	0922	0929	0936	0943	0949	**7**
5	80990	80996	81003	81010	81017	**7**
6	1057	1064	1070	1077	1084	**7**
7	1124	1131	1137	1144	1151	**7**
8	1191	1198	1204	1211	1218	**7**
9	1258	1265	1271	1278	1285	**7**
650	81325	81331	81338	81345	81351	**7**
1	1391	1398	1405	1411	1418	**7**
2	1458	1465	1471	1478	1485	**7**
3	1525	1531	1538	1544	1551	**7**
4	1591	1598	1604	1611	1617	**7**
5	81657	81664	81671	81677	81684	**7**
6	1723	1730	1737	1743	1750	**7**
7	1790	1796	1803	1809	1816	**7**
8	1856	1862	1869	1875	1882	**7**
9	1921	1928	1935	1941	1948	**7**
660	81987	81994	82000	82007	82014	**7**
1	2053	2060	2066	2073	2079	**7**
2	2119	2125	2132	2138	2145	**7**
3	2184	2191	2197	2204	2210	**7**
4	2249	2256	2263	2269	2276	**7**
5	82315	82321	82328	82334	82341	**7**
6	2380	2387	2393	2400	2406	**7**
7	2445	2452	2458	2465	2471	**6**
8	2510	2517	2523	2530	2536	**6**
9	2575	2582	2588	2595	2601	**6**
670	82640	82646	82653	82659	82666	**6**
1	2705	2711	2718	2724	2730	**6**
2	2769	2776	2782	2789	2795	**6**
3	2834	2840	2847	2853	2860	**6**
4	2898	2905	2911	2918	2924	**6**
5	82963	82969	82975	82982	82988	**6**
6	3027	3033	3040	3046	3052	**6**
7	3091	3097	3104	3110	3117	**6**
8	3155	3161	3168	3174	3181	**6**
9	3219	3225	3232	3238	3245	**6**
680	83283	83289	83296	83302	83308	**6**
1	3347	3353	3359	3366	3372	**6**
2	3410	3417	3423	3429	3436	**6**
3	3474	3480	3487	3493	3499	**6**
4	3537	3544	3550	3556	3563	**6**
5	83601	83607	83613	83620	83620	**6**
6	3664	3670	3677	3683	3689	**6**
7	3727	3734	3740	3746	3753	**6**
8	3790	3797	3803	3809	3816	**6**
9	3853	3860	3866	3872	3879	**6**
690	83916	83923	83929	83935	83942	**6**
1	3979	3985	3992	3998	4004	**6**
2	4042	4048	4055	4061	4067	**6**
3	4105	4111	4117	4123	4130	**6**
4	4167	4173	4180	4186	4192	**6**
5	84230	84236	84242	84248	84255	**6**
6	4292	4298	4305	4311	4317	**6**
7	4354	4361	4367	4373	4379	**6**
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9	4479	4485	4491	4497	4504	**6**

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700	84510	84516	84522	84528	84535	6
1	4572	4578	4584	4590	4597	6
2	4634	4640	4646	4652	4658	6
3	4696	4702	4708	4714	4720	6
4	4757	4763	4770	4776	4782	6
5	84819	84825	84831	84837	84844	6
6	4880	4887	4893	4899	4905	6
7	4942	4948	4954	4960	4967	6
8	5003	5009	5016	5022	5028	6
9	5065	5071	5077	5083	5089	6
710	85126	85132	85138	85144	85150	6
1	5187	5193	5199	5205	5211	6
2	5248	5254	5260	5266	5272	6
3	5309	5315	5321	5327	5333	6
4	5370	5376	5382	5388	5394	6
5	85431	85437	85443	85449	85455	6
6	5491	5497	5503	5509	5516	6
7	5552	5558	5564	5570	5576	6
8	5612	5618	5625	5631	5637	6
9	5673	5679	5685	5691	5697	6
720	85733	85739	85745	85751	85757	6
1	5794	5800	5806	5812	5818	6
2	5854	5860	5866	5872	5878	6
3	5914	5920	5926	5932	5938	6
4	5974	5980	5986	5992	5998	6
5	86034	86040	86046	86052	86058	6
6	6094	6100	6106	6112	6118	6
7	6153	6159	6165	6171	6177	6
8	6213	6219	6225	6231	6237	6
9	6273	6279	6285	6291	6297	6
730	86332	86338	86344	86350	86356	6
1	6392	6398	6404	6410	6415	6
2	6451	6457	6463	6469	6475	6
3	6510	6516	6522	6528	6534	6
4	6570	6576	6581	6587	6593	6
5	86629	86635	86641	86646	86652	6
6	6688	6694	6700	6705	6711	6
7	6747	6753	6759	6764	6770	6
8	6806	6812	6817	6823	6829	6
9	6864	6870	6876	6882	6888	6
740	86923	86929	86935	86941	86947	6
1	6982	6988	6994	6999	7005	6
2	7040	7046	7052	7058	7064	6
3	7099	7105	7111	7116	7122	6
4	7157	7163	7169	7175	7181	6
5	87216	87221	87227	87233	87239	6
6	7274	7280	7286	7291	7297	6
7	7332	7338	7344	7349	7355	6
8	7390	7396	7402	7408	7413	6
9	7448	7454	7460	7466	7471	6
750	87506	87512	87518	87523	87529	6
1	7564	7570	7576	7581	7587	6
2	7622	7628	7633	7639	7645	6
3	7679	7685	7691	7697	7703	6
4	7737	7743	7749	7754	7760	6
5	87795	87800	87806	87812	87818	6
6	7852	7858	7864	7869	7875	6
7	7910	7915	7921	7927	7933	6
8	7967	7973	7978	7984	7990	6
9	8024	8030	8036	8041	8047	6

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700	84541	84547	84553	84559	84566	6
1	4603	4609	4615	4621	4628	6
2	4665	4671	4677	4683	4689	6
3	4726	4733	4739	4745	4751	6
4	4788	4794	4800	4807	4813	6
5	84850	84856	84862	84868	84874	6
6	4911	4917	4924	4930	4936	6
7	4973	4979	4985	4991	4997	6
8	5034	5040	5046	5052	5058	6
9	5095	5101	5107	5114	5120	6
710	85156	85163	85169	85175	85181	6
1	5217	5224	5230	5236	5242	6
2	5278	5285	5291	5297	5303	6
3	5339	5345	5352	5358	5364	6
4	5400	5406	5412	5418	5425	6
5	85461	85467	85473	85479	85485	6
6	5522	5528	5534	5540	5546	6
7	5582	5588	5594	5600	5606	6
8	5643	5649	5655	5661	5667	6
9	5703	5709	5715	5721	5727	6
720	85763	85769	85775	85781	85788	6
1	5824	5830	5836	5842	5848	6
2	5884	5890	5896	5902	5908	6
3	5944	5950	5956	5962	5968	6
4	6004	6010	6016	6022	6028	6
5	86064	86070	86076	86082	86088	6
6	6124	6130	6136	6141	6147	6
7	6183	6189	6195	6201	6207	6
8	6243	6249	6255	6261	6267	6
9	6303	6308	6314	6320	6326	6
730	86362	86368	86374	86380	86386	6
1	6421	6427	6433	6439	6445	6
2	6481	6487	6493	6499	6504	6
3	6540	6546	6552	6558	6564	6
4	6599	6605	6611	6617	6623	6
5	86658	86646	86670	86676	86682	6
6	6717	6723	6729	6735	6741	6
7	6776	6782	6788	6794	6800	6
8	6835	6841	6847	6853	6859	6
9	6894	6900	6906	6911	6917	6
740	86953	86958	86964	86970	86976	6
1	7011	7017	7023	7029	7035	6
2	7070	7075	7081	7087	7093	6
3	7128	7134	7140	7146	7151	6
4	7186	7192	7198	7204	7210	6
5	87245	87251	87256	87262	87268	6
6	7303	7309	7315	7320	7326	6
7	7361	7367	7373	7379	7384	6
8	7419	7425	7431	7437	7442	6
9	7477	7483	7489	7495	7500	6
750	87535	87541	87547	87552	87558	6
1	7593	7599	7604	7610	7616	6
2	7651	7656	7662	7668	7674	6
3	7708	7714	7720	7726	7731	6
4	7766	7772	7777	7783	7789	6
5	87823	87829	87835	87841	87846	6
6	7881	7887	7892	7898	7904	6
7	7938	7944	7950	7955	7961	6
8	7996	8001	8007	8013	8018	6
9	8053	8058	8064	8070	8076	6

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2	8195	8201	8207	8213	8218	6
3	8252	8258	8264	8270	8275	6
4	8309	8315	8321	8326	8332	6
5	88366	88372	88377	88383	88389	6
6	8423	8429	8434	8440	8446	6
7	8480	8485	8491	8497	8502	6
8	8536	8542	8547	8553	8559	6
9	8593	8598	8604	8610	8615	6
770	88649	88655	88660	88666	88672	6
1	8705	8711	8717	8722	8728	6
2	8762	8767	8773	8779	8784	6
3	8818	8824	8829	8835	8840	6
4	8874	8880	8885	8891	8897	6
5	88930	88936	88941	88947	88953	6
6	8986	8992	8997	9003	9009	6
7	9042	9048	9053	9059	9064	6
8	9098	9104	9109	9115	9120	6
9	9154	9159	9165	9170	9176	6
780	89209	89215	89221	89226	89232	6
1	9265	9271	9276	9282	9287	6
2	9321	9326	9332	9337	9343	6
3	9376	9382	9387	9393	9398	6
4	9432	9437	9443	9448	9454	6
5	89487	89492	89498	89504	89509	6
6	9542	9548	9553	9559	9564	5
7	9597	9603	9609	9614	9620	5
8	9653	9658	9664	9669	9675	5
9	9708	9713	9719	9724	9730	5
790	89763	89768	89774	89779	89785	5
1	9818	9823	9829	9834	9840	5
2	9873	9878	9883	9889	9894	5
3	9927	9933	9938	9944	9949	5
4	9982	9988	9993	9998	90004	5
5	90037	90042	90048	90053	90059	5
6	0091	0097	0102	0108	0113	5
7	0146	0151	0157	0162	0168	5
8	0200	0206	0211	0217	0222	5
9	0255	0260	0266	0271	0276	5
800	90309	90314	90320	90325	90331	5
1	0363	0369	0374	0380	0385	5
2	0417	0423	0428	0434	0439	5
3	0472	0477	0482	0488	0493	5
4	0526	0531	0536	0542	0547	5
5	90580	90585	90590	90596	90601	5
6	0634	0639	0644	0650	0655	5
7	0687	0693	0698	0703	0709	5
8	0741	0747	0752	0757	0763	5
9	0795	0800	0806	0811	0816	5
810	90849	90854	90859	90865	90870	5
1	0902	0907	0913	0918	0924	5
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3	1009	1014	1020	1025	1030	5
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5	91116	91121	91126	91132	91137	5
6	1169	1174	1180	1185	1190	5
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3	8281	8287	8292	8298	8304	6
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5	88395	88400	88406	88412	88417	6
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7	8508	8513	8519	8525	8530	6
8	8564	8570	8576	8581	8587	6
9	8621	8627	8632	8638	8643	6
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1	8734	8739	8745	8750	8756	6
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4	8902	8908	8913	8919	8925	6
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6	9014	9020	9025	9031	9037	6
7	9070	9076	9081	9087	9092	6
8	9126	9131	9137	9143	9148	6
9	9182	9187	9193	9198	9204	6
780	89237	89243	89248	89254	89260	6
1	9293	9298	9304	9310	9315	6
2	9348	9354	9360	9365	9371	6
3	9404	9409	9415	9421	9426	6
4	9459	9465	9470	9476	9481	6
5	89515	89520	89526	89531	89537	6
6	9570	9575	9581	9586	9592	5
7	9625	9631	9636	9642	9647	5
8	9680	9686	9691	9697	9702	5
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790	89790	89796	89801	89807	89812	5
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2	9900	9905	9911	9916	9922	5
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6	0119	0124	0129	0135	0140	5
7	0173	0179	0184	0189	0195	5
8	0227	0233	0238	0244	0249	5
9	0282	0287	0293	0298	0304	5
800	90336	90342	90347	90352	90358	5
1	0390	0396	0401	0407	0412	5
2	0445	0450	0455	0461	0466	5
3	0499	0504	0509	0515	0520	5
4	0553	0558	0563	0569	0574	5
5	90607	90612	90617	90623	90628	5
6	0660	0666	0671	0677	0682	5
7	0714	0720	0725	0730	0736	5
8	0768	0773	0779	0784	0789	5
9	0822	0827	0832	0838	0843	5
810	90875	90881	90886	90891	90897	5
1	0929	0934	0940	0945	0950	5
2	0982	0988	0993	0998	1004	5
3	1036	1041	1046	1052	1057	5
4	1089	1094	1100	1105	1110	5
5	91142	91148	91153	91158	91164	5
6	1196	1201	1206	1212	1217	5
7	1249	1254	1259	1265	1270	5
8	1302	1307	1312	1318	1323	5
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820	91381	91387	91392	91397	91403	5
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2	1487	1492	1498	1503	1508	5
3	1540	1545	1551	1556	1561	5
4	1593	1598	1603	1609	1614	5
5	91645	91651	91656	91661	91666	5
6	1698	1703	1709	1714	1719	5
7	1751	1756	1761	1766	1772	5
8	1803	1808	1814	1819	1824	5
9	1855	1861	1866	1871	1876	5
830	91908	91913	91918	91924	91929	5
1	1960	1965	1971	1976	1981	5
2	2012	2018	2023	2028	2033	5
3	2065	2070	2075	2080	2085	5
4	2117	2122	2127	2132	2137	5
5	92169	92174	92179	92184	92189	5
6	2221	2226	2231	2236	2241	5
7	2273	2278	2283	2288	2293	5
8	2324	2330	2335	2340	2345	5
9	2376	2381	2387	2392	2397	5
840	92428	92433	92438	92443	92449	5
1	2480	2485	2490	2495	2500	5
2	2531	2536	2542	2547	2552	5
3	2583	2588	2593	2598	2603	5
4	2634	2639	2645	2650	2655	5
5	92686	92691	92696	92701	92706	5
6	2737	2742	2747	2752	2758	5
7	2788	2793	2799	2804	2809	5
8	2840	2845	2850	2855	2860	5
9	2891	2896	2901	2906	2911	5
850	92942	92947	92952	92957	92962	5
1	2993	2998	3003	3008	3013	5
2	3044	3049	3054	3059	3064	5
3	3095	3100	3105	3110	3115	5
4	3146	3151	3156	3161	3166	5
5	93197	93202	93207	93212	93217	5
6	3247	3252	3258	3263	3268	5
7	3298	3303	3308	3313	3318	5
8	3349	3354	3359	3364	3369	5
9	3399	3404	3409	3414	3420	5
860	93450	93455	93460	93465	93470	5
1	3500	3505	3510	3515	3520	5
2	3551	3556	3561	3566	3571	5
3	3601	3606	3611	3616	3621	5
4	3651	3656	3661	3666	3671	5
5	93702	93707	93712	93717	93722	5
6	3752	3757	3762	3767	3772	5
7	3802	3807	3812	3817	3822	5
8	3852	3857	3862	3867	3872	5
9	3902	3907	3912	3917	3922	5
870	93952	93957	93962	93967	93972	5
1	4002	4007	4012	4017	4022	5
2	4052	4057	4062	4067	4072	5
3	4101	4106	4111	4116	4121	5
4	4151	4156	4161	4166	4171	5
5	94201	94206	94211	94216	94221	5
6	4250	4255	4260	4265	4270	5
7	4300	4305	4310	4315	4320	5
8	4349	4354	4359	4364	4369	5
9	4399	4404	4409	4414	4419	5

N	5	6	7	8	9	D
820	91408	91413	91418	91424	91429	5
1	1461	1466	1471	1477	1482	5
2	1514	1519	1524	1529	1535	5
3	1566	1572	1577	1582	1587	5
4	1619	1624	1630	1635	1640	5
5	91672	91677	91682	91687	91693	5
6	1724	1730	1735	1740	1745	5
7	1777	1782	1787	1793	1798	5
8	1829	1834	1840	1845	1850	5
9	1882	1887	1892	1897	1903	5
830	91934	91939	91944	91950	91955	5
1	1986	1991	1997	2002	2007	5
2	2038	2044	2049	2054	2059	5
3	2091	2096	2101	2106	2111	5
4	2143	2148	2153	2158	2163	5
5	92195	92200	92205	92210	92215	5
6	2247	2252	2257	2262	2267	5
7	2298	2304	2309	2314	2319	5
8	2350	2355	2361	2366	2371	5
9	2402	2407	2412	2418	2423	5
840	92454	92459	92464	92469	92474	5
1	2505	2511	2516	2521	2526	5
2	2557	2562	2567	2572	2578	5
3	2609	2614	2619	2624	2629	5
4	2660	2665	2670	2675	2681	5
5	92711	92716	92722	92727	92732	5
6	2763	2768	2773	2778	2783	5
7	2814	2819	2824	2829	2834	5
8	2865	2870	2875	2881	2886	5
9	2916	2921	2927	2932	2937	5
850	92967	92973	92978	92983	92988	5
1	3018	3024	3029	3034	3039	5
2	3069	3075	3080	3085	3090	5
3	3120	3125	3131	3136	3141	5
4	3171	3176	3181	3186	3192	5
5	93222	93227	93232	93237	93242	5
6	3273	3278	3283	3288	3293	5
7	3323	3328	3334	3339	3344	5
8	3374	3379	3384	3389	3394	5
9	3425	3430	3435	3440	3445	5
860	93475	93480	93485	93490	93495	5
1	3526	3531	3536	3541	3546	5
2	3576	3581	3586	3591	3596	5
3	3626	3631	3636	3641	3646	5
4	3676	3682	3687	3692	3697	5
5	93727	93732	93737	93742	93747	5
6	3777	3782	3787	3792	3797	5
7	3827	3832	3837	3842	3847	5
8	3877	3882	3887	3892	3897	5
9	3927	3932	3937	3942	3947	5
870	93977	93982	93987	93992	93997	5
1	4027	4032	4037	4042	4047	5
2	4077	4082	4086	4091	4096	5
3	4126	4131	4136	4141	4146	5
4	4176	4181	4186	4191	4196	5
5	94226	94231	94236	94240	94245	5
6	4275	4280	4285	4290	4295	5
7	4325	4330	4335	4340	4345	5
8	4374	4379	4384	4389	4394	5
9	4424	4429	4433	4438	4443	5

N	0	1	2	3	4	D
880	94448	94453	94458	94463	94468	5
1	4498	4503	4507	4512	4517	5
2	4547	4552	4557	4562	4567	5
3	4596	4601	4606	4611	4616	5
4	4645	4650	4655	4660	4665	5
5	94694	94699	94704	94709	94714	5
6	4743	4748	4753	4758	4763	5
7	4792	4797	4802	4807	4812	5
8	4841	4846	4851	4856	4861	5
9	4890	4895	4900	4905	4910	5
890	94939	94944	94949	94954	94959	5
1	4988	4993	4998	5002	5007	5
2	5036	5041	5046	5051	5056	5
3	5085	5090	5095	5100	5105	5
4	5134	5139	5143	5148	5153	5
5	95182	95187	95192	95197	95202	5
6	5231	5236	5240	5245	5250	5
7	5279	5284	5289	5294	5299	5
8	5328	5332	5337	5342	5347	5
9	5376	5381	5386	5390	5395	5
900	95424	95429	95434	95439	95444	5
1	5472	5477	5482	5487	5492	5
2	5521	5525	5530	5535	5540	5
3	5569	5574	5578	5583	5588	5
4	5617	5622	5626	5631	5636	5
5	95665	95670	95674	95679	95684	5
6	5713	5718	5722	5727	5732	5
7	5761	5766	5770	5775	5780	5
8	5809	5813	5818	5823	5828	5
9	5856	5861	5866	5871	5875	5
910	95904	95909	95914	95918	95923	5
1	5952	5957	5961	5966	5971	5
2	5999	6004	6009	6014	6019	5
3	6047	6052	6057	6061	6066	5
4	6095	6099	6104	6109	6114	5
5	96142	96147	96152	96156	96161	5
6	6190	6194	6199	6204	6209	5
7	6237	6242	6246	6251	6256	5
8	6284	6289	6294	6298	6303	5
9	6332	6336	6341	6346	6350	5
920	96379	96384	96388	96393	96398	5
1	6426	6431	6435	6440	6445	5
2	6473	6478	6483	6487	6492	5
3	6520	6525	6530	6534	6539	5
4	6567	6572	6577	6581	6586	5
5	96614	96619	96624	96628	96633	5
6	6661	6666	6670	6675	6680	5
7	6708	6713	6717	6722	6727	5
8	6755	6759	6764	6769	6774	5
9	6802	6806	6811	6816	6820	5
930	96848	96853	96858	96862	96867	5
1	6895	6900	6904	6909	6914	5
2	6942	6946	6951	6956	6960	5
3	6988	6993	6997	7002	7007	5
4	7035	7039	7044	7049	7053	5
5	97081	97086	97090	97095	97100	5
6	7128	7132	7137	7142	7146	5
7	7174	7179	7183	7188	7192	5
8	7220	7225	7230	7234	7239	5
9	7267	7271	7276	7280	7285	5

N	5	6	7	8	9	D
880	94473	94478	94483	94488	94493	5
1	4522	4527	4532	4537	4542	5
2	4571	4576	4581	4586	4591	5
3	4621	4626	4630	4635	4640	5
4	4670	4675	4680	4685	4689	5
5	94719	94724	94729	94734	94738	5
6	4768	4773	4778	4783	4787	5
7	4817	4822	4827	4832	4836	5
8	4866	4871	4876	4880	4885	5
9	4915	4919	4924	4929	4934	5
890	94963	94968	94973	94978	94983	5
1	5012	5017	5022	5027	5032	5
2	5061	5066	5071	5075	5080	5
3	5109	5114	5119	5124	5129	5
4	5158	5163	5168	5173	5177	5
5	95207	95211	95216	95221	95226	5
6	5255	5260	5265	5270	5274	5
7	5303	5308	5313	5318	5323	5
8	5352	5357	5361	5366	5371	5
9	5400	5405	5410	5415	5419	5
900	95448	95453	95458	95463	95468	5
1	5497	5501	5506	5511	5516	5
2	5545	5550	5554	5559	5564	5
3	5593	5598	5602	5607	5612	5
4	5641	5646	5650	5655	5660	5
5	95689	95694	95698	95703	95708	5
6	5737	5742	5746	5751	5756	5
7	5785	5789	5794	5799	5804	5
8	5832	5837	5842	5847	5852	5
9	5880	5885	5890	5895	5899	5
910	95928	95933	95938	95942	95947	5
1	5976	5980	5985	5990	5995	5
2	6023	6028	6033	6038	6042	5
3	6071	6076	6080	6085	6090	5
4	6118	6123	6128	6133	6137	5
5	96166	96171	96175	96180	96185	5
6	6213	6218	6223	6227	6232	5
7	6261	6265	6270	6275	6280	5
8	6308	6313	6317	6322	6327	5
9	6355	6360	6365	6369	6374	5
920	96402	96407	96412	96417	96421	5
1	6450	6454	6459	6464	6468	5
2	6497	6501	6506	6511	6515	5
3	6544	6548	6553	6558	6562	5
4	6591	6595	6600	6605	6609	5
5	96638	96642	96647	96652	96656	5
6	6685	6689	6694	6699	6703	5
7	6731	6736	6741	6745	6750	5
8	6778	6783	6788	6792	6797	5
9	6825	6830	6834	6839	6844	5
930	96872	96876	96881	96886	96890	5
1	6918	6923	6928	4932	6937	5
2	6965	6970	6974	6979	6984	5
3	7011	7016	7021	7025	7030	5
4	7058	7063	7067	7072	7077	5
5	97104	97109	97114	97118	97123	5
6	7151	7155	7160	7165	7169	5
7	7197	7202	7206	7211	7216	5
8	7243	7248	7253	7257	7262	5
9	7290	7294	7299	7304	7308	5

N	0	1	2	3	4	D
940	97313	97317	97322	97327	97331	5
1	7359	7364	7368	7373	7377	5
2	7405	7410	7414	7419	7424	5
3	7451	7456	7460	7465	7470	5
4	7497	7502	7506	7511	7516	5
5	97543	97548	97552	97557	97562	5
6	7589	7594	7598	7603	7607	5
7	7635	7640	7644	7649	7653	5
8	7681	7685	7690	7695	7699	5
9	7727	7731	7736	7740	7745	5
950	97772	97777	97782	97786	97791	5
1	7818	7823	7827	7832	7836	5
2	7864	7868	7873	7877	7882	5
3	7909	7914	7918	7923	7928	5
4	7955	7959	7964	7968	7973	5
5	98000	98005	98009	98014	98019	5
6	8046	8050	8055	8059	8064	5
7	8091	8096	8100	8105	8109	5
8	8137	8141	8146	8150	8155	5
9	8182	8186	8191	8195	8200	5
960	98227	98232	98236	98241	98245	5
1	8272	8277	8281	8286	8290	5
2	8318	8322	8327	8331	8336	4
3	8363	8367	8372	8376	8381	4
4	8408	8412	8417	8421	8426	4
5	98453	98457	98462	98466	98471	4
6	8498	8502	8507	8511	8516	4
7	8543	8547	8552	8556	8561	4
8	8588	8592	8597	8601	8605	4
9	8632	8637	8641	8646	8650	4
970	98677	98682	98686	98691	98695	4
1	8722	8726	8731	8735	8740	4
2	8767	8771	8776	8780	8784	4
3	8811	8816	8820	8825	8829	4
4	8856	8860	8865	8869	8874	4
5	98900	98905	98909	98914	98918	4
6	8945	8949	8954	8958	8963	4
7	8989	8994	8998	9003	9007	4
8	9034	9038	9043	9047	9052	4
9	9078	9083	9087	9092	9096	4
980	99123	99127	99131	99136	99140	4
1	9167	9171	9176	9180	9185	4
2	9211	9216	9220	9224	9229	4
3	9255	9260	9264	9269	9273	4
4	9300	9304	9308	9313	9317	4
5	99344	99348	99352	99357	99361	4
6	9388	9392	9396	9401	9405	4
7	9432	9436	9441	9445	9449	4
8	9476	9480	9484	9489	9493	4
9	9520	9524	9528	9533	9537	4
990	99564	99568	99572	99577	99581	4
1	9607	9612	9616	9621	9625	4
2	9651	9656	9660	9664	9669	4
3	9695	9699	9704	9708	9712	4
4	9739	9743	9747	9752	9756	4
5	99782	99787	99791	99795	99800	4
6	9826	9830	9835	9839	9843	4
7	9870	9874	9878	9883	9887	4
8	9913	9917	9922	9926	9930	4
9	9957	9961	9965	9970	9974	4
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N	5	6	7	8	9	D
940	97336	97340	97345	97350	97354	5
1	7382	7387	7391	7396	7400	5
2	7428	7433	7437	7442	7447	5
3	7474	7479	7483	7488	7493	5
4	7520	7525	7529	7534	7539	5
5	97566	97571	97575	97580	97585	5
6	7612	7617	7621	7626	7630	5
7	7658	7663	7667	7672	7676	5
8	7704	7708	7713	7717	7722	5
9	7749	7754	7759	7763	7768	5
950	97795	97800	97804	97809	97813	5
1	7841	7845	7850	7855	7859	5
2	7886	7891	7896	7900	7905	5
3	7932	7937	7941	7946	7950	5
4	7978	7982	7987	7991	7996	5
5	98023	98028	98032	98037	98041	5
6	8068	8073	8078	8082	8087	5
7	8114	8118	8123	8127	8132	5
8	8159	8164	8168	8173	8177	5
9	8204	8209	8214	8218	8223	5
960	98250	98254	98259	98263	98268	5
1	8295	8299	8304	8308	8313	5
2	8340	8345	8349	8354	8358	4
3	8385	8390	8394	8399	8403	4
4	8430	8435	8439	8444	8448	4
5	98475	98480	98484	98489	98493	4
6	8520	8525	8529	8534	8538	4
7	8565	8570	8574	8579	8583	4
8	8610	8614	8619	8623	8628	4
9	8655	8659	8664	8668	8673	4
970	98700	98704	98709	98713	98717	4
1	8744	8749	8753	8758	8762	4
2	8789	8793	8798	8802	8807	4
3	8834	8838	8843	8847	8851	4
4	8878	8883	8887	8892	8896	4
5	98923	98927	98932	98936	98941	4
6	8967	8972	8976	8981	8985	4
7	9012	9016	9021	9025	9029	4
8	9056	9061	9065	9069	9074	4
9	9100	9105	9109	9114	9118	4
980	99145	99149	99154	99158	99162	4
1	9189	9193	9198	9202	9207	4
2	9233	9238	9242	9247	9251	4
3	9277	9282	9286	9291	9295	4
4	9322	9326	9330	9335	9339	4
5	99366	99370	99374	99379	99383	4
6	9410	9414	9419	9423	9427	4
7	9454	9458	9463	9467	9471	4
8	9498	9502	9506	9511	9515	4
9	9542	9546	9550	9555	9559	4
990	99585	99590	99594	99599	99603	4
1	9629	9634	9638	9642	9647	4
2	9673	9677	9682	9686	9691	4
3	9717	9721	9726	9730	9734	4
4	9760	9765	9769	9774	9778	4
5	99804	99808	99813	99817	99822	4
6	9848	9852	9856	9861	9865	4
7	9891	9896	9900	9904	9909	4
8	9935	9939	9944	9948	9952	4
9	9978	9983	9987	9991	9996	4
1000	00022	00026	00030	00035	00039	4

	179°	178°	177°	176°	175°
Sin	0°	1°	2°	3°	4°
0′	−∞	8.24186	8.54282	8.71880	8.84358
1	6.46373	24903	54642	72120	84539
2	76476	25609	54999	72359	84718
3	94085	26304	55354	72597	84897
4	7.06579	26988	55705	72834	85075
5	7.16270	8.27661	8.56054	8.73069	8.85252
6	24188	28324	56400	73303	85429
7	30882	28977	56743	73535	85605
8	36682	29621	57084	73767	85780
9	41797	30255	57421	73997	85955
10	7.46373	8.30879	8.57757	8.74226	8.86128
11	50512	31495	58089	74454	86301
12	54291	32103	58419	74680	86474
13	57767	32702	58747	74906	86645
14	60985	33292	59072	75130	86816
15	7.63982	8.33875	8.59395	8.75353	8.86987
16	66784	34450	59715	75575	87156
17	69417	35018	60033	75795	87325
18	71900	35578	60349	76015	87494
19	74248	36131	60662	76234	87661
20	7.76475	8.36678	8.60973	8.76451	8.87829
21	78594	37217	61282	76667	87995
22	80615	37750	61589	76883	88161
23	82545	38276	61894	77097	88326
24	84393	38796	62196	77310	88490
25	7.86166	8.39310	8.62497	8.77522	8.88654
26	87870	39818	62795	77733	88817
27	89509	40320	63091	77943	88980
28	91088	40816	63385	78152	89142
29	92612	41307	63678	78360	89304
30	7.94084	8.41792	8.63968	8.78568	8.89464
31	95508	42272	64256	78774	89625
32	96887	42746	64543	78979	89784
33	98223	43216	64827	79183	89943
34	99520	43680	65110	79386	90102
35	8.00779	8.44139	8.65391	8.79588	8.90260
36	02002	44594	65670	79789	90417
37	03192	45044	65947	79990	90574
38	04350	45489	66223	80189	90730
39	05478	45930	66497	80388	90885
40	8.06578	8.46366	8.66769	8.80585	8.91040
41	07650	46799	67039	80782	91195
42	08696	47226	67308	80978	91349
43	09718	47650	67575	81173	91502
44	10717	48069	67841	81367	91655
45	8.11693	8.48485	8.68104	8.81560	8.91807
46	12647	48896	68367	81752	91959
47	13581	49304	68627	81944	92110
48	14495	49708	68886	82134	92261
49	15391	50108	69144	82324	92411
50	8.16268	8.50504	8.69400	8.82513	8.92561
51	17128	50897	69654	82701	92710
52	17971	51287	69907	82888	92859
53	18798	51673	70159	83075	93007
54	19610	52055	70409	83261	93154
55	8.20407	8.52434	8.70658	8.83446	8.93301
56	21189	52810	70905	83630	93448
57	21958	53183	71151	83813	93594
58	22713	53552	71395	83996	93740
59	23456	53919	71638	84177	93885
60	8.24186	8.54282	8.71880	8.84358	8.94030
	89°	88°	87°	86°	85°
Cos	90°	91°	92°	93°	94°

174°	173°	172°	171°	170°	Sin
5°	6°	7°	8°	9°	
8.94030	9.01923	9.08589	9.14356	9.19433	60′
94174	02043	08692	14445	19513	59
94317	02163	08795	14535	19592	58
94461	02283	08897	14624	19672	57
94603	02402	08999	14714	19751	56
8.94746	9.02520	9.09101	9.14803	9.19830	55
94887	02639	09202	14891	19909	54
95029	02757	09304	14980	19988	53
95170	02874	09405	15069	20067	52
95310	02992	09506	15157	20145	51
8.95450	9.03109	9.09606	9.15245	9.20223	50
95589	03226	09707	15333	20302	49
95728	03342	09807	15421	20380	48
95867	03458	09907	15508	20458	47
96005	03574	10006	15596	20535	46
8.96143	9.03690	9.10106	9.15683	9.20613	45
96280	03805	10205	15770	20691	44
96417	03920	10304	15857	20768	43
96553	04034	10402	15944	20845	42
96689	04149	10501	16030	20922	41
8.96825	9.04262	9.10599	9.16116	9.20999	40
96960	04376	10697	16203	21076	39
97095	04490	10795	16289	21153	38
97229	04603	10893	16374	21229	37
97363	04715	10990	16460	21306	36
8.97496	9.04828	9.11087	9.16545	9.21382	35
97629	04940	11184	16631	21458	34
97762	05052	11281	16716	21534	33
97894	05164	11377	16801	21610	32
98026	05275	11474	16886	21685	31
8.98157	9.05386	9.11570	9.16970	9.21761	30
98288	05497	11666	17055	21836	29
98419	05607	11761	17139	21912	28
98549	05717	11857	17223	21987	27
98679	05827	11952	17307	22062	26
8.98808	9.05937	9.12047	9.17391	9.22137	25
98937	06046	12142	17474	22211	24
99066	06155	12236	17558	22286	23
99194	06264	12331	17641	22361	22
99322	06372	12425	17724	22435	21
8.99450	9.06481	9.12519	9.17807	9.22509	20
99577	06589	12612	17890	22583	19
99704	06696	12706	17973	22657	18
99830	06804	12799	18055	22731	17
99956	06911	12892	18137	22805	16
9.00082	9.07018	9.12985	9.18220	9.22878	15
00207	07124	13078	18302	22952	14
00332	07231	13171	18383	23025	13
00456	07337	13263	18465	23098	12
00581	07442	13355	18547	23171	11
9.00704	9.07548	9.13447	9.18628	9.23244	10
00828	07653	13539	18709	23317	9
00951	07758	13630	18790	23390	8
01074	07863	13722	18871	23462	7
01196	07968	13813	18952	23535	6
9.01318	9.08072	9.13904	9.19033	9.23607	5
01440	08176	13994	19113	23679	4
01561	08280	14085	19193	23752	3
01682	08383	14175	19273	23823	2
01803	08486	14266	19353	23895	1
9.01923	9.08589	9.14356	9.19433	9.23967	0
84°	83°	82°	81°	80°	Cos
95°	96°	97°	98°	99°	

	169°	168°	167°	166°	165°
Sin	**10°**	**11°**	**12°**	**13°**	**14°**
0′	9.23967	9.28060	9.31788	9.35209	9.38368
1	24039	28125	31847	35263	38418
2	24110	28190	31907	35318	38469
3	24181	28254	31966	35373	38519
4	24253	28319	32025	35427	38570
5	9.24324	9.28384	9.32084	9.35481	9.38620
6	24395	28448	32143	35536	38670
7	24466	28512	32202	35590	38721
8	24536	28577	32261	35644	38771
9	24607	28641	32319	35698	38821
10	9.24677	9.28705	9.32378	9.35752	9.38871
11	24748	28769	32437	35806	38921
12	24818	28833	32495	35860	38971
13	24888	28896	32553	35914	39021
14	24958	28960	32612	35968	39071
15	9.25028	9.29024	9.32670	9.36022	9.39121
16	25098	29087	32728	36075	39170
17	25168	29150	32786	36129	39220
18	25237	29214	32844	36182	39270
19	25307	29277	32902	36236	39319
20	9.25376	9.29340	9.32960	9.36289	9.39369
21	25445	29403	33018	36342	39418
22	25514	29466	33075	36395	39467
23	25583	29529	33133	36449	39517
24	25652	29591	33190	36502	39566
25	9.25721	9.29654	9.33248	9.36555	9.39615
26	25790	29716	33305	36608	39664
27	25858	29779	33362	36660	39713
28	25927	29841	33420	36713	39762
29	25995	29903	33477	36766	39811
30	9.26063	9.29966	9.33534	9.36819	9.39860
31	26131	30028	33591	36871	39909
32	26199	30090	33647	36924	39958
33	26267	30151	33704	36976	40006
34	26335	30213	33761	37028	40055
35	9.26403	9.30275	9.33818	9.37081	9.40103
36	26470	30336	33874	37133	40152
37	26538	30398	33931	37185	40200
38	26605	30459	33987	37237	40249
39	26672	30521	34043	37289	40297
40	9.26739	9.30582	9.34100	9.37341	9.40346
41	26806	30643	34156	37393	40394
42	26873	30704	34212	37445	40442
43	26940	30765	34268	37497	40490
44	27007	30826	34324	37549	40538
45	9.27073	9.30887	9.34380	9.37600	9.40586
46	27140	30947	34436	37652	40634
47	27206	31008	34491	37703	40682
48	27273	31068	34547	37755	40730
49	27339	31129	34602	37806	40778
50	9.27405	9.31189	9.34658	9.37858	9.40825
51	27471	31250	34713	37909	40873
52	27537	31310	34769	37960	40921
53	27602	31370	34824	38011	40968
54	27668	31430	34879	38062	41016
55	9.27734	9.31490	9.34934	9.38113	9.41063
56	27799	31549	34989	38164	41111
57	27864	31609	35044	38215	41158
58	27930	31669	35099	38266	41205
59	27995	31728	35154	38317	41252
60	9.28060	9.31788	9.35209	9.38368	9.41300
	79°	**78°**	**77°**	**76°**	**75°**
Cos	100°	101°	102°	103°	104°

164°	163°	162°	161°	160°	Sin
15°	**16°**	**17°**	**18°**	**19°**	
9.41300	9.44034	9.46594	9.48998	9.51264	**60′**
41347	44078	46635	49037	51301	**59**
41394	44122	46676	49076	51338	**58**
41441	44166	46717	49115	51374	**57**
41488	44210	46758	49153	51411	**56**
9.41535	9.44253	9.46800	9.49192	9.51447	**55**
41582	44297	46841	49231	51484	**54**
41628	44341	46882	49269	51520	**53**
41675	44385	46923	49308	51557	**52**
41722	44428	46964	49347	51593	**51**
9.41768	9.44472	9.47005	9.49385	9.51629	**50**
41815	44516	47045	49424	51666	**49**
41861	44559	47086	49462	51702	**48**
41908	44602	47127	49500	51738	**47**
41954	44646	47168	49539	51774	**46**
9.42001	9.44689	9.47209	9.49577	9.51811	**45**
42047	44733	47249	49615	51847	**44**
42093	44776	47290	49654	51883	**43**
42140	44819	47330	49692	51919	**42**
42186	44862	47371	49730	51955	**41**
9.42232	9.44905	9.47411	9.49768	9.51991	**40**
42278	44948	47452	49806	52027	**39**
42324	44992	47492	49844	52063	**38**
42370	45035	47533	49882	52099	**37**
42416	45077	47573	49920	52135	**36**
9.42461	9.45120	9.47613	9.49958	9.52171	**35**
42507	45163	47654	49996	52207	**34**
42553	45206	47694	50034	52242	**33**
42599	45249	47734	50072	52278	**32**
42644	45292	47774	50110	52314	**31**
9.42690	9.45334	9.47814	9.50148	9.52350	**30**
42735	45377	47854	50185	52385	**29**
42781	45419	47894	50223	52421	**28**
42826	45462	47934	50261	52456	**27**
42872	45504	47974	50298	52492	**26**
9.42917	9.45547	9.48014	9.50336	9.52527	**25**
42962	45589	48054	50374	52563	**24**
43008	45632	48094	50411	52598	**23**
43053	45674	48133	50449	52634	**22**
43098	45716	48173	50486	52669	**21**
9.43143	9.45758	9.48213	9.50523	9.52705	**20**
43188	45801	48252	50561	52740	**19**
43233	45843	48292	50598	52775	**18**
43278	45885	48332	50635	52811	**17**
43323	45927	48371	50673	52846	**16**
9.43367	9.45969	9.48411	9.50710	9.52881	**15**
43412	46011	48450	50747	52916	**14**
43457	46053	48490	50784	52951	**13**
43502	46095	48529	50821	52986	**12**
43546	46136	48568	50858	53021	**11**
9.43591	9.46178	9.48607	9.50896	9.53056	**10**
43635	46220	48647	50933	53092	**9**
43680	46262	48686	50970	53126	**8**
43724	46303	48725	51007	53161	**7**
43769	46345	48764	51043	53196	**6**
9.43813	9.46386	9.48803	9.51080	9.53231	**5**
43857	46428	48842	51117	53266	**4**
43901	46469	48881	51154	53301	**3**
43946	46511	48920	51191	53336	**2**
43990	46552	48959	51227	53370	**1**
9.44034	9.46594	9.48998	9.51264	9.53405	**0**
74°	**73°**	**72°**	**71°**	**70°**	Cos
105°	106°	107°	108°	109°	

	159°	158°	157°	156°	155°
Sin	20°	21°	22°	23°	24°
0′	9.53405	9.55433	9.57358	9.59188	9.60931
1	53440	55466	57389	59218	60960
2	53475	55499	57420	59247	60988
3	53509	55532	57451	59277	61016
4	53544	55564	57482	59307	61045
5	9.53578	9.55597	9.57514	9.59336	9.61073
6	53613	55630	57545	59366	61101
7	53647	55663	57576	59396	61129
8	53682	55695	57607	59425	61158
9	53716	55728	57638	59455	61186
10	9.53751	9.55761	9.57669	9.59484	9.61214
11	53785	55793	57700	59514	61242
12	53819	55826	57731	59543	61270
13	53854	55858	57762	59573	61298
14	53888	55891	57793	59602	61326
15	9.53922	9.55923	9.57824	9.59632	9.61354
16	53957	55956	57855	59661	61382
17	53991	55988	57885	59690	61411
18	54025	56021	57916	59720	61438
19	54059	56053	57947	59749	61466
20	9.54093	9.56085	9.57978	9.59778	9.61494
21	54127	56118	58008	59808	61522
22	54161	56150	58039	59837	61550
23	54195	56182	58070	59866	61578
24	54229	56215	58101	59895	61606
25	9.54263	9.56247	9.58131	9.59924	9.61634
26	54297	56279	58162	59954	61662
27	54331	56311	58192	59983	61689
28	54365	56343	58223	60012	61717
29	54399	56375	58253	60041	61745
30	9.54433	9.56408	9.58284	9.60070	9.61773
31	54466	56440	58314	60099	61800
32	54500	56472	58345	60128	61828
33	54534	56504	58375	60157	61856
34	54567	56536	58406	60186	61883
35	9.54601	9.56568	9.58436	9.60215	9.61911
36	54635	56599	58467	60244	61939
37	54668	56631	58497	60273	61966
38	54702	56663	58527	60302	61994
39	54735	56695	58557	60331	62021
40	9.54769	9.56727	9.58588	9.60359	9.62049
41	54802	56759	58618	60388	62076
42	54836	56790	58648	60417	62104
43	54869	56822	58678	60446	62131
44	54903	56854	58709	60474	62159
45	9.54936	9.56886	9.58739	9.60503	9.62186
46	54969	56917	58769	60532	62214
47	55003	56949	58799	60561	62241
48	55036	56980	58829	60589	62268
49	55069	57012	58859	60618	62296
50	9.55102	9.57044	9.58889	9.60646	9.62323
51	55136	57075	58919	60675	62350
52	55169	57107	58949	60704	62377
53	55202	57138	58979	60732	62405
54	55235	57169	59009	60761	62432
55	9.55268	9.57201	9.59039	9.60789	9.62459
56	55301	57232	59069	60818	62486
57	55334	57264	59098	60846	62513
58	55367	57295	59128	60875	62541
59	55400	57326	59158	60903	62568
60	9.55433	9.57358	9.59188	9.60931	9.62595
	69°	68°	67°	66°	65°
Cos	110°	111°	112°	113°	114°

154°	153°	152°	151°	150°	Sin
25°	26°	27°	28°	29°	
9.62595	9.64184	9.65705	9.67161	9.68557	60′
62622	64210	65729	67185	68580	59
62649	64236	65754	67208	68603	58
62676	64262	65779	67232	68625	57
62703	64288	65804	67256	68648	56
9.62730	9.64313	9.65828	9.67280	9.68671	55
62757	64339	65853	67303	68694	54
62784	64365	65878	67327	68716	53
62811	64391	65902	67350	68739	52
62838	64417	65927	67374	68762	51
9.62865	9.64442	9.65952	9.67398	9.68784	50
62892	64468	65976	67421	68807	49
62918	64494	66001	67445	68829	48
62945	64519	66025	67468	68852	47
62972	64545	66050	67492	68875	46
9.62999	9.64571	9.66075	9.67515	9.68897	45
63026	64596	66099	67539	68920	44
63052	64622	66124	67562	68942	43
63079	64647	66148	67586	68965	42
63106	64673	66173	67609	68987	41
9.63133	9.64698	9.66197	9.67633	9.69010	40
63159	64724	66221	67656	69032	39
63186	64749	66246	67680	69055	38
63213	64775	66270	67703	69077	37
63239	64800	66295	67726	69100	36
9.63266	9.64826	9.66319	9.67750	9.69122	35
63292	64851	66343	67773	69144	34
63319	64877	66368	67796	69167	33
63345	64902	66392	67820	69189	32
63372	64927	66416	67843	69212	31
9.63398	9.64953	9.66441	9.67866	9.69234	30
63425	64978	66465	67890	69256	29
63451	65003	66489	67913	69279	28
63478	65029	66513	67936	69301	27
63504	65054	66537	67959	69323	26
9.63531	9.65079	9.66562	9.67982	9.69345	25
63557	65104	66586	68006	69368	24
63583	65130	66610	68029	69390	23
63610	65155	66634	68052	69412	22
63636	65180	66658	68075	69434	21
9.63662	9.65205	9.66682	9.68098	9.69456	20
63689	65230	66706	68121	69479	19
63715	65255	66731	68144	69501	18
63741	65281	66755	68167	69523	17
63767	65306	66779	68190	69545	16
9.63794	9.65331	9.66803	9.68213	9.69567	15
63820	65356	66827	68237	69589	14
63846	65381	66851	68260	69611	13
63872	65406	66875	68283	69633	12
63898	65431	66899	68305	69655	11
9.63924	9.65456	9.66922	9.68328	9.69677	10
63950	65481	66946	68351	69699	9
63976	65506	66970	68374	69721	8
64002	65531	66994	68397	69743	7
64028	65556	67018	68420	69765	6
9.64054	9.65580	9.67042	9.68443	9.69787	5
64080	65605	67066	68466	69809	4
64106	65630	67090	68489	69831	3
64132	65655	67113	68512	69853	2
64158	65680	67137	68534	69875	1
9.64184	9.65705	9.67161	9.68557	9.69897	0
64°	63°	62°	61°	60°	Cos
115°	116°	117°	118°	119°	

	149°	148°	147°	146°	145°
Sin	30°	31°	32°	33°	34°
0′	9.69897	9.71184	9.72421	9.73611	9.74756
1	69919	71205	72441	73630	74775
2	69941	71226	72461	73650	74794
3	69963	71247	72482	73669	74812
4	69984	71268	72502	73689	74831
5	9.70006	9.71289	9.72522	9.73708	9.74850
6	70028	71310	72542	73727	74868
7	70050	71331	72562	73747	74887
8	70072	71352	72582	73766	74906
9	70093	71373	72602	73785	74924
10	9.70115	9.71393	9.72622	9.73805	9.74943
11	70137	71414	72643	73824	74961
12	70159	71435	72663	73843	74980
13	70180	71456	72683	73863	74999
14	70202	71477	72703	73882	75017
15	9.70224	9.71498	9.72723	9.73901	9.75036
16	70245	71519	72743	73921	75054
17	70267	71539	72763	73940	75073
18	70288	71560	72783	73959	75091
19	70310	71581	72803	73978	75110
20	9.70332	9.71602	9.72823	9.73997	9.75128
21	70353	71622	72843	74017	75147
22	70375	71643	72863	74036	75165
23	70396	71664	72883	74055	75184
24	70418	71685	72902	74074	75202
25	9.70439	9.71705	9.72922	9.74093	9.75221
26	70461	71726	72942	74113	75239
27	70482	71747	72962	74132	75258
28	70504	71767	72982	74151	75276
29	70525	71788	73002	74170	75294
30	9.70547	9.71809	9.73022	9.74189	9.75313
31	70568	71829	73041	74208	75331
32	70590	71850	73061	74227	75350
33	70611	71870	73081	74246	75368
34	70633	71891	73101	74265	75386
35	9.70654	9.71911	9.73121	9.74284	9.75405
36	70675	71932	73140	74303	75423
37	70697	71952	73160	74322	75441
38	70718	71973	73180	74341	75459
39	70739	71994	73200	74360	75478
40	9.70761	9.72014	9.73219	9.74379	9.75496
41	70782	72034	73239	74398	75514
42	70803	72055	73259	74417	75533
43	70824	72075	73278	74436	75551
44	70846	72096	73298	74455	75569
45	9.70867	9.72116	9.73318	9.74474	9.75587
46	70888	72137	73337	74493	75605
47	70909	72157	73357	74512	75624
48	70931	72177	73377	74531	75642
49	70952	72198	73396	74549	75660
50	9.70973	9.72218	9.73416	9.74568	9.75678
51	70994	72238	73435	74587	75696
52	71015	72259	73455	74606	75714
53	71036	72279	73474	74625	75733
54	71058	72299	73494	74644	75751
55	9.71079	9.72320	9.73513	9.74662	9.75769
56	71100	72340	73533	74681	75787
57	71121	72360	73552	74700	75805
58	71142	72381	73572	74719	75823
59	71163	72401	73591	74737	75841
60	9.71184	9.72421	9.73611	9.74756	9.75859
	59°	58°	57°	56°	55°
Cos	120°	121°	122°	123°	124°

144°	143°	142°	141°	140°	Sin
35°	36°	37°	38°	39°	
9.75859	9.76922	9.77946	9.78934	9.79887	60′
75877	76939	77963	78950	79903	59
75895	76957	77980	78967	79918	58
75913	76974	77997	78983	79934	57
75931	76991	78013	78999	79950	56
9.75949	9.77009	9.78030	9.79015	9.79965	55
75967	77026	78047	79031	79981	54
75985	77043	78063	79047	79996	53
76003	77061	78080	79063	80012	52
76021	77078	78097	79079	80027	51
9.76039	9.77095	9.78113	9.79095	9.80043	50
76057	77112	78130	79111	80058	49
76075	77130	78147	79128	80074	48
76093	77147	78163	79144	80089	47
76111	77164	78180	79160	80105	46
9.76129	9.77181	9.78197	9.79176	9.80120	45
76146	77199	78213	79192	80136	44
76164	77216	78230	79208	80151	43
76182	77233	78246	79224	80166	42
76200	77250	78263	79240	80182	41
9.76218	9.77268	9.78280	9.79256	9.80197	40
76236	77285	78296	79272	80213	39
76253	77302	78313	79288	80228	38
76271	77319	78329	79304	80244	37
76289	77336	78346	79319	80259	36
9.76307	9.77353	9.78362	9.79335	9.80274	35
76324	77370	78379	79351	80290	34
76342	77387	78395	79367	80305	33
76360	77405	78412	79383	80320	32
76378	77422	78428	79399	80336	31
9.76395	9.77439	9.78445	9.79415	9.80351	30
76413	77456	78461	79431	80366	29
76431	77473	78478	79447	80382	28
76448	77490	78494	79463	80397	27
76466	77507	78510	79478	80412	26
9.76484	9.77524	9.78527	9.79494	9.80428	25
76501	77541	78543	79510	80443	24
76519	77558	78560	79526	80458	23
76537	77575	78576	79542	80473	22
76554	77592	78592	79558	80489	21
9.76572	9.77609	9.78609	9.79573	9.80504	20
76590	77626	78625	79589	80519	19
76607	77643	78642	79605	80534	18
76625	77660	78658	79621	80550	17
76642	77677	78674	79636	80565	16
9.76660	9.77694	9.78691	9.79652	9.80580	15
76677	77711	78707	79668	80595	14
76695	77728	78723	79684	80610	13
76712	77744	78739	79699	80625	12
76730	77761	78756	79715	80641	11
9.76747	9.77778	9.78772	9.79731	9.80656	10
76765	77795	78788	79746	80671	9
76782	77812	78805	79762	80686	8
76800	77829	78821	79778	80701	7
76817	77846	78837	79793	80716	6
9.76835	9.77862	9.78853	9.79809	9.80731	5
76852	77879	78869	79825	80746	4
76870	77896	78886	79840	80762	3
76887	77913	78902	79856	80777	2
76904	77930	78918	79872	80792	1
9.76922	9.77946	.78934	9.79887	9.80807	0
54°	53°	52°	51°	50°	Cos
125°	126°	127°	128°	129°	

	139°	138°	137°	136°	135°
Sin	40°	41°	42°	43°	44°
0′	9.80807	9.81694	9.82551	9.83378	9.84177
1	80822	81709	82565	83392	84190
2	80837	81723	82579	83405	84203
3	80852	81738	82593	83419	84216
4	80867	81752	82607	83432	84229
5	9.80882	9.81767	9.82621	9.83446	9.84242
6	80897	81781	82635	83459	84255
7	80912	81796	82649	83473	84269
8	80927	81810	82663	83486	84282
9	80942	81825	82677	83500	84295
10	9.80957	9.81839	9.82691	9.83513	9.84308
11	80972	81854	82705	83527	84321
12	80987	81868	82719	83540	84334
13	81002	81882	82733	83554	84347
14	81017	81897	82747	83567	84360
15	9.81032	9.81911	9.82761	9.83581	9.84373
16	81047	81926	82775	83594	84385
17	81061	81940	82788	83608	84398
18	81076	81955	82802	83621	84411
19	81091	81969	82816	83634	84424
20	9.81106	9.81983	9.82830	9.83648	9.84437
21	81121	81998	82844	83661	84450
22	81136	82012	82858	83674	84463
23	81151	82026	82872	83688	84476
24	81166	82041	82885	83701	84489
25	9.81180	9.82055	9.82899	9.83715	9.84502
26	81195	82069	82913	83728	84515
27	81210	82084	82927	83741	84528
28	81225	82098	82941	83755	84540
29	81240	82112	82955	83768	84553
30	9.81254	9.82126	9.82968	9.83781	9.84566
31	81269	82141	82982	83795	84579
32	81284	82155	82996	83808	84592
33	81299	82169	83010	83821	84605
34	81314	82184	83023	83834	84618
35	9.81328	9.82198	9.83037	9.83848	9.84630
36	81343	82212	83051	83861	84643
37	81358	82226	83065	83874	84656
38	81372	82240	83078	83887	84669
39	81387	82255	83092	83901	84682
40	9.81402	9.82269	9.83106	9.83914	9.84694
41	81417	82283	83120	83927	84707
42	81431	82297	83133	83940	84720
43	81446	82311	83147	83954	84733
44	81461	82326	83161	83967	84745
45	9.81475	9.82340	9.83174	9.83980	9.84758
46	81490	82354	83188	83993	84771
47	81505	82368	83202	84006	84784
48	81519	82382	83215	84020	84796
49	81534	82396	83229	84033	84809
50	9.81549	9.82410	9.83242	9.84046	9.84822
51	81563	82424	83256	84059	84835
52	81578	82439	83270	84072	84847
53	81592	82453	83283	84085	84860
54	81607	82467	83297	84098	84873
55	9.81622	9.82481	9.83310	9.84112	9.84885
56	81636	82495	83324	84125	84898
57	81651	82509	83338	84138	84911
58	81665	82523	83351	84151	84923
59	81680	82537	83365	84164	84936
60	9.81694	9.82551	9.83378	9.84177	9.84949
	49°	48°	47°	46°	45°
Cos	130°	131°	132°	133°	134°

134°	133°	132°	131°	130°	Sin
45°	46°	47°	48°	49°	
9.84949	9.85693	9.86413	9.87107	9.87778	60′
84961	85706	86425	87119	87789	59
84974	85718	86436	87130	87800	58
84986	85730	86448	87141	87811	57
84999	85742	86460	87153	87822	56
9.85012	9.85754	9.86472	9.87164	9.87833	55
85024	85766	86483	87175	87844	54
85037	85779	86495	87187	87855	53
85049	85791	86507	87198	87866	52
85062	85803	86518	87209	87877	51
9.85074	9.85815	9.86530	9.87221	9.87887	50
85087	85827	86542	87232	87898	49
85100	85839	86554	87243	87909	48
85112	85851	86565	87255	87920	47
85125	85864	86577	87266	87931	46
9.85137	9.85876	9.86589	9.87277	9.87942	45
85150	85888	86600	87288	87953	44
85162	85900	86612	87300	87964	43
85175	85912	86624	87311	87975	42
85187	85924	86635	87322	87985	41
9.85200	9.85936	9.86647	9.87334	9.87996	40
85212	85948	86659	87345	88007	39
85225	85960	86670	87356	88018	38
85237	85972	86682	87367	88029	37
85250	85984	86694	87378	88040	36
9.85262	9.85996	9.86705	9.87390	9.88051	35
85274	86008	86717	87401	88061	34
85287	86020	86728	87412	88072	33
85299	86032	86740	87423	88083	32
85312	86044	86752	87434	88094	31
9.85324	9.86056	9.86763	9.87446	9.88105	30
85337	86068	86775	87457	88115	29
85349	86080	86786	87468	88126	28
85361	86092	86798	87479	88137	27
85374	86104	86809	87490	88148	26
9.85386	9.86116	9.86821	9.87501	9.88158	25
85399	86128	86832	87513	88169	24
85411	86140	86844	87524	88180	23
85423	86152	86855	87535	88191	22
85436	86164	86867	87546	88201	21
9.85448	9.86176	9.86879	9.87557	9.88212	20
85460	86188	86890	87568	88223	19
85473	86200	86902	87579	88234	18
85485	86211	86913	87590	88244	17
85497	86223	86924	87601	88255	16
9.85510	9.86235	9.86936	9.87613	9.88266	15
85522	86247	86947	87624	88276	14
85534	86259	86959	87635	88287	13
85547	86271	86970	87646	88298	12
85559	86283	86982	87657	88308	11
9.85571	9.86295	9.86993	9.87668	9.88319	10
85583	86306	87005	87679	88330	9
85596	86318	87016	87690	88340	8
85608	86330	87028	87701	88351	7
85620	86342	87039	87712	88362	6
9.85632	9.86354	9.87050	9.87723	9.88372	5
85645	86366	87062	87734	88383	4
85657	86377	87073	87745	88394	3
85669	86389	87085	87756	88404	2
85681	86401	87096	87767	88415	1
9.85693	9.86413	9.87107	9.87778	9.88425	0
44°	43°	42°	41°	40°	Cos
135°	136°	137°	138°	139°	

	129°	128°	127°	126°	125°
Sin	50°	51°	52°	53°	54°
0′	9.88425	9.89050	9.89653	9.90235	9.90796
1	88436	89060	89663	90244	90805
2	88447	89071	89673	90254	90814
3	88457	89081	89683	90263	90823
4	88468	89091	89693	90273	90832
5	9.88478	9.89101	9.89702	9.90282	9.90842
6	88489	89112	89712	90292	90851
7	88499	89122	89722	90301	90860
8	88510	89132	89732	90311	90869
9	88521	89142	89742	90320	90878
10	9.88531	9.89152	9.89752	9.90330	9.90887
11	88542	89162	89761	90339	90896
12	88552	89173	89771	90349	90906
13	88563	89183	89781	90358	90915
14	88573	89193	89791	90368	90924
15	9.88584	9.89203	9.89801	9.90377	9.90933
16	88594	89213	89810	90386	90942
17	88605	89223	89820	90396	90951
18	88615	89233	89830	90405	90960
19	88626	89244	89840	90415	90969
20	9.88636	9.89254	9.89849	9.90424	9.90978
21	88647	89264	89859	90434	90987
22	88657	89274	89869	90443	90996
23	88668	89284	89879	90452	91005
24	88678	89294	89888	90462	91014
25	9.88688	9.89304	9.89898	9.90471	9.91023
26	88699	89314	89908	90480	91033
27	88709	89324	89918	90490	91042
28	88720	89334	89927	90499	91051
29	88730	89344	89937	90509	91060
30	9.88741	9.89354	9.89947	9.90518	9.91069
31	88751	89364	89956	90527	91078
32	88761	89375	89966	90537	91087
33	88772	89385	89976	90546	91096
34	88782	89395	89985	90555	91105
35	9.88793	9.89405	9.89995	9.90565	9.91114
36	88803	89415	90005	90574	91123
37	88813	89425	90014	90583	91132
38	88824	89435	90024	90592	91141
39	88834	89445	90034	90602	91149
40	9.88844	9.89455	9.90043	9.90611	9.91158
41	88855	89465	90053	90620	91167
42	88865	89475	90063	90630	91176
43	88875	89485	90072	90639	91185
44	88886	89495	90082	90648	91194
45	9.88896	9.89504	9.90091	9.90657	9.91203
46	88906	89514	90101	90667	91212
47	88917	89524	90111	90676	91221
48	88927	89534	90120	90685	91230
49	88937	89544	90130	90694	91239
50	9.88948	9.89554	9.90139	9.90704	9.91248
51	88958	89564	90149	90713	91257
52	88968	89574	90159	90722	91266
53	88978	89584	90168	90731	91274
54	88989	89594	90178	90741	91283
55	9.88999	9.89604	9.90187	9.90750	9.91292
56	89009	89614	90197	90759	91301
57	89020	89624	90206	90768	91310
58	89030	89633	90216	90777	91319
59	89040	89643	90225	90787	91328
60	9.89050	9.89653	9.90235	9.90796	9.91336
	39°	38°	37°	36°	35°
Cos	140°	141°	142°	143°	144°

124°	123°	122°	121°	120°	Sin
55°	**56°**	**57°**	**58°**	**59°**	
9.91336	9.91857	9.92359	9.92842	9.93307	**60′**
91345	91866	92367	92850	93314	**59**
91354	91874	92376	92858	93322	**58**
91363	91883	92384	92866	93329	**57**
91372	91891	92392	92874	93337	**56**
9.91381	9.91900	9.92400	9.92881	9.93344	**55**
91389	91908	92408	92889	93352	**54**
91398	91917	92416	92897	93360	**53**
91407	91925	92425	92905	93367	**52**
91416	91934	92433	92913	93375	**51**
9.91425	9.91942	9.92441	9.92921	9.93382	**50**
91433	91951	92449	92929	93390	**49**
91442	91959	92457	92936	93397	**48**
91451	91968	92465	92944	93405	**47**
91460	91976	92473	92952	93412	**46**
9.91469	9.91985	9.92482	9.92960	9.93420	**45**
91477	91993	92490	92968	93427	**44**
91486	92002	92498	92976	93435	**43**
91495	92010	92506	92983	93442	**42**
91504	92018	92514	92991	93450	**41**
9.91512	9.92027	9.92522	9.92999	9.93457	**40**
91521	92035	92530	93007	93465	**39**
91530	92044	92538	93014	93472	**38**
91538	92052	92546	93022	93480	**37**
91547	92060	92555	93030	93487	**36**
9.91556	9.92069	9.92563	9.93038	9.93495	**35**
91565	92077	92571	93046	93502	**34**
91573	92086	92579	93053	93510	**33**
91582	92094	92587	93061	93517	**32**
91591	92102	92595	93069	93525	**31**
9.91599	9.92111	9.92603	9.93077	9.93532	**30**
91608	92119	92611	93084	93539	**29**
91617	92127	92619	93092	93547	**28**
91625	92136	92627	93100	93554	**27**
91634	92144	92635	93108	93562	**26**
9.91643	9.92152	9.92643	9.93115	9.93569	**25**
91651	92161	92651	93123	93577	**24**
91660	92169	92659	93131	93584	**23**
91669	92177	92667	93138	93591	**22**
91677	92186	92675	93146	93599	**21**
9.91686	9.92194	9.92683	9.93154	9.93606	**20**
91695	92202	92691	93161	93614	**19**
91703	92211	92699	93169	93621	**18**
91712	92219	92707	93177	93628	**17**
91720	92227	92715	93184	93636	**16**
9.91729	9.92235	9.92723	9.93192	9.93643	**15**
91738	92244	92731	93200	93650	**14**
91746	92252	92739	93207	93658	**13**
91755	92260	92747	93215	93665	**12**
91763	92269	92755	93223	93673	**11**
9.91772	9.92277	9.92763	9.93230	9.93680	**10**
91781	92285	92771	93238	93687	**9**
91789	92293	92779	93246	93695	**8**
91798	92302	92787	93253	93702	**7**
91806	92310	92795	93261	93709	**6**
9.91815	9.92318	9.92803	9.93269	9.93717	**5**
91823	92326	92810	93276	93724	**4**
91832	92335	92818	93284	93731	**3**
91840	92343	92826	93291	93738	**2**
91849	92351	92834	93299	93746	**1**
9.91857	9.92359	9.92842	9.93307	9.93753	**0**
34°	**33°**	**32°**	**31°**	**30°**	Cos
145°	146°	147°	148°	149°	

	119°	118°	117°	116°	115°
Sin	60°	61°	62°	63°	64°
0′	9.93753	9.94182	9.94593	9.94988	9.95366
1	93760	94189	94600	94995	95372
2	93768	94196	94607	95001	95378
3	93775	94203	94614	95007	95384
4	93782	94210	94620	95014	95391
5	9.93789	9.94217	9.94627	9.95020	9.95397
6	93797	94224	94634	95027	95403
7	93804	94231	94640	95033	95409
8	93811	94238	94647	95039	95415
9	93819	94245	94654	95046	95421
10	9.93826	9.94252	9.94660	9.95052	9.95427
11	93833	94259	94667	95059	95434
12	93840	94266	94674	95065	95440
13	93847	94273	94680	95071	95446
14	93855	94279	94687	95078	95452
15	9.93862	9.94286	9.94694	9.95084	9.95458
16	93869	94293	94700	95090	95464
17	93876	94300	94707	95097	95470
18	93884	94307	94714	95103	95476
19	93891	94314	94720	95110	95482
20	9.93898	9.94321	9.94727	9.95116	9.95488
21	93905	94328	94734	95122	95494
22	93912	94335	94740	95129	95500
23	93920	94342	94747	95135	95507
24	93927	94349	94753	95141	95513
25	9.93934	9.94355	9.94760	9.95148	9.95519
26	93941	94362	94767	95154	95525
27	93948	94369	94773	95160	95531
28	93955	94376	94780	95167	95537
29	93963	94383	94786	95173	95543
30	9.93970	9.94390	9.94793	9.95179	9.95549
31	93977	94397	94799	95185	95555
32	93984	94404	94806	95192	95561
33	93991	94410	94813	95198	95567
34	93998	94417	94819	95204	95573
35	9.94005	9.94424	9.94826	9.95211	9.95579
36	94012	94431	94832	95217	95585
37	94020	94438	94839	95223	95591
38	94027	94445	94845	95229	95597
39	94034	94451	94852	95236	95603
40	9.94041	9.94458	9.94858	9.95242	9.95609
41	94048	94465	94865	95248	95615
42	94055	94472	94871	95254	95621
43	94062	94479	94878	95261	95627
44	94069	94485	94885	95267	95633
45	9.94076	9.94492	9.94891	9.95273	9.95639
46	94083	94499	94898	95279	95645
47	94090	94506	94904	95286	95651
48	94098	94513	94911	95292	95657
49	94105	94519	94917	95298	95663
50	9.94112	9.94526	9.94923	9.95304	9.95668
51	94119	94533	94930	95310	95674
52	94126	94540	94936	95317	95680
53	94133	94546	94943	95323	95686
54	94140	94553	94949	95329	95692
55	9.94147	9.94560	9.94956	9.95335	9.95698
56	94154	94567	94962	95341	95704
57	94161	94573	94969	95348	95710
58	94168	94580	94975	95354	95716
59	94175	94587	94982	95360	95722
60	9.94182	9.94593	9.94988	9.95366	9.95728
	29°	28°	27°	26°	25°
Cos	150°	151°	152°	153°	154°

114°	113°	112°	111°	110°	Sin
65°	66°	67°	68°	69°	
9.95728	9.96073	9.96403	9.96717	9.97015	60′
95733	96079	96408	96722	97020	59
95739	96084	96413	96727	97025	58
95745	96090	96419	96732	97030	57
95751	96095	96424	96737	97035	56
9.95757	9.96101	9.96429	9.96742	9.97039	55
95763	96107	96435	96747	97044	54
95769	96112	96440	96752	97049	53
95775	96118	96445	96757	97054	52
95780	96123	96451	96762	97059	51
9.95786	9.96129	9.96456	9.96767	9.97063	50
95792	96135	96461	96772	97068	49
95798	96140	96467	96778	97073	48
95804	96146	96472	96783	97078	47
95810	96151	96477	96788	97083	46
9.95815	9.96157	9.96483	9.96793	9.97087	45
95821	96162	96488	96798	97092	44
95827	96168	96493	96803	97097	43
95833	96174	96498	96808	97102	42
95839	96179	96504	96813	97107	41
9.95844	9.96185	9.96509	9.96818	9.97111	40
95850	96190	96514	96823	97116	39
95856	96196	96520	96828	97121	38
95862	96201	96525	96833	97126	37
95868	96207	96530	96838	97130	36
9.95873	9.96212	9.96535	9.96843	9.97135	35
95879	96218	96541	96848	97140	34
95885	96223	96546	96853	97145	33
95891	96229	96551	96858	97149	32
95897	96234	96556	96863	97154	31
9.95902	9.96240	9.96562	9.96868	9.97159	30
95908	96245	96567	96873	97163	29
95914	96251	96572	96878	97168	28
95920	96256	96577	96883	97173	27
95925	96262	96582	96888	97178	26
9.95931	9.96267	9.96588	9.96893	9.97182	25
95937	96273	96593	96898	97187	24
95942	96278	96598	96903	97192	23
95948	96284	96603	96907	97196	22
95954	96289	96608	96912	97201	21
9.95960	9.96294	9.96614	9.96917	9.97206	20
95965	96300	96619	96922	97210	19
95971	96305	96624	96927	97215	18
95977	96311	96629	96932	97220	17
95982	96316	96634	96937	97224	16
9.95988	9.96322	9.96640	9.96942	9.97229	15
95994	96327	96645	96947	97234	14
96000	96333	96650	96952	97238	13
96005	96338	96655	96957	97243	12
96011	96343	96660	96962	97248	11
9.96017	9.96349	9.96665	9.96966	9.97252	10
96022	96354	96670	96971	97257	9
96028	96360	96676	96976	97262	8
96034	96365	96681	96981	97266	7
96039	96370	96686	96986	97271	6
9.96045	9.96376	9.96691	9.96991	9.97276	5
96050	96381	96696	96996	97280	4
96056	96387	96701	97001	97285	3
96062	96392	96706	97005	97289	2
96067	96397	96711	97010	97294	1
9.96073	9.96403	9.96717	9.97015	9.97299	0
24°	23°	22°	21°	20°	Cos
155°	156°	157°	158°	159°	

	109°	108°	107°	106°	105°
Sin	70°	71°	72°	73°	74°
0′	9.97299	9.97567	9.97821	9.98060	9.98284
1	97303	97571	97825	98063	98288
2	97308	97576	97829	98067	98291
3	97312	97580	97833	98071	98295
4	97317	97584	97837	98075	98299
5	9.97322	9.97589	9.97841	9.98079	9.98302
6	97326	97593	97845	98083	98306
7	97331	97597	97849	98087	98309
8	97335	97602	97853	98090	98313
9	97340	97606	97857	98094	98317
10	9.97344	9.97610	9.97861	9.98098	9.98320
11	97349	97615	97866	98102	98324
12	97353	97619	97870	98106	98327
13	97358	97623	97874	98110	98331
14	97363	97628	97878	98113	98334
15	9.97367	9.97632	9.97882	9.98117	9.98338
16	97372	97636	97886	98121	98342
17	97376	97640	97890	98125	98345
18	97381	97645	97894	98129	98349
19	97385	97649	97898	98132	98352
20	9.97390	9.97653	9.97902	9.98136	9.98356
21	97394	97657	97906	98140	98359
22	97399	97662	97910	98144	98363
23	97403	97666	97914	98147	98366
24	97408	97670	97918	98151	98370
25	9.97412	9.97674	9.97922	9.98155	9.98373
26	97417	97679	97926	98159	98377
27	97421	97683	97930	98162	98381
28	97426	97687	97934	98166	98384
29	97430	97691	97938	98170	98388
30	9.97435	9.97696	9.97942	9.98174	9.98391
31	97439	97700	97946	98177	98395
32	97444	97704	97950	98181	98398
33	97448	97708	97954	98185	98402
34	97453	97713	97958	98189	98405
35	9.97457	9.97717	9.97962	9.98192	9.98409
36	97461	97721	97966	98196	98412
37	97466	97725	97970	98200	98415
38	97470	97729	97974	98204	98419
39	97475	97734	97978	98207	98422
40	9.97479	9.97738	9.97982	9.98211	9.98426
41	97484	97742	97986	98215	98429
42	97488	97746	97989	98218	98433
43	97492	97750	97993	98222	98436
44	97497	97754	97997	98226	98440
45	9.97501	9.97759	9.98001	9.98229	9.98443
46	97506	97763	98005	98233	98447
47	97510	97767	98009	98237	98450
48	97515	97771	98013	98240	98453
49	97519	97775	98017	98244	98457
50	9.97523	9.97779	9.98021	9.98248	9.98460
51	97528	97784	98025	98251	98464
52	97532	97788	98029	98255	98467
53	97536	97792	98032	98259	98471
54	97541	97796	98036	98262	98474
55	9.97545	9.97800	9.98040	9.98266	9.98477
56	97550	97804	98044	98270	98481
57	97554	97808	98048	98273	98484
58	97558	97812	98052	98277	98488
59	97563	97817	98056	98281	98491
60	9.97567	9.97821	9.98060	9.98284	9.98494
	19°	18°	17°	16°	15°
Cos	160°	161°	162°	163°	164°

104°	103°	102°	101°	100°	Sin
75°	**76°**	**77°**	**78°**	**79°**	
9.98494	9.98690	9.98872	9.99040	9.99195	**60′**
98498	98694	98875	99043	99197	**59**
98501	98697	98878	99046	99200	**58**
98505	98700	98881	99048	99202	**57**
98508	98703	98884	99051	99204	**56**
9.98511	9.98706	9.98887	9.99054	9.99207	**55**
98515	98709	98890	99056	99209	**54**
98518	98712	98893	99059	99212	**53**
98521	98715	98896	99062	99214	**52**
98525	98719	98898	99064	99217	**51**
9.98528	9.98722	9.98901	9.99067	9.99219	**50**
98531	98725	98904	99070	99221	**49**
98535	98728	98907	99072	99224	**48**
98538	98731	98910	99075	99226	**47**
98541	98734	98913	99078	99229	**46**
9.98545	9.98737	9.98916	9.99080	9.99231	**45**
98548	98740	98919	99083	99233	**44**
98551	98743	98921	99086	99236	**43**
98555	98746	98924	99088	99238	**42**
98558	98750	98927	99091	99241	**41**
9.98561	9.98753	9.98930	9.99093	9.99243	**40**
98565	98756	98933	99096	99245	**39**
98568	98759	98936	99099	99248	**38**
98571	98762	98938	99101	99250	**37**
98574	98765	98941	99104	99252	**36**
9.98578	9.98768	9.98944	9.99106	9.99255	**35**
98581	98771	98947	99109	99257	**34**
98584	98774	98950	99112	99260	**33**
98588	98777	98953	99114	99262	**32**
98591	98780	98955	99117	99264	**31**
9.98594	9.98783	9.98958	9.99119	9.99267	**30**
98597	98786	98961	99122	99269	**29**
98601	98789	98964	99124	99271	**28**
98604	98792	98967	99127	99274	**27**
98607	98795	98969	99130	99276	**26**
9.98610	9.98798	9.98972	9.99132	9.99278	**25**
98614	98801	98975	99135	99281	**24**
98617	98804	98978	99137	99283	**23**
98620	98807	98980	99140	99285	**22**
98623	98810	98983	99142	99288	**21**
9.98627	9.98813	9.98986	9.99145	9.99290	**20**
98630	98816	98989	99147	99292	**19**
98633	98819	98991	99150	99294	**18**
98636	98822	98994	99152	99297	**17**
98640	98825	98997	99155	99299	**16**
9.98643	9.98828	9.99000	9.99157	9.99301	**15**
98646	98831	99002	99160	99304	**14**
98649	98834	99005	99162	99306	**13**
98652	98837	99008	99165	99308	**12**
98656	98840	99011	99167	99310	**11**
9.98659	9.98843	9.99013	9.99170	9.99313	**10**
.98662	98846	99016	99172	99315	**9**
98665	98849	99019	99175	99317	**8**
98668	98852	99022	99177	99319	**7**
98671	98855	99024	99180	99322	**6**
9.98675	9.98858	9.99027	9.99182	9.99324	**5**
98678	98861	99030	99185	99326	**4**
98681	98864	99032	99187	99328	**3**
98684	98867	99035	99190	99331	**2**
98687	98869	99038	99192	99333	**1**
9.98690	9.98872	9.99040	9.99195	9.99335	**0**
14°	**13°**	**12°**	**11°**	**10°**	Cos
165°	166°	167°	168°	169°	

	99°	98°	97°	96°	95°
Sin	80°	81°	82°	83°	84°
0′	9.99335	9.99462	9.99575	9.99675	9.99761
1	99337	99464	99577	99677	99763
2	99340	99466	99579	99678	99764
3	99342	99468	99581	99680	99765
4	99344	99470	99582	99681	99767
5	9.99346	9.99472	9.99584	9.99683	9.99768
6	99348	99474	99586	99684	99769
7	99351	99476	99588	99686	99771
8	99353	99478	99589	99687	99772
9	99355	99480	99591	99689	99773
10	9.99357	9.99482	9.99593	9.99690	9.99775
11	99359	99484	99595	99692	99776
12	99362	99486	99596	99693	99777
13	99364	99488	99598	99695	99778
14	99366	99490	99600	99696	99780
15	9.99368	9.99492	9.99601	9.99698	9.99781
16	99370	99494	99603	99699	99782
17	99372	99495	99605	99701	99783
18	99375	99497	99607	99702	99785
19	99377	99499	99608	99704	99786
20	9.99379	9.99501	9.99610	9.99705	9.99787
21	99381	99503	99612	99707	99788
22	99383	99505	99613	99708	99790
23	99385	99507	99615	99710	99791
24	99388	99509	99617	99711	99792
25	9.99390	9.99511	9.99618	9.99713	9.99793
26	99392	99513	99620	99714	99795
27	99394	99515	99622	99716	99796
28	99396	99517	99624	99717	99797
29	99398	99518	99625	99718	99798
30	9.99400	9.99520	9.99627	9.99720	9.99800
31	99402	99522	99629	99721	99801
32	99404	99524	99630	99723	99802
33	99407	99526	99632	99724	99803
34	99409	99528	99633	99726	99804
35	9.99411	9.99530	9.99635	9.99727	9.99806
36	99413	99532	99637	99728	99807
37	99415	99533	99638	99730	99808
38	99417	99535	99640	99731	99809
39	99419	99537	99642	99733	99810
40	9.99421	9.99539	9.99643	9.99734	9.99812
41	99423	99541	99645	99736	99813
42	99425	99543	99647	99737	99814
43	99427	99545	99648	99738	99815
44	99429	99546	99650	99740	99816
45	9.99432	9.99548	9.99651	9.99741	9.99817
46	99434	99550	99653	99742	99819
47	99436	99552	99655	99744	99820
48	99438	99554	99656	99745	99821
49	99440	99556	99658	99747	99822
50	9.99442	9.99557	9.99659	9.99748	9.99823
51	99444	99559	99661	99749	99824
52	99446	99561	99663	99751	99825
53	99448	99563	99664	99752	99827
54	99450	99565	99666	99753	99828
55	9.99452	9.99566	9.99667	9.99755	9.99829
56	99454	99568	99669	99756	99830
57	99456	99570	99670	99757	99831
58	99458	99572	99672	99759	99832
59	99460	99574	99674	99760	99833
60	9.99462	9.99575	9.99675	9.99761	9.99834
	9°	8°	7°	6°	5°
Cos	170°	171°	172°	173°	174°

94°	93°	92°	91°	90°	Sin
85°	86°	87°	88°	89°	
9.99834	9.99894	9.99940	9.99974	9.99993	60′
99836	99895	99941	99974	99994	59
99837	99896	99942	99974	99994	58
99838	99897	99942	99975	99994	57
99839	99898	99943	99975	99994	56
9.99840	9.99898	9.99944	9.99976	9.99994	55
99841	99899	99944	99976	99995	54
99842	99900	99945	99977	99995	53
99843	99901	99946	99977	99995	52
99844	99902	99946	99977	99995	51
9.99845	9.99903	9.99947	9.99978	9.99995	50
99846	99904	99948	99978	99996	49
99847	99904	99948	99979	99996	48
99848	99905	99949	99979	99996	47
99850	99906	99949	99979	99996	46
9.99851	9.99907	9.99950	9.99980	9.99996	45
99852	99908	99951	99980	99996	44
99853	99909	99951	99981	99997	43
99854	99909	99952	99981	99997	42
99855	99910	99952	99981	99997	41
9.99856	9.99911	9.99953	9.99982	9.99997	40
99857	99912	99954	99982	99997	39
99858	99913	99954	99982	99997	38
99859	99913	99955	99983	99997	37
99860	99914	99955	99983	99998	36
9.99861	9.99915	9.99956	9.99983	9.99998	35
99862	99916	99956	99984	99998	34
99863	99917	99957	99984	99998	33
99864	99917	99958	99984	99998	32
99865	99918	99958	99985	99998	31
9.99866	9.99919	9.99959	9.99985	9.99998	30
99867	99920	99959	99985	99998	29
99868	99920	99960	99986	99999	28
99869	99921	99960	99986	99999	27
99870	99922	99961	99986	99999	26
9.99871	9.99923	9.99961	9.99987	9.99999	25
99872	99923	99962	99987	99999	24
99873	99924	99962	99987	99999	23
99874	99925	99963	99988	99999	22
99875	99926	99963	99988	99999	21
9.99876	9.99926	9.99964	9.99988	9.99999	20
99877	99927	99964	99989	99999	19
99878	99928	99965	99989	99999	18
99879	99929	99966	99989	99999	17
99879	99929	99966	99989	00000	16
9.99880	9.99930	9.99967	9.99990	0.00000	15
99881	99931	99967	99990	00000	14
99882	99932	99967	99990	00000	13
99883	99932	99968	99990	00000	12
99884	99933	99968	99991	00000	11
9.99885	9.99934	9.99969	9 99991	0.00000	10
99886	99934	99969	99991	00000	9
99887	99935	99970	99992	00000	8
99888	99936	99970	99992	00000	7
99889	99936	99971	99992	00000	6
9.99890	9.99937	9.99971	9.99992	0.00000	5
99891	99938	99972	99992	00000	4
99891	99938	99972	99993	00000	3
99892	99939	99973	99993	00000	2
99893	99940	99973	99993	00000	1
9.99894	9.99940	9.99974	9.99993	0.00000	0
4°	3°	2°	1°	0°	Cos
175°	176°	177°	178°	179°	

	179°	178°	177°	176°	175°
Tan	0°	1°	2°	3°	4°
0′	− ∞	8.24192	8.54308	8.71940	8.84464
1	6.46373	24910	54669	72181	84646
2	76476	25616	55027	72420	84826
3	94085	26312	55382	72659	85006
4	7.06579	26996	55734	72896	85185
5	7.16270	8.27669	8.56083	8.73132	8.85363
6	24188	28332	56429	73366	85540
7	30882	28986	56773	73600	85717
8	36682	29629	57114	73832	85893
9	41797	30263	57452	74063	86069
10	7.46373	8.30888	8.57788	8.74292	8.86243
11	50512	31505	58121	74521	86417
12	54291	32112	58451	74748	86591
13	57767	32711	58779	74974	86763
14	60986	33302	59105	75199	86935
15	7.63982	8.33886	8.59428	8.75423	8.87106
16	66785	34461	59749	75645	87277
17	69418	35029	60068	75867	87447
18	71900	35590	60384	76087	87616
19	74248	36143	60698	76306	87785
20	7.76476	8.36689	8.61009	8.76525	8.87953
21	78595	37229	61319	76742	88120
22	80615	37762	61626	76958	88287
23	82546	38289	61931	77173	88453
24	84394	38809	62234	77387	88618
25	7.86167	8.39323	8.62535	8.77600	8.88783
26	87871	39832	62834	77811	88948
27	89510	40334	63131	78022	89111
28	91089	40830	63426	78232	89274
29	92613	41321	63718	78441	89437
30	7.94086	8.41807	8.64009	8.78649	8.89598
31	95510	42287	64298	78855	89760
32	96889	42762	64585	79061	89920
33	98225	43232	64870	79266	90080
34	99522	43696	65154	79470	90240
35	8.00781	8.44156	8.65435	8.79673	8.90399
36	02004	44611	65715	79875	90557
37	03194	45061	65993	80076	90715
38	04353	45507	66269	80277	90872
39	05481	45948	66543	80476	91029
40	8.06581	8.46385	8.66816	8.80674	8.91185
41	07653	46817	67087	80872	91340
42	08700	47245	67356	81068	91495
43	09722	47669	67624	81264	91650
44	10720	48089	67890	81459	91803
45	8.11696	8.48505	8.68154	8.81653	8.91957
46	12651	48917	68417	81846	92110
47	13585	49325	68678	82038	92262
48	14500	49729	68938	82230	92414
49	15395	50130	69196	82420	92565
50	8.16273	8.50527	8.69453	8.82610	8.92716
51	17133	50920	69708	82799	92866
52	17976	51310	69962	82987	93016
53	18804	51696	70214	83175	93165
54	19616	52079	70465	83361	93313
55	8.20413	8.52459	8.70714	8.83547	8.93462
56	21195	52835	70962	83732	93609
57	21964	53208	71208	83916	93756
58	22720	53578	71453	84100	93903
59	23462	53945	71697	84282	94049
60	8.24192	8.54308	8.71940	8.84464	8.94195
	89°	88°	87°	86°	85°
Cot	90°	91°	92°	93°	94°

174°	173°	172°	171°	170°	Tan
5°	**6°**	**7°**	**8°**	**9°**	
8.94195	9.02162	9.08914	9.14780	9.19971	**60′**
94340	02283	09019	14872	20053	**59**
94485	02404	09123	14963	20134	**58**
94630	02525	09227	15054	20216	**57**
94773	02645	09330	15145	20297	**56**
8.94917	9.02766	9.09434	9.15236	9.20378	**55**
95060	02885	09537	15327	20459	**54**
95202	03005	09640	15417	20540	**53**
95344	03124	09742	15508	20621	**52**
95486	03242	09845	15598	20701	**51**
8.95627	9.03361	9.09947	9.15688	9.20782	**50**
95767	03479	10049	15777	20862	**49**
95908	03597	10150	15867	20942	**48**
96047	03714	10252	15956	21022	**47**
96187	03832	10353	16046	21102	**46**
8.96325	9.03948	9.10454	9.16135	9.21182	**45**
96464	04065	10555	16224	21261	**44**
96602	04181	10656	16312	21341	**43**
96739	04297	10756	16401	21420	**42**
96877	04413	10856	16489	21499	**41**
8.97013	9.04528	9.10956	9.16577	9.21578	**40**
97150	04643	11056	16665	21657	**39**
97285	04758	11155	16753	21736	**38**
97421	04873	11254	16841	21814	**37**
97556	04987	11353	16928	21893	**36**
8.97691	9.05101	9.11452	9.17016	9.21971	**35**
97825	05214	11551	17103	22049	**34**
97959	05328	11649	17190	22127	**33**
98092	05441	11747	17277	22205	**32**
98225	05553	11845	17363	22283	**31**
8.98358	9.05666	9.11943	9.17450	9.22361	**30**
98490	05778	12040	17536	22438	**29**
98622	05890	12138	17622	22516	**28**
98753	06002	12235	17708	22593	**27**
98884	06113	12332	17794	22670	**26**
8.99015	9.06224	9.12428	9.17880	9.22747	**25**
99145	06335	12525	17965	22824	**24**
99275	06445	12621	18051	22901	**23**
99405	06556	12717	18136	22977	**22**
99534	06666	12813	18221	23054	**21**
8.99662	9.06775	9.12909	9.18306	9.23130	**20**
99791	06885	13004	18391	23206	**19**
99919	06994	13099	18475	23283	**18**
9.00046	07103	13194	18560	23359	**17**
00174	07211	13289	18644	23435	**16**
9.00301	9.07320	9.13384	9.18728	9.23510	**15**
00427	07428	13478	18812	23586	**14**
00553	07536	13573	18896	23661	**13**
00679	07643	13667	18979	23737	**12**
00805	07751	13761	19063	23812	**11**
9.00930	9.07858	9.13854	9.19146	9.23887	**10**
01055	07964	13948	19229	23962	**9**
01179	08071	14041	19312	24037	**8**
01303	08177	14134	19395	24112	**7**
01427	08283	14227	19478	24186	**6**
9.01550	9.08389	9.14320	9.19561	9.24261	**5**
01673	08495	14412	19643	24335	**4**
01796	08600	14504	19725	24410	**3**
01918	08705	14597	19807	24484	**2**
02040	08810	14688	19889	24558	**1**
9.02162	9.08914	9.14780	9.19971	9.24632	**0**
84°	**83°**	**82°**	**81°**	**80°**	Cot
95°	96°	97°	98°	99°	

	169°	168°	167°	166°	165°
Tan	10°	11°	12°	13°	14°
0′	9.24632	9.28865	9.32747	9.36336	9.39677
1	24706	28933	32810	36394	39731
2	24779	29000	32872	36452	39785
3	24853	29067	32933	36509	39838
4	24926	29134	32995	36566	39892
5	9.25000	9.29201	9.33057	9.36624	9.39945
6	25073	29268	33119	36681	39999
7	25146	29335	33180	36738	40052
8	25219	29402	33242	36795	40106
9	25292	29468	33303	36852	40159
10	9.25365	9.29535	9.33365	9.36909	9.40212
11	25437	29601	33426	36966	40266
12	25510	29668	33487	37023	40319
13	25582	29734	33548	37080	40372
14	25655	29800	33609	37137	40425
15	9.25727	9.29866	9.33670	9.37193	9.40478
16	25799	29932	33731	37250	40531
17	25871	29998	33792	37306	40584
18	25943	30064	33853	37363	40636
19	26015	30130	33913	37419	40689
20	9.26086	9.30195	9.33974	9.37476	9.40742
21	26158	30261	34034	37532	40795
22	26229	30326	34095	37588	40847
23	26301	30391	34155	37644	40900
24	26372	30457	34215	37700	40952
25	9.26443	9.30522	9.34276	9.37756	9.41005
26	26514	30587	34336	37812	41057
27	26585	30652	34396	37868	41109
28	26655	30717	34456	37924	41161
29	26726	30782	34516	37980	41214
30	9.26797	9.30846	9.34576	9.38035	9.41266
31	26867	30911	34635	38091	41318
32	26937	30975	34695	38147	41370
33	27008	31040	34755	38202	41422
34	27078	31104	34814	38257	41474
35	9.27148	9.31168	9.34874	9.38313	9.41526
36	27218	31233	34933	38368	41578
37	27288	31297	34992	38423	41629
38	27357	31361	35051	38479	41681
39	27427	31425	35111	38534	41733
40	9.27496	9.31489	9.35170	9.38589	9.41784
41	27566	31552	35229	38644	41836
42	27635	31616	35288	38699	41887
43	27704	31679	35347	38754	41939
44	27773	31743	35405	38808	41990
45	9.27842	9.31806	9.35464	9.38863	9.42041
46	27911	31870	35523	38918	42093
47	27980	31933	35581	38972	42144
48	28049	31996	35640	39027	42195
49	28117	32059	35698	39082	42246
50	9.28186	9.32122	9.35757	9.39136	9.42297
51	28254	32185	35815	39190	42348
52	28323	32248	35873	39245	42399
53	28391	32311	35931	39299	42450
54	28459	32373	35989	39353	42501
55	9.28527	9.32436	9.36047	9.39407	9.42552
56	28595	32498	36105	39461	42603
57	28662	32561	36163	39515	42653
58	28730	32623	36221	39569	42704
59	28798	32685	36279	39623	42755
60	9.28865	9.32747	9.36336	9.39677	9.42805
	79°	78°	77°	76°	75°
Cot	100°	101°	102°	103°	104°

164°	163°	162°	161°	160°	Tan
15°	**16°**	**17°**	**18°**	**19°**	
9.42805	9.45750	9.48534	9.51178	9.53697	**60′**
42856	45797	48579	51221	53738	**59**
42906	45845	48624	51264	53779	**58**
42957	45892	48669	51306	53820	**57**
43007	45940	48714	51349	53861	**56**
9.43057	9.45987	9.48759	9.51392	9.53902	**55**
43108	46035	48804	51435	53943	**54**
43158	46082	48849	51478	53984	**53**
43208	46130	48894	51520	54025	**52**
43258	46177	48939	51563	54065	**51**
9.43308	9.46224	9.48984	9.51606	9.54106	**50**
43358	46271	49029	51648	54147	**49**
43408	46319	49073	51691	54187	**48**
43458	46366	49118	51734	54228	**47**
43508	46413	49163	51776	54269	**46**
9.43558	9.46460	9.49207	9.51819	9.54309	**45**
43607	46507	49252	51861	54350	**44**
43657	46554	49296	51903	54390	**43**
43707	46601	49341	51946	54431	**42**
43756	46648	49385	51988	54471	**41**
9.43806	9.46694	9.49430	9.52031	9.54512	**40**
43855	46741	49474	52073	54552	**39**
43905	46788	49519	52115	54593	**38**
43954	46835	49563	52157	54633	**37**
44004	46881	49607	52200	54673	**36**
9.44053	9.46928	9.49652	9.52242	9.54714	**35**
44102	46975	49696	52284	54754	**34**
44151	47021	49740	52326	54794	**33**
44201	47068	49784	52368	54835	**32**
44250	47114	49828	52410	54875	**31**
9.44299	9.47160	9.49872	9.52452	9.54915	**30**
44348	47207	49916	52494	54955	**29**
44397	47253	49960	52536	54995	**28**
44446	47299	50004	52578	55035	**27**
44495	47346	50048	52620	55075	**26**
9.44544	9.47392	9.50092	9.52661	9.55115	**25**
44592	47438	50136	52703	55155	**24**
44641	47484	50180	52745	55195	**23**
44690	47530	50223	52787	55235	**22**
44738	47576	50267	52829	55275	**21**
9.44787	9.47622	9.50311	9.52870	9.55315	**20**
44836	47668	50355	52912	55355	**19**
44884	47714	50398	52953	55395	**18**
44933	47760	50442	52995	55434	**17**
44981	47806	50485	53037	55474	**16**
9.45029	9.47852	9.50529	9.53078	9.55514	**15**
45078	47897	50572	53120	55554	**14**
45126	47943	50616	53161	55593	**13**
45174	47989	50659	53202	55633	**12**
45222	48035	50703	53244	55673	**11**
9.45271	9.48080	9.50746	9.53285	9.55712	**10**
45319	48126	50789	53327	55752	**9**
45367	48171	50833	53368	55791	**8**
45415	48217	50876	53409	55831	**7**
45463	48262	50919	53450	55870	**6**
9.45511	9.48307	9.50962	9.53492	9.55910	**5**
45559	48353	51005	53533	55949	**4**
45606	48398	51048	53574	55989	**3**
45654	48443	51092	53615	56028	**2**
45702	48489	51135	53656	56067	**1**
9.45750	9.48534	9.51178	9.53697	9.56107	**0**
74°	**73°**	**72°**	**71°**	**70°**	Cot
105°	106°	107°	108°	109°	

	159°	158°	157°	156°	155°
Tan	20°	21°	22°	23°	24°
0′	9.56107	9.58418	9.60641	9.62785	9.64858
1	56146	58455	60677	62820	64892
2	56185	58493	60714	62855	64926
3	56224	58531	60750	62890	64960
4	56264	58569	60786	62926	64994
5	9.56303	9.58606	9.60823	9.62961	9.65028
6	56342	58644	60859	62996	65062
7	56381	58681	60895	63031	65096
8	56420	58719	60931	63066	65130
9	56459	58757	60967	63101	65164
10	9.56498	9.58794	9.61004	9.63135	9.65197
11	56537	58832	61040	63170	65231
12	56576	58869	61076	63205	65265
13	56615	58907	61112	63240	65299
14	56654	58944	61148	63275	65333
15	9.56693	9.58981	9.61184	9.63310	9.65366
16	56732	59019	61220	63345	65400
17	56771	59056	61256	63379	65434
18	56810	59094	61292	63414	65467
19	56849	59131	61328	63449	65501
20	9.56887	9.59168	9.61364	9.63484	9.65535
21	56926	59205	61400	63519	65568
22	56965	59243	61436	63553	65602
23	57004	59280	61472	63588	65636
24	57042	59317	61508	63623	65669
25	9.75081	9.59354	9.61544	9.63657	9.65703
26	57120	59391	61579	63692	65736
27	57158	59429	61615	63726	65770
28	57197	59466	61651	63761	65803
29	57235	59503	61687	63796	65837
30	9.57274	9.59540	9.61722	9.63830	9.65870
31	57312	59577	61758	63865	65904
32	57351	59614	61794	63899	65937
33	57389	59651	61830	63934	65971
34	57428	59688	61865	63968	66004
35	9.57466	9.59725	9.61901	9.64003	9.66038
36	57504	59762	61936	64037	66071
37	57543	59799	61972	64072	66104
38	57581	59835	62008	64106	66138
39	57619	59872	62043	64140	66171
40	9.57658	9.59909	9.62079	9.64175	9.66204
41	57696	59946	62114	64209	66238
42	57734	59983	62150	64243	66271
43	57772	60019	62185	64278	66304
44	57810	60056	62221	64312	66337
45	9.57849	9.60093	9.62256	9.64346	9.66371
46	57887	60130	62292	64381	66404
47	57925	60166	62327	64415	66437
48	57963	60203	62362	64449	66470
49	58001	60240	62398	64483	66503
50	9.58039	9.60276	9.62433	9.64517	9.66537
51	58077	60313	62468	64552	66570
52	58115	60349	62504	64586	66603
53	58153	60386	62539	64620	66636
54	58191	60422	62574	64654	66669
55	9.58229	9.60459	9.62609	9.64688	9.66702
56	58267	60495	62645	64722	66735
57	58304	60532	62680	64756	66768
58	58342	60568	62715	64790	66801
59	58380	60605	62750	64824	66834
60	9.58418	9.60641	9.62785	9.64858	9.66867
	69°	68°	67°	66°	65°
Cot	110°	111°	112°	113°	114°

154°	153°	152°	151°	150°	Tan
25°	26°	27°	28°	29°	
9.66867	9.68818	9.70717	9.72567	9.74375	60′
66900	68850	70748	72598	74405	59
66933	68882	70779	72628	74435	58
66966	68914	70810	72659	74465	57
66999	68946	70841	72689	74494	56
9.67032	9.68978	9.70873	9.72720	9.74524	55
67065	69010	70904	72750	74554	54
67098	69042	70935	72780	74583	53
67131	69074	70966	72811	74613	52
67163	69106	70997	72841	74643	51
9.67196	9.69138	9.71028	9.72872	9.74673	50
67229	69170	71059	72902	74702	49
67262	69202	71090	72932	74732	48
67295	69234	71121	72963	74762	47
67327	69266	71153	72993	74791	46
9.67360	9.69298	9.71184	9.73023	9.74821	45
67393	69329	71215	73054	74851	44
67426	69361	71246	73084	74880	43
67458	69393	71277	73114	74910	42
67491	69425	71308	73144	74939	41
9.67524	9.69457	9.71339	9.73175	9.74969	40
67556	69488	71370	73205	74998	39
67589	69520	71401	73235	75028	38
67622	69552	71431	73265	75058	37
67654	69584	71462	73295	75087	36
9.67687	9.69615	9.71493	9.73326	9.75117	35
67719	69647	71524	73356	75146	34
67752	69679	71555	73386	75176	33
67785	69710	71586	73416	75205	32
67817	69742	71617	73446	75235	31
9.67850	9.69774	9.71648	9.73476	9.75264	30
67882	69805	71679	73507	75294	29
67915	69837	71709	73537	75323	28
67947	69868	71740	73567	75353	27
67980	69900	71771	73597	75382	26
9.68012	9.69932	9.71802	9.73627	9.75411	25
68044	69963	71833	73657	75441	24
68077	69995	71863	73687	75470	23
68109	70026	71894	73717	75500	22
68142	70058	71925	73747	75529	21
9.68174	9.70089	9.71955	9.73777	9.75558	20
68206	70121	71986	73807	75588	19
68239	70152	72017	73837	75617	18
68271	70184	72048	73867	75647	17
68303	70215	72078	73897	75676	16
9.68336	9.70247	9.72109	9.73927	9.75705	15
68368	70278	72140	73957	75735	14
68400	70309	72170	73987	75764	13
68432	70341	72201	74017	75793	12
68465	70372	72231	74047	75822	11
9.68497	9.70404	9.72262	9.74077	9.75852	10
68529	70435	72293	74107	75881	9
68561	70466	72323	74137	75910	8
68593	70498	72354	74166	75939	7
68626	70529	72384	74196	75969	6
9.68658	9.70560	9.72415	9.74226	9.75998	5
68690	70592	72445	74256	76027	4
68722	70623	72476	74286	76056	3
68754	70654	72506	74316	76086	2
68786	70685	72537	74345	76115	1
9.68818	9.70717	9.72567	9.74375	9.76144	0
64°	63°	62°	61°	60°	Cot
115°	116°	117°	118°	119°	

	149°	148°	147°	146°	145°
Tan	30°	31°	32°	33°	34°
0′	9.76144	9.77877	9.79579	9.81252	9.82899
1	76173	77906	79607	81279	82926
2	76202	77935	79635	81307	82953
3	76231	77963	79663	81335	82980
4	76261	77992	79691	81362	83008
5	9.76290	9.78020	9.79719	9.81390	9.83035
6	76319	78049	79747	81418	83062
7	76348	78077	79776	81445	83089
8	76377	78106	79804	81473	83117
9	76406	78135	79832	81500	83144
10	9.76435	9.78163	9.79860	9.81528	9.83171
11	76464	78192	79888	81556	83198
12	76493	78220	79916	81583	83225
13	76522	78249	79944	81611	83252
14	76551	78277	79972	81638	83280
15	9.76580	9.78306	9.80000	9.81666	9.83307
16	76609	78334	80028	81693	83334
17	76639	78363	80056	81721	83361
18	76668	78391	80084	81748	83388
19	76697	78419	80112	81776	83415
20	9.76725	9.78448	9.80140	9.81803	9.83442
21	76754	78476	80168	81831	83470
22	76783	78505	80195	81858	83497
23	76812	78533	80223	81886	83524
24	76841	78562	80251	81913	83551
25	9.76870	9.78590	9.80279	9.81941	9.83578
26	76899	78618	80307	81968	83605
27	76928	78647	80335	81996	83632
28	76957	78675	80363	82023	83659
29	76986	78704	80391	82051	83683
30	9.77015	9.78732	9.80419	9.82078	9.83713
31	77044	78760	80447	82106	83740
32	77073	78789	80474	82133	83768
33	77101	78817	80502	82161	83795
34	77130	78845	80530	82188	83822
35	9.77159	9.78874	9.80558	9.82215	9.83849
36	77188	78902	80586	82243	83876
37	77217	78930	80614	82270	83903
38	77246	78959	80642	82298	83930
39	77274	78987	80669	82325	83957
40	9.77303	9.79015	9.80697	9.82352	9.83984
41	77332	79043	80725	82380	84011
42	77361	79072	80753	82407	84038
43	77390	79100	80781	82435	84065
44	77418	79128	80808	82462	84092
45	9.77447	9.79156	9.80836	9.82489	9.84119
46	77476	79185	80864	82517	84146
47	77505	79213	80892	82544	84173
48	77533	79241	80919	82571	84200
49	77562	79269	80947	82599	84227
50	9.77591	9.79297	9.80975	9.82626	9.84254
51	77619	79326	81003	82653	84280
52	77648	79354	81030	82681	84307
53	77677	79382	81058	82708	84334
54	77706	79410	81086	82735	84361
55	9.77734	9.79438	9.81113	9.82762	9.84388
56	77763	79466	81141	82790	84415
57	77791	79495	81169	82817	84442
58	77820	79523	81196	82844	84469
59	77849	79551	81224	82871	84496
60	9.77877	9.79579	9.81252	9.82899	9.84523
	59°	58°	57°	56°	55°
Cot	120°	121°	122°	123°	124°

144°	143°	142°	141°	140°	Tan
35°	**36°**	**37°**	**38°**	**39°**	
9.84523	9.86126	9.87711	9.89281	9.90837	**60′**
84550	86153	87738	89307	90863	**59**
84576	86179	87764	89333	90889	**58**
84603	86206	87790	89359	90914	**57**
84630	86232	87817	89385	90940	**56**
9.84657	9.86259	9.87843	9.89411	9.90966	**55**
84684	86285	87869	89437	90992	**54**
84711	86312	87895	89463	91018	**53**
84738	86338	87922	89489	91043	**52**
84764	86365	87948	89515	91069	**51**
9.84791	9.86392	9.87974	9.89541	9.91095	**50**
84818	86418	88000	89567	91121	**49**
84845	86445	88027	89593	91147	**48**
84872	86471	88053	89619	91172	**47**
84899	86498	88079	89645	91198	**46**
9.84925	9.86524	9.88105	9.89671	9.91224	**45**
84952	86551	88131	89697	91250	**44**
84979	86577	88158	89723	91276	**43**
85006	86603	88184	89749	91301	**42**
85033	86630	88210	89775	91327	**41**
9.85059	9.86656	9.88236	9.89801	9.91353	**40**
85086	86683	88262	89827	91379	**39**
85113	86709	88289	89853	91404	**38**
85140	86736	88315	89879	91430	**37**
85166	86762	88341	89905	91456	**36**
9.85193	9.86789	9.88367	9.89931	9.91482	**35**
85220	86815	88393	89957	91507	**34**
85247	86842	88420	89983	91533	**33**
85273	86868	88446	90009	91559	**32**
85300	86894	88472	90035	91585	**31**
9.85327	9.86921	9.88498	9.90061	9.91610	**30**
85354	86947	88524	90086	91636	**29**
85380	86974	88550	90112	91662	**28**
85407	87000	88577	90138	91688	**27**
85434	87027	88603	90164	91713	**26**
9.85460	9.87053	9.88629	9.90190	9.91739	**25**
85487	87079	88655	90216	91765	**24**
85514	87106	88681	90242	91791	**23**
85540	87132	88707	90268	91816	**22**
85567	87158	88733	90294	91842	**21**
9.85594	9.87185	9.88759	9.90320	9.91868	**20**
85620	87211	88786	90346	91893	**19**
85647	87238	88812	90371	91919	**18**
85674	87264	88838	90379	91945	**17**
85700	87290	88864	90423	91971	**16**
9.85727	9.87317	9.88890	9.90449	9.91996	**15**
85754	87343	88916	90475	92022	**14**
85780	87369	88942	90501	92048	**13**
85807	87396	88968	90527	92073	**12**
85834	87422	88994	90553	92099	**11**
9.85860	9.87448	9.89020	9.90578	9.92125	**10**
85887	87475	89046	90604	92150	**9**
85913	87501	89073	90630	92176	**8**
85940	87527	89099	90656	92202	**7**
85967	87554	89125	90682	92227	**6**
9.85993	9.87580	9.89151	9.90708	9.92253	**5**
86020	87606	89177	90734	92279	**4**
86046	87633	89203	90759	92304	**3**
86073	87659	89229	90785	92330	**2**
86100	87685	89255	90811	92356	**1**
9.86126	9.87711	9.89281	9.90837	9.92381	**0**
54°	**53°**	**52°**	**51°**	**50°**	Cot
125°	126°	127°	128°	129°	

	139°	138°	137°	136°	135°
Tan	**40°**	**41°**	**42°**	**43°**	**44°**
0′	9.92381	9.93916	9.95444	9.96966	9.98484
1	92407	93942	95469	96991	98509
2	92433	93967	95495	97016	98534
3	92458	93993	95520	97042	98560
4	92484	94018	95545	97067	98585
5	9.92510	9.94044	9.95571	9.97092	9.98610
6	92535	94069	95596	97118	98635
7	92561	94095	95622	97143	98661
8	92587	94120	95647	97168	98686
9	92612	94146	95672	97193	98711
10	9.92638	9.94171	9.95698	9.97219	9.98737
11	92663	94197	95723	97244	98762
12	92689	94222	95748	97269	98787
13	92715	94248	95774	97295	98812
14	92740	94273	95799	97320	98838
15	9.92766	9.94299	9.95825	9.97345	9.98863
16	92792	94324	95850	97371	98888
17	92817	94350	95875	97396	98913
18	92843	94375	95901	97421	98939
19	92868	94401	95926	97447	98964
20	9.92894	9.94426	9.95952	9.97472	9.98989
21	92920	94452	95977	97497	99015
22	92945	94477	96002	97523	99040
23	92971	94503	96028	97548	99065
24	92996	94528	96053	97573	99090
25	9.93022	9.94554	9.96078	9.97598	9.99116
26	93048	94579	96104	97624	99141
27	93073	94604	96129	97649	99166
28	93099	94630	96155	97674	99191
29	93124	94655	96180	97700	99217
30	9.93150	9.94681	9.96205	9.97725	9.99242
31	93175	94706	96231	97750	99267
32	93201	94732	96256	97776	99293
33	93227	94757	96281	97801	99318
34	93252	94783	96307	97826	99343
35	9.93278	9.94808	9.96332	9.97851	9.99368
36	93303	94834	96357	97877	99394
37	93329	94859	96383	97902	99419
38	93354	94884	96408	97927	99444
39	93380	94910	96433	97953	99469
40	9.93406	9.94935	9.96459	9.97978	9.99495
41	93431	94961	96484	98003	99520
42	93457	94986	98510	98029	99545
43	93482	95012	96535	98054	99570
44	93508	95037	96560	98079	99596
45	9.93533	9.95062	9.96586	9.98104	9.99621
46	93559	95088	96611	98130	99646
47	93584	95113	96636	98155	99672
48	93610	95139	96662	98180	99697
49	93636	95164	96687	98206	99722
50	9.93661	9.95190	9.96712	9.98231	9.99747
51	93687	95215	96738	98256	99773
52	93712	95240	96763	98281	99798
53	93738	95266	96788	98307	99823
54	93763	95291	96814	98332	99848
55	9.93789	9.95317	9.96839	9.98357	9.99874
56	93814	95342	96864	98383	99899
57	93840	95368	96890	98408	99924
58	93865	95393	96915	98433	99949
59	93891	95418	96940	98458	99975
60	9.93916	9.95444	9.96966	9.98484	0.00000
	49°	**48°**	**47°**	**46°**	**45°**
Cot	130°	131°	132°	133°	134°

134°	133°	132°	131°	130°	Tan
45°	**46°**	**47°**	**48°**	**49°**	
0.00000	0.01516	0.03034	0.04556	0.06084	**60′**
00025	01542	03060	04582	06109	**59**
00051	01567	03085	04607	06135	**58**
00076	01592	03110	04632	06160	**57**
00101	01617	03136	04658	06186	**56**
0.00126	0.01643	0.03161	0.04683	0.06211	**55**
00152	01668	03186	04709	06237	**54**
00177	01693	03212	04734	06262	**53**
00202	01719	03237	04760	06288	**52**
00227	01744	03262	04785	06313	**51**
0.00253	0.01769	0.03288	0.04810	0.06339	**50**
00278	01794	03313	04836	06364	**49**
00303	01820	03338	04861	06390	**48**
00328	01845	03364	04887	06416	**47**
00354	01870	03389	04912	06441	**46**
0.00379	0.01896	0.03414	0.04938	0.06467	**45**
00404	01921	03440	04963	06492	**44**
00430	01946	03465	04988	06518	**43**
00455	01971	03490	05014	06543	**42**
00480	01997	03516	05039	06569	**41**
0.00505	0.02022	0.03541	0.05065	0.06594	**40**
00531	02047	03567	05090	06620	**39**
00556	02073	03592	05116	06646	**38**
00581	02098	03617	05141	06671	**37**
00606	02123	03643	05166	06697	**36**
0.00632	0.02149	0.03668	0.05192	0.06722	**35**
00657	02174	03693	05217	06748	**34**
00682	02199	03719	05243	06773	**33**
00707	02224	03744	05268	06799	**32**
00733	02250	03769	05294	06825	**31**
0.00758	0.02275	0.03795	0.05319	0.06850	**30**
00783	02300	03820	05345	06876	**29**
00809	02326	03845	05370	06901	**28**
00834	02351	03871	05396	06927	**27**
00859	02376	03896	05421	06952	**26**
0.00884	0.02402	0.03922	0.05446	0.06978	**25**
00910	02427	03947	05472	07004	**24**
00935	02452	03972	05497	07029	**23**
00960	02477	03998	05523	07055	**22**
00985	02503	04023	05548	07080	**21**
0.01011	0.02528	0.04048	0.05574	0.07106	**20**
01036	02553	04074	05599	07132	**19**
01061	02579	04099	05625	07157	**18**
01087	02604	04125	05650	07183	**17**
01112	02629	04150	05676	07208	**16**
0.01137	0.02655	0.04175	0.05701	0.07234	**15**
01162	02680	04201	05727	07260	**14**
01188	02705	04226	05752	07285	**13**
01213	02731	04252	05778	07311	**12**
01238	02756	04277	05803	07337	**11**
0.01263	0.02781	0.04302	0.05829	0.07362	**10**
01289	02807	04328	05854	07388	**9**
01314	02832	04353	05880	07413	**8**
01339	02857	04378	05905	07439	**7**
01365	02882	04404	05931	07465	**6**
0.01390	0.02908	0.04429	0.05956	0.07490	**5**
01415	02933	04455	05982	07516	**4**
01440	02958	04480	06007	07542	**3**
01466	02984	04505	06033	07567	**2**
01491	03009	04531	06058	07593	**1**
0.01516	0.03034	0.04556	0.06084	0.07619	**0**
44°	**43°**	**42°**	**41°**	**40°**	Cot
135°	136°	137°	138°	139°	

	129°	128°	127°	126°	125°
Tan	50°	51°	52°	53°	54°
0′	0.07619	0.09163	0.10719	0.12289	0.13874
1	07644	09189	10745	12315	13900
2	07670	09215	10771	12341	13927
3	07696	09241	10797	12367	13954
4	07721	09266	10823	12394	13980
5	0.07747	0.09292	0.10849	0.12420	0.14007
6	07773	09318	10875	12446	14033
7	07798	09344	10901	12473	14060
8	07824	09370	10927	12499	14087
9	07850	09396	10954	12525	14113
10	0.07875	0.09422	0.10980	0.12552	0.14140
11	07901	09447	11006	12578	14166
12	07927	09473	11032	12604	14193
13	07952	09499	11058	12631	14220
14	07978	09525	11084	12657	14246
15	0.08004	0.09551	0.11110	0.12683	0.14273
16	08029	09577	11136	12710	14300
17	08055	09603	11162	12736	14326
18	08081	09629	11188	12762	14353
19	08107	09654	11214	12789	14380
20	0.08132	0.09680	0.11241	0.12815	0.14406
21	08158	09706	11267	12842	14433
22	08184	09732	11293	12868	14460
23	08209	09758	11319	12894	14486
24	08235	09784	11345	12921	14513
25	0.08261	0.09810	0.11371	0.12947	0.14540
26	08287	09836	11397	12973	14566
27	08312	09862	11423	13000	14593
28	08338	09888	11450	13026	14620
29	08364	09914	11476	13053	14646
30	0.08390	0.09939	0.11502	0.13079	0.14673
31	08415	09965	11528	13106	14700
32	08441	09991	11554	13132	14727
33	08467	10017	11580	13158	14753
34	08493	10043	11607	13185	14780
35	0.08518	**0.**10069	0.11633	0.13211	0.14807
36	08544	10095	11659	13238	14834
37	08570	10121	11685	13264	14860
38	08596	10147	11711	13291	14887
39	08621	10173	11738	13317	14914
40	0.08647	0.10199	0.11764	0.13344	0.14941
41	08673	10225	11790	13370	14967
42	08699	10251	11816	13397	14994
43	08724	10277	11842	13423	15021
44	08750	10303	11869	13449	15048
45	0.08776	0.10329	0.11895	0.13476	0.15075
46	08802	10355	11921	13502	15101
47	08828	10381	11947	13529	15128
48	08853	10407	11973	13555	15155
49	08879	10433	12000	13582	15182
50	0.08905	0.10459	0.12026	0.13608	0.15209
51	08931	10485	12052	13635	15236
52	08957	10511	12078	13662	15262
53	08982	10537	12105	13688	15289
54	09008	10563	12131	13715	15316
55	0.09034	0.10589	0.12157	0.13741	0.15343
56	09060	10615	12183	13768	15370
57	09086	10641	12210	13794	15397
58	09111	10667	12236	13821	15424
59	09137	10693	12262	13847	15450
60	0.09163	0.10719	0.12289	0.13874	0.15477
	39°	38°	37°	36°	35°
Cot	140°	141°	142°	143°	144°

124°	123°	122°	121°	120°	Tan
55°	**56°**	**57°**	**58°**	**59°**	
0.15477	0.17101	0.18748	0.20421	0.22123	**60′**
15504	17129	18776	20449	22151	**59**
15531	17156	18804	20477	22180	**58**
15558	17183	18831	20505	22209	**57**
15585	17210	18859	20534	22237	**56**
0.15612	0.17238	0.18887	0.20562	0.22266	**55**
15639	17265	18914	20590	22294	**54**
15666	17292	18942	20618	22323	**53**
15693	17319	18970	20646	22352	**52**
15720	17347	18997	20674	22381	**51**
0.15746	0.17374	0.19025	0.20703	0.22409	**50**
15773	17401	19053	20731	22438	**49**
15800	17429	19081	20759	23467	**48**
15827	17456	19108	20787	23495	**47**
15854	17483	19136	20815	23524	**46**
0.15881	0.17511	0.19164	0.20844	0.23553	**45**
15908	17538	19192	20872	23582	**44**
15935	17565	19219	20900	23610	**43**
15962	17593	19247	20928	23639	**42**
15989	17620	19275	20957	22668	**41**
0.16016	0.17648	0.19303	0.20985	0.22697	**40**
16043	17675	19331	21013	22726	**39**
16070	17702	19358	21041	22754	**38**
16097	17730	19386	21070	22783	**37**
16124	17757	19414	21098	22812	**36**
0.16151	0.17785	0.19442	0.21126	0.22841	**35**
16178	17812	19470	21155	22870	**34**
16205	17839	19498	21183	22899	**33**
16232	17867	19526	21211	22927	**32**
16260	17894	19553	21240	22956	**31**
0.16287	0.17922	0.19581	0.21268	0.22985	**30**
16314	17949	19609	21296	23014	**29**
16341	17977	19637	21325	23043	**28**
16368	18004	19665	21353	23072	**27**
16395	18032	19693	21382	23101	**26**
0.16422	0.18059	0.19721	0.21410	0.23130	**25**
16449	18087	19749	21438	23159	**24**
16476	18114	19777	21467	23188	**23**
16503	18142	19805	21495	23217	**22**
16530	18169	19832	21524	23246	**21**
0.16558	0.18197	0.19860	0.21552	0.23275	**20**
16585	18224	19888	21581	23303	**19**
16612	18252	19916	21609	23332	**18**
16639	18279	19944	21637	23361	**17**
16666	18307	19972	21666	23391	**16**
0.16693	0.18334	0.20000	0.21694	0.23420	**15**
16720	18362	20028	21723	23449	**14**
16748	18389	20056	21751	23478	**13**
16775	18417	20084	21780	23507	**12**
16802	18444	20112	21808	23536	**11**
0.16829	0.18472	0.20140	0.21837	0.23565	**10**
16856	18500	20168	21865	23594	**9**
16883	18527	20196	21894	23623	**8**
16911	18555	20224	21923	23652	**7**
16938	18582	20253	21951	23681	**6**
0.16965	0.18610	0.20281	0.21980	0.23710	**5**
16992	18638	20309	22008	23739	**4**
17020	18665	20337	22037	23769	**3**
17047	18693	20365	22065	23798	**2**
17074	18721	20393	22094	23827	**1**
0.17101	0.18748	0.20421	0.22123	0.23856	**0**
34°	**33°**	**32°**	**31°**	**30°**	Cot
145°	146°	147°	148°	149°	

	119°	118°	117°	116°	115°
Tan	60°	61°	62°	63°	64°
0′	0.23856	0.25625	0.27433	0.29283	0.31182
1	23885	25655	27463	29315	31214
2	23914	25684	27494	29346	31246
3	23944	25714	27524	29377	31278
4	23973	25744	27555	29408	31310
5	0.24002	0.25774	0.27585	0.29440	0.31342
6	24031	25804	27616	29471	31374
7	24061	25834	27646	29502	31407
8	24090	25863	27677	29534	31439
9	24119	25893	27707	29565	31471
10	0.24148	0.25923	0.27738	0.29596	0.31503
11	24178	25953	27769	29628	31535
12	24207	25983	27799	29659	31568
13	24236	26013	27830	29691	31600
14	24265	26043	27860	29722	31632
15	0.24295	0.26073	0.27891	0.29753	0.31664
16	24324	26103	27922	29785	31697
17	24353	26133	27952	29816	31729
18	24383	26163	27983	29848	31761
19	24412	26193	28014	29879	31794
20	0.24442	0.26223	0.28045	0.29911	0.31826
21	24471	26253	28075	29942	31858
22	24500	26283	28106	29974	31891
23	24530	26313	28137	30005	31923
24	24559	26343	28167	30037	31956
25	0.24589	0.26373	0.28198	0.30068	0.31988
26	24618	26403	28229	30100	32020
27	24647	26433	28260	30132	32053
28	24677	26463	28291	30163	32085
29	24706	26493	28321	30195	32118
30	0.24736	0.26524	0.28352	0.30226	0.32150
31	24765	26554	28383	30258	32183
32	24795	26584	28414	30290	32215
33	24824	26614	28445	30321	32248
34	24854	26644	28476	30353	32281
35	0.24883	0.26674	0.28507	0.30385	0.32313
36	24913	26705	28538	30416	32346
37	24942	26735	28569	30448	32378
38	24972	26765	28599	30480	32411
39	25002	26795	28630	30512	32444
40	0.25031	0.26825	0.28661	0.30543	0.32476
41	25061	26856	28692	30575	32509
42	25090	26886	28723	30607	32542
43	25120	26916	28754	30639	32574
44	25149	26946	28785	30671	32607
45	0.25179	0.26977	0.28816	0.30702	0.32640
46	25209	27007	28847	30734	32673
47	25238	27037	28879	30766	32705
48	25268	27068	28910	30798	32738
49	25298	27098	28941	30830	32771
50	0.25327	0.27128	0.28972	0.30862	0.32804
51	25357	27159	29003	30894	32837
52	25387	27189	29034	30926	32869
53	25417	27220	29065	30958	32902
54	25446	27250	29096	30990	32935
55	0.25476	0.27280	0.29127	0.31022	0.32968
56	25506	27311	29159	31054	33001
57	25535	27341	29190	31086	33034
58	25565	27372	29221	31118	33067
59	25595	27402	29252	31150	33100
60	0.25625	0.27433	0.29283	0.31182	0.33133
	29°	28°	27°	26°	25°
Cot	150°	151°	152°	153°	154°

114°	113°	112°	111°	110°	Tan
65°	**66°**	**67°**	**68°**	**69°**	
0.33133	0.35142	0.37215	0.39359	0.41582	**60′**
33166	35176	37250	39395	41620	**59**
33199	35210	37285	39432	41658	**58**
33232	35244	37320	39468	41696	**57**
33265	35278	37355	39505	41733	**56**
0.33298	0.35312	0.37391	0.39541	0.41771	**55**
33331	35346	37426	39578	41809	**54**
33364	35380	37461	39614	41847	**53**
33397	35414	37496	39651	41885	**52**
33430	35448	37532	39687	41923	**51**
0.33463	0.35483	0.37567	0.39724	0.41961	**50**
33497	35517	37602	39760	41999	**49**
33530	35551	37638	39797	42037	**48**
33563	35585	37673	39834	42075	**47**
33596	35619	37708	39870	42113	**46**
0.33629	0.35654	0.37744	0.39907	0.42151	**45**
33663	35688	37779	39944	42190	**44**
33696	35722	37815	39981	42228	**43**
33729	35757	37850	40017	42266	**42**
33762	35791	37886	40054	42304	**41**
0.33796	0.35825	0.37921	0.40091	0.42342	**40**
33829	35860	37957	40128	42381	**39**
33862	35894	37992	40165	42419	**38**
33896	35928	38028	40210	42457	**37**
33929	35963	38064	40238	42496	**36**
0.33962	0.35997	0.38099	0.40275	0.42534	**35**
33996	36032	38135	40312	42572	**34**
34029	36066	38170	40349	42611	**33**
34063	36101	38206	40386	42649	**32**
34096	36135	38242	40423	42688	**31**
0.34130	0.36170	0.38278	0.40460	0.42726	**30**
34163	36204	38313	40497	42765	**29**
34197	36239	38349	40534	42803	**28**
34230	36274	38385	40571	42842	**27**
34264	36308	38421	40609	42880	**26**
0.34297	0.36343	0.38456	0.40646	0.42919	**25**
34331	36377	38492	40683	42958	**24**
34364	36412	38528	40720	42996	**23**
34398	36447	38564	40757	43035	**22**
34432	36481	38600	40795	43074	**21**
0.34465	0.36516	0.38636	0.40832	0.43113	**20**
34499	36551	38672	40869	43151	**19**
34533	36586	38708	40906	43190	**18**
34566	36621	38744	40944	43229	**17**
34600	36655	36780	40981	43268	**16**
0.34634	0.36690	0.38816	0.41019	0.43307	**15**
34667	36725	38852	41056	43346	**14**
34701	36760	38888	41093	43385	**13**
34735	36795	38924	41131	43424	**12**
34769	36830	38960	41168	43463	**11**
0.34803	0.36865	0.38996	0.41206	0.43502	**10**
34836	36899	39033	41243	43541	**9**
34870	36934	39069	41281	43580	**8**
34904	36969	39105	41319	43619	**7**
34938	37004	39141	41356	43658	**6**
0.34972	0.37039	0.39177	0.41394	0.43697	**5**
35006	37074	39214	41431	43736	**4**
35040	37110	39250	41469	43776	**3**
35074	37145	39286	41507	43815	**2**
35108	37180	39323	41545	43854	**1**
0.35142	0.37215	0.39359	0.41582	0.43893	**0**
24°	**23°**	**22°**	**21°**	**20°**	Cot
155°	156°	157°	158°	159°	

	109°	108°	107°	106°	105°
Tan	70°	71°	72°	73°	74°
0′	0.43893	0.46303	0.48822	0.51466	0.54250
1	43933	46344	48865	51511	54298
2	43972	46385	48908	51557	54346
3	44011	46426	48952	51602	54394
4	44051	46467	48995	51647	54441
5	0.44090	0.46508	0.49038	0.51693	0.54489
6	44130	46550	49081	51738	54537
7	44169	46591	49124	51783	54585
8	44209	46632	49167	51829	54633
9	44248	46673	49211	51874	54681
10	0.44288	0.46715	0.49254	0.51920	0.54729
11	44327	46756	49297	51965	54778
12	44367	46798	49341	52011	54826
13	44407	46839	49384	52057	54874
14	44446	46880	49428	52103	54922
15	0.44486	0.46922	0.49471	0.52148	0.54971
16	44526	46963	49515	52194	55019
17	44566	47005	49558	52240	55067
18	44605	47047	49602	52286	55116
19	44645	47088	49645	52332	55164
20	0.44685	0.47130	0.49689	0.52378	0.55213
21	44725	47171	49733	52424	55262
22	44765	47213	49777	52470	55310
23	44805	47255	49820	52516	55359
24	44845	47297	49864	52562	55408
25	0.44885	0.47339	0.49908	0.52608	0.55456
26	44925	47380	49952	52654	55505
27	44965	47422	49996	52701	55554
28	45005	47464	50040	52747	55603
29	45045	47506	50084	52793	55652
30	0.45085	0.47548	0.50128	0.52840	0.55701
31	45125	47590	50172	52886	55750
32	45165	47632	50216	52932	55799
33	45206	47674	50260	52979	55849
34	45246	47716	50304	53025	55898
35	0.45286	0.47758	0.50348	0.53072	0.55947
36	45327	47800	50393	53119	55996
37	45367	47843	50437	53165	56046
38	45407	47885	50481	53212	56095
39	45448	47927	50526	53259	56145
40	0.45488	0.47969	0.50570	0.53306	0.56194
41	45529	48012	50615	53352	56244
42	45569	48054	50659	53399	56293
43	45610	48097	50704	53446	56343
44	45650	48139	50748	53493	56393
45	0.45691	0.48181	0.50793	0.53540	0.56442
46	45731	48224	50837	53587	56492
47	45772	48266	50882	53634	56542
48	45813	48309	50927	53681	56592
49	45853	48352	50971	53729	56642
50	0.45894	0.48394	0.51016	0.53776	0.56692
51	45935	48437	51061	53823	56742
52	45975	48480	51106	53870	56792
53	46016	48522	51151	53918	56842
54	46057	48565	51196	53965	56892
55	0.46098	0.48608	0.51241	0.54013	0.56943
56	46139	48651	51286	54060	56993
57	46180	48694	51331	54108	57043
58	46221	48736	51376	54155	57094
59	46262	48779	51421	54203	57144
60	0.46303	0.48822	0.51466	0.54250	0.57195
	19°	18°	17°	16°	15°
Cot	160°	161°	162°	163°	164°

104°	103°	102°	101°	100°	Tan
75°	76°	77°	78°	79°	
0.57195	0.60323	0.63664	0.67253	0.71135	60′
57245	60377	63721	67315	71202	59
57296	60431	63779	67377	71270	58
57347	60485	63837	67439	71338	57
57397	60539	63895	67502	71405	56
0.57448	0.60593	0.63953	0.67564	0.71473	55
57499	60647	64011	67627	71541	54
57550	60701	64069	67689	71609	53
57601	60755	64127	67752	71677	52
57652	60810	64185	67815	71746	51
0.57703	0.60864	0.64243	0.67878	0.71814	50
57754	60918	64302	67941	71883	49
57805	60973	64360	68004	71951	48
57856	61028	64419	68067	72020	47
57907	61082	64477	68130	72089	46
0.57959	0.61137	0.64536	0.68194	0.72158	45
58010	61192	64595	68257	72227	44
58061	61246	64653	68321	72296	43
58113	61301	64712	68384	72365	42
58164	61356	64771	68448	72434	41
0.58216	0.61411	0.64830	0.68511	0.72504	40
58267	61466	64889	68575	72573	39
58319	61521	64949	68639	72643	38
58371	61577	65008	68703	72712	37
58422	61632	65067	68767	72782	36
0.58474	0.61687	0.65126	0.68832	0.72852	35
58526	61743	65186	68896	72922	34
58578	61798	65245	68960	72992	33
58630	61853	65305	69025	73063	32
58682	61909	65365	69089	73133	31
0.58734	0.61965	0.65424	0.69154	0.73203	30
58786	62020	65484	69218	73274	29
58839	62076	65544	69283	73345	28
58891	62132	65604	69348	73415	27
58943	62188	65664	69413	73486	26
0.58995	0.62244	0.65724	0.69478	0.73557	25
59048	62300	65785	69543	73628	24
59100	62356	65845	69609	73699	23
59153	62412	65905	69674	73771	22
59205	62468	65966	69739	73842	21
0.59258	0.62524	0.66026	0.69805	0.73914	20
59311	62581	66087	69870	73985	19
59364	62637	66147	69936	74057	18
59416	62694	66208	70002	74129	17
59469	62750	66269	70068	74201	16
0.59522	0.62807	0.66330	0.70134	0.74273	15
59575	62863	66391	70200	74345	14
59628	62920	66452	70266	74418	13
59681	62977	66513	70332	74490	12
59734	63034	66574	70399	74563	11
0.59788	0.63091	0.66635	0.70465	0.74635	10
59841	63148	66697	70532	74708	9
59894	63205	66758	70598	74781	8
59948	63262	66820	70665	74854	7
60001	63319	66881	70732	74927	6
0.60055	0.63376	0.66943	0.70799	0.75000	5
60108	63434	67005	70866	75074	4
60162	63491	67067	70933	75147	3
60215	63548	67128	71000	75221	2
60269	63606	67190	71067	75294	1
0.60323	0.63664	0.67253	0.71135	0.75368	0
14°	13°	12°	11°	10°	Cot
165°	166°	167°	168°	169°	

	99°	98°	97°	96°	95°
Tan	80°	81°	82°	83°	84°
0′	0.75368	0.80029	0.85220	0.91086	0.97838
1	75442	80111	85312	91190	97960
2	75516	80193	85403	91295	98082
3	75590	80275	85496	91400	98204
4	75665	80357	85588	91505	98327
5	0.75739	0.80439	0.85680	0.91611	0.98450
6	75814	80522	85773	91717	98573
7	75888	80605	85866	91823	98697
8	75963	80688	85959	91929	98821
9	76038	80771	86052	92036	98945
10	0.76113	0.80854	0.86146	0.92142	0.99070
11	76188	80937	86239	92249	99195
12	76263	81021	86333	92357	99321
13	76339	81104	86427	92464	99447
14	76414	81188	86522	92572	99573
15	0.76490	0.81272	0.86616	0.92680	0.99699
16	76565	81356	86711	92789	99826
17	76641	81440	86806	92897	99954
18	76717	81525	86901	93006	1.00081
19	76794	81609	86996	93115	00209
20	0.76870	0.81694	0.87091	0.93225	1.00338
21	76946	81779	87187	93334	00466
22	77023	81864	87283	93444	00595
23	77099	81949	87379	93555	00725
24	77176	82035	87475	93665	00855
25	0.77253	0.82120	0.87572	0.93776	1.00985
26	77330	82206	87668	93887	01116
27	77407	82292	87765	93998	01247
28	77484	82378	87862	94110	01378
29	77562	82464	87960	94222	01510
30	0.77639	0.82550	0.88057	0.94334	1.01642
31	77717	82637	88155	94447	01775
32	77795	82723	88253	94559	01908
33	77873	82810	88351	94672	02041
34	77951	82897	88449	94786	02175
35	0.78029	0.82984	0.88548	0.94899	1.02309
36	78107	83072	88647	95013	02444
37	78186	83159	88746	95127	02579
38	78264	83247	88845	95242	02715
39	78343	83335	88944	95357	02850
40	0.78422	0.83423	0.89044	0.95472	1.02987
41	78501	83511	89144	95587	03123
42	78580	83599	89244	95703	03261
43	78659	83688	89344	95819	03398
44	78739	83776	89445	95935	03536
45	0.78818	0.83865	0.89546	0.96052	1.03675
46	78898	83954	89647	96168	03813
47	78978	84044	89748	96286	03953
48	79058	84133	89850	96403	04092
49	79138	84223	89951	96521	04233
50	0.79218	0.84312	0.90053	0.96639	1.04373
51	79299	84402	90155	96758	04514
52	79379	84492	90258	96876	04656
53	79460	84583	90360	96995	04798
54	79541	84673	90463	97115	04940
55	0.79622	0.84764	0.90566	0.97234	1.05083
56	79703	84855	90670	97355	05227
57	79784	84946	90773	97475	05370
58	79866	85037	90877	97596	05515
59	79947	85128	90981	97717	05660
60	0.80029	0.85220	0.91086	0.97838	1.05805
	9°	8°	7°	6°	5°
Cot	170°	171°	172°	153°	174°

94°	93°	92°	91°	90°	Tan
85°	**86°**	**87°**	**88°**	**89°**	
1.05805	1.15536	1.28060	1.45692	1.75808	**60′**
05951	15718	28303	46055	76538	**59**
06097	15900	28547	46422	77280	**58**
06244	16084	28792	46792	78036	**57**
06391	16268	29038	47165	78805	**56**
1.06538	1.16453	1.29286	1.47541	1.79587	**55**
06687	16639	29535	47921	80384	**54**
06835	16825	29786	48304	81196	**53**
06984	17013	30038	48690	82024	**52**
07134	17201	30292	49080	82867	**51**
1.07284	1.17390	1.30547	1.49473	1.83727	**50**
07435	17580	30804	49870	84605	**49**
07586	17770	31062	50271	85500	**48**
07738	17962	31322	50675	86415	**47**
07890	18154	31583	51083	87349	**46**
1.08043	1.18347	1.31846	1.51495	1.88304	**45**
08197	18541	32110	51911	89280	**44**
08350	18736	32376	52331	90278	**43**
08505	18932	32644	52755	91300	**42**
08660	19128	32913	53183	92347	**41**
1.08815	1.19326	1.33184	1.53615	1.93419	**40**
08971	19524	33457	54052	94519	**39**
09128	19723	33731	54493	95647	**38**
09285	19924	34007	54939	96806	**37**
09443	20125	34285	55389	97996	**36**
1.09601	1.20327	1.34565	1.55844	1.99219	**35**
09760	20530	34846	56304	2.00478	**34**
09920	20734	35130	56768	01175	**33**
10080	20939	35415	57238	03111	**32**
10240	21145	35702	57713	04490	**31**
1.10402	1.21351	1.35991	1.58193	2.05914	**30**
10563	21559	36282	58679	07387	**29**
10726	21768	36574	59170	08911	**28**
10889	21978	36869	59666	10490	**27**
11052	22189	37166	60168	12129	**26**
1.11217	1.22400	1.37465	1.60677	2.13833	**25**
11382	22613	37766	61191	15606	**24**
11547	22827	38069	61711	17454	**23**
11713	23042	38374	62238	19385	**22**
11880	23258	38681	62771	21405	**21**
1.12047	1.23475	1.38991	1.63311	2.23524	**20**
12215	23694	39302	63857	25752	**19**
12384	23913	39616	64410	28100	**18**
12553	24133	39932	64971	30582	**17**
12723	24355	40251	65539	33215	**16**
1.12894	1.24577	1.40572	1.66114	2.36018	**15**
13065	24801	40895	66698	39014	**14**
13237	25026	41221	67289	42233	**13**
13409	25252	41549	67888	45709	**12**
13583	25479	41879	68495	49488	**11**
1.13757	1.25708	1.42212	1.69112	2.53627	**10**
13931	25937	42548	69737	58203	**9**
14107	26168	42886	70371	63318	**8**
14283	26400	43227	71014	69118	**7**
14460	26634	43571	71668	75812	**6**
1.14637	1.26868	1.43917	1.72331	2.83730	**5**
14815	27104	44266	73004	2.93421	**4**
14994	27341	44618	73688	3.05915	**3**
15174	27580	44973	74384	3.23524	**2**
15354	27819	45331	75090	3.53627	**1**
1.15536	1.28060	1.45692	1.75808	∞	**0**
4°	**3°**	**2°**	**1°**	**0°**	Cot
175°	176°	177°	178°	179°	

	179°	178°	177°	176°	175°
Sin	0°	1°	2°	3°	4°
0′	.00000	.01745	.03490	.05234	.06976
1	029	774	519	263	.07005
2	058	803	548	292	034
3	087	832	577	321	063
4	116	862	606	350	092
5	.00145	.01891	.03635	.05379	.07121
6	175	920	664	408	150
7	204	949	693	437	179
8	233	978	723	466	208
9	262	.02007	752	495	237
10	.00291	.02036	.03781	.05524	.07266
11	320	065	810	553	295
12	349	094	839	582	324
13	378	123	868	611	353
14	407	152	897	640	382
15	.00436	.02181	.03926	.05669	.07411
16	465	211	955	698	440
17	495	240	984	727	469
18	524	269	.04013	756	498
19	553	298	042	785	527
20	.00582	.02327	.04071	.05814	.07556
21	611	356	100	844	585
22	640	385	129	873	614
23	669	414	159	902	643
24	698	443	188	931	672
25	.00727	.02472	.04217	.05960	.07701
26	756	501	246	989	730
27	785	530	275	.06018	759
28	814	560	304	047	788
29	844	589	333	076	817
30	.00873	.02618	.04362	.06105	.07846
31	902	647	391	134	875
32	931	676	420	163	904
33	960	705	449	192	933
34	989	734	478	221	962
35	.01018	.02763	.04507	.06250	.07991
36	047	792	536	279	.08020
37	076	821	565	308	049
38	105	850	594	337	078
39	134	879	623	366	107
40	.01164	.02908	.04653	.06395	.08136
41	193	938	682	424	165
42	222	967	711	453	194
43	251	996	740	482	223
44	280	.03025	769	511	252
45	.01309	.03054	.04798	.06540	.08281
46	338	083	827	569	310
47	367	112	856	598	339
48	396	141	885	627	368
49	425	170	914	656	397
50	.01454	.03199	.04943	.06685	.08426
51	483	228	972	714	455
52	513	257	.05001	743	484
53	542	286	030	773	513
54	571	316	059	802	542
55	.01600	.03345	.05088	.06831	.08571
56	629	374	117	860	600
57	658	403	146	889	629
58	687	432	175	918	658
59	716	461	205	947	687
60	.01745	.03490	.05234	.06976	.08716
	89°	88°	87°	86°	85°
Cos	90°	91°	92°	93°	94°

174°	173°	172°	171°	170°	Sin
5°	**6°**	**7°**	**8°**	**9°**	
.08716	.10453	.12187	.13917	.15643	**60′**
745	482	216	946	672	**59**
774	511	245	975	701	**58**
803	540	274	.14004	730	**57**
831	569	302	033	758	**56**
.08860	.10597	.12331	.14061	.15787	**55**
889	626	360	090	816	**54**
918	655	389	119	845	**53**
947	684	418	148	873	**52**
976	713	447	177	902	**51**
.09005	.10742	.12476	.14205	.15931	**50**
034	771	504	234	959	**49**
063	800	533	263	988	**48**
092	829	562	292	.16017	**47**
121	858	591	320	046	**46**
.09150	.10887	.12620	.14349	.16074	**45**
179	916	649	378	103	**44**
208	945	678	407	132	**43**
237	973	706	436	160	**42**
266	11002	735	464	189	**41**
.09295	.11031	.12764	.14493	.16218	**40**
324	060	793	522	246	**39**
353	089	822	551	275	**38**
382	118	851	580	304	**37**
411	147	880	608	333	**36**
.09440	.11176	.12908	.14637	.16361	**35**
469	205	937	666	390	**34**
498	234	966	695	419	**33**
527	263	995	723	447	**32**
556	291	.13024	752	476	**31**
.09585	.11320	.13053	.14781	.16505	**30**
614	349	081	810	533	**29**
642	378	110	838	562	**28**
671	407	139	867	591	**27**
700	436	168	896	620	**26**
.09729	.11465	.13197	.14925	.16648	**25**
758	494	226	954	677	**24**
787	523	254	982	706	**23**
816	552	283	.15011	734	**22**
845	580	312	040	763	**21**
.09874	.11609	.13341	.15069	.16792	**20**
903	638	370	097	820	**19**
932	667	399	126	849	**18**
961	696	427	155	878	**17**
990	725	456	184	906	**16**
.10019	.11754	.13485	.15212	.16935	**15**
048	783	514	241	964	**14**
077	812	543	270	992	**13**
106	840	572	299	.17021	**12**
135	869	600	327	050	**11**
.10164	.11898	.13629	.15356	.17078	**10**
192	927	658	385	107	**9**
221	956	687	414	136	**8**
250	985	716	442	164	**7**
279	.12014	744	471	193	**6**
.10308	.12043	.13773	.15500	.17222	**5**
337	071	802	529	250	**4**
366	100	831	557	279	**3**
395	129	860	586	308	**2**
424	158	889	615	336	**1**
.10453	.12187	.13917	.15643	.17365	**0**
84°	**83°**	**82°**	**81°**	**80°**	Cos
95°	96°	97°	98°	99°	

•	169°	168°	167°	166°	165°
Sin	10°	11°	12°	13°	14°
0′	.17365	.19081	.20791	.22495	.24192
1	393	109	820	523	220
2	422	138	848	552	249
3	451	167	877	580	277
4	479	195	905	608	305
5	.17508	.19224	.20933	.22637	.24333
6	537	252	962	665	362
7	565	281	990	693	390
8	594	309	.21019	722	418
9	623	338	047	750	446
10	.17651	.19366	.21076	.22778	.24474
11	680	395	104	807	503
12	708	423	132	835	531
13	737	452	161	863	559
14	766	481	189	892	587
15	.17794	.19509	.21218	.22920	.24615
16	823	538	246	948	644
17	852	566	275	977	672
18	880	595	303	.23005	700
19	909	623	331	033	728
20	.17937	.19652	.21360	.23062	.24756
21	966	680	388	090	784
22	995	709	417	118	813
23	.18023	737	445	146	841
24	052	766	474	175	869
25	.18081	.19794	.21502	.23203	.24897
26	109	823	530	231	925
27	138	851	559	260	954
28	166	880	587	288	982
29	195	908	616	316	.25010
30	.18224	.19937	.21644	.23345	.25038
31	252	965	672	373	066
32	281	994	701	401	094
33	309	.20022	729	429	122
34	338	051	758	458	151
35	.18367	.20079	.21786	.23486	.25179
36	395	108	814	514	207
37	424	136	843	542	235
38	452	165	871	571	263
39	481	193	899	599	291
40	.18509	.20222	.21928	.23627	.25320
41	538	250	956	656	348
42	567	279	985	684	376
43	595	307	.22013	712	404
44	624	336	041	740	432
45	.18652	.20364	.22070	.23769	.25460
46	681	393	098	797	488
47	710	421	126	825	516
48	738	450	155	853	545
49	767	478	183	882	573
50	.18795	.20507	.22212	.23910	.25601
51	824	535	240	938	629
52	852	563	268	966	657
53	881	592	297	995	685
54	910	620	325	.24023	713
55	.18938	.20649	.22353	.24051	.25741
56	967	677	382	079	769
57	995	706	410	108	798
58	.19024	734	438	136	826
59	052	763	467	164	854
60	.19081	.20791	.22495	.24192	.25882
	79°	78°	77°	76°	75°
Cos	100°	101°	102°	103°	104°

164°	163°	162°	161°	160°	Sin
15°	**16°**	**17°**	**18°**	**19°**	
.25882	.27564	.29237	.30902	.32557	**60′**
910	592	265	929	584	**59**
938	620	293	957	612	**58**
966	648	321	985	639	**57**
994	676	348	.31012	667	**56**
.26022	.27704	.29376	.31040	.32694	**55**
050	731	404	068	722	**54**
079	759	432	095	749	**53**
107	787	460	123	777	**52**
135	815	487	151	804	**51**
.26163	.27843	.29515	.31178	.32832	**50**
191	871	543	206	859	**49**
219	899	571	233	887	**48**
247	927	599	261	914	**47**
275	955	626	289	942	**46**
.26303	.27983	.29654	.31316	.32969	**45**
331	.28011	682	344	997	**44**
359	039	710	372	.33024	**43**
387	067	737	399	051	**42**
415	095	765	427	079	**41**
.26443	.28123	.29793	.31454	.33106	**40**
471	150	821	482	134	**39**
500	178	849	510	161	**38**
528	206	876	537	189	**37**
556	234	904	565	216	**36**
.26584	.28262	.29932	.31593	.33244	**35**
612	290	960	620	271	**34**
640	318	987	648	298	**33**
668	346	.30015	675	326	**32**
696	374	043	703	353	**31**
.26724	.28402	.30071	.31730	.33381	**30**
752	429	098	758	408	**29**
780	457	126	786	436	**28**
808	485	154	813	463	**27**
836	513	182	841	490	**26**
.26864	.28541	.30209	.31868	.33518	**25**
892	569	237	896	545	**24**
920	597	265	923	573	**23**
948	625	292	951	600	**22**
976	652	320	979	627	**21**
.27004	.28680	.30348	.32006	.33655	**20**
032	708	376	034	682	**19**
060	736	403	061	710	**18**
088	764	431	089	737	**17**
116	792	459	116	764	**16**
.27144	.28820	.30486	.32144	.33792	**15**
172	847	514	171	819	**14**
200	875	542	199	846	**13**
228	903	570	227	874	**12**
256	931	597	254	901	**11**
.27284	.28959	.30625	.32282	.33929	**10**
312	987	653	309	956	**9**
340	.29015	680	337	983	**8**
368	042	708	364	.34011	**7**
396	070	736	392	038	**6**
.27424	.29098	.30763	.32419	.34065	**5**
452	126	791	447	093	**4**
480	154	819	474	120	**3**
508	182	846	502	147	**2**
536	209	874	529	175	**1**
.27564	.29237	.30902	.32557	.34202	**0**
74°	**73°**	**72°**	**71°**	**70°**	Cos
105°	106°	107°	108°	109°	

	159°	158°	157°	156°	155°
Sin	**20°**	**21°**	**22°**	**23°**	**24°**
0′	.34202	.35837	.37461	.39073	.40674
1	229	864	488	100	700
2	257	891	515	127	727
3	284	918	542	153	753
4	311	945	569	180	780
5	.34339	.35973	.37595	.39207	.40806
6	366	.36000	622	234	833
7	393	027	649	260	860
8	421	054	676	287	886
9	448	081	703	314	913
10	.34475	.36108	.37730	.39341	.40939
11	503	135	757	367	966
12	530	162	784	394	992
13	557	190	811	421	.41019
14	584	217	838	448	045
15	.34612	.36244	.37865	.39474	.41072
16	639	271	892	501	098
17	666	298	919	528	125
18	694	325	946	555	151
19	721	352	973	581	178
20	.34748	.36379	.37999	.39608	.41204
21	775	406	.38026	635	231
22	803	434	053	661	257
23	830	461	080	688	284
24	857	488	107	715	310
25	.34884	.36515	.38134	.39741	.41337
26	912	542	161	768	363
27	939	569	188	795	390
28	966	596	215	822	416
29	993	623	241	848	443
30	.35021	.36650	.38268	.39875	.41469
31	048	677	295	902	496
32	075	704	322	928	522
33	102	731	349	955	549
34	130	758	376	982	575
35	.35157	.36785	.38403	.40008	.41602
36	184	812	430	035	628
37	211	839	456	062	655
38	239	867	483	088	681
39	266	894	510	115	707
40	.35293	.36921	.38537	.40141	.41734
41	320	948	564	168	760
42	347	975	591	195	787
43	375	.37002	617	221	813
44	402	029	644	248	840
45	.35429	.37056	.38671	.40275	.41866
46	456	083	698	301	892
47	484	110	725	328	919
48	511	137	752	355	945
49	538	164	778	381	972
50	.35565	.37191	.38805	.40408	.41998
51	592	218	832	434	.42024
52	619	245	859	461	051
53	647	272	886	488	077
54	674	299	912	514	104
55	.35701	.37326	.38939	.40541	.42130
56	728	353	966	567	156
57	755	380	993	594	183
58	782	407	.39020	621	209
59	810	434	046	647	235
60	.35837	.37461	.39073	.40674	.42262
	69°	**68°**	**67°**	**66°**	**65°**
Cos	110°	111°	112°	113°	114°

154°	153°	152°	151°	150°	Sin
25°	**26°**	**27°**	**28°**	**29°**	
.42262	.43837	.45399	.46947	.48481	**60′**
288	863	425	973	506	**59**
315	889	451	999	532	**58**
341	916	477	47024	557	**57**
367	942	503	050	583	**56**
.42394	.43968	.45529	.47076	.48608	**55**
420	994	554	101	634	**54**
446	.44020	580	127	659	**53**
473	046	606	153	684	**52**
499	072	632	178	710	**51**
.42525	.44098	.45658	.47204	.48735	**50**
552	124	684	229	761	**49**
578	151	710	255	786	**48**
604	177	736	281	811	**47**
631	203	762	306	837	**46**
.42657	.44229	.45787	.47332	.48862	**45**
683	255	813	358	888	**44**
709	281	839	383	913	**43**
736	307	865	409	938	**42**
762	333	891	434	964	**41**
.42788	.44359	.45917	.47460	.48989	**40**
815	385	942	486	.49014	**39**
841	411	968	511	040	**38**
867	437	994	537	065	**37**
894	464	.46020	562	090	**36**
.42920	.44490	.46046	.47588	.49116	**35**
946	516	072	614	141	**34**
972	542	097	639	166	**33**
999	568	123	665	192	**32**
.43025	594	149	690	217	**31**
.43051	.44620	.46175	.47716	.49242	**30**
077	646	201	741	268	**29**
104	672	226	767	293	**28**
130	698	252	793	318	**27**
156	724	278	818	344	**26**
.43182	.44750	.46304	.47844	.49369	**25**
209	776	330	869	394	**24**
235	802	355	895	419	**23**
261	828	381	920	445	**22**
287	854	407	946	470	**21**
.43313	.44880	.46433	.47971	.49495	**20**
340	906	458	997	521	**19**
366	932	484	.48022	546	**18**
392	958	510	048	571	**17**
418	984	536	073	596	**16**
.43445	.45010	.46561	.48099	.49622	**15**
471	036	587	124	647	**14**
497	062	613	150	672	**13**
523	088	639	175	697	**12**
549	114	664	201	723	**11**
.43575	.45140	.46690	.48226	.49748	**10**
602	166	716	252	773	**9**
628	192	742	277	798	**8**
654	218	767	303	824	**7**
680	243	793	328	849	**6**
.43706	.45269	.46819	.48354	.49874	**5**
733	295	844	379	899	**4**
759	321	870	405	924	**3**
785	347	896	430	950	**2**
811	373	921	456	975	**1**
.43837	.45399	.46947	.48481	.50000	**0**
64°	**63°**	**62°**	**61°**	**60°**	Cos
115°	116°	117°	118°	119°	

	149°	148°	147°	146°	145°
Sin	30°	31°	32°	33°	34°
0′	.50000	.51504	.52992	.54464	.55919
1	025	529	.53017	488	943
2	050	554	041	513	968
3	076	579	066	537	992
4	101	604	091	561	.56016
5	.50126	.51628	.53115	.54586	.56040
6	151	653	140	610	064
7	176	678	164	635	088
8	201	703	189	659	112
9	227	728	214	683	136
10	.50252	.51753	.53238	.54708	.56160
11	277	778	263	732	184
12	302	803	288	756	208
13	327	828	312	781	232
14	352	852	337	805	256
15	.50377	.51877	.53361	.54829	.56280
16	403	902	386	854	305
17	428	927	411	878	329
18	453	952	435	902	353
19	478	977	460	927	377
20	.50503	.52002	.53484	.54951	.56401
21	528	026	509	975	425
22	553	051	534	999	449
23	578	076	558	.55024	473
24	603	101	583	048	497
25	.50628	.52126	.53607	.55072	.56521
26	654	151	632	097	545
27	679	175	656	121	569
28	704	200	681	145	593
29	729	225	705	169	617
30	.50754	.52250	.53730	.55194	.56641
31	779	275	754	218	665
32	804	299	779	242	689
33	829	324	804	266	713
34	854	349	828	291	736
35	.50879	.52374	.53853	.55315	.56760
36	904	399	877	339	784
37	929	423	902	363	808
38	954	448	926	388	832
39	979	473	951	412	856
40	.51004	.52498	.53975	.55436	.56880
41	029	522	.54000	460	904
42	054	547	024	484	928
43	079	572	049	509	952
44	104	597	073	533	976
45	.51129	.52621	.54097	.55557	.57000
46	154	646	122	581	024
47	179	671	146	605	047
48	204	696	171	630	071
49	229	720	195	654	095
50	.51254	.52745	.54220	.55678	.57119
51	279	770	244	702	143
52	304	794	269	726	167
53	329	819	293	750	191
54	354	844	317	775	215
55	.51379	.52869	.54342	.55799	.57238
56	404	893	366	823	262
57	429	918	391	847	286
58	454	943	415	871	310
59	479	967	440	895	334
60	.51504	.52992	.54464	.55919	.57358
	59°	58°	57°	56°	55°
Cos	120°	121°	122°	123°	124°

·144°	143°	142°	141°	140°	Sin
35°	**36°**	**37°**	**38°**	**39°**	
.57358	.58779	.60182	.61566	.62932	**60′**
381	802	205	589	955	**59**
405	826	228	612	977	**58**
429	849	251	635	.63000	**57**
453	873	274	658	022	**56**
.57477	.58896	.60298	.61681	.63045	**55**
501	920	321	704	068	**54**
524	943	344	726	090	**53**
548	967	367	749	113	**52**
572	990	390	772	135	**51**
.57596	.59014	.60414	.61795	.63158	**50**
619	037	437	818	180	**49**
643	061	460	841	203	**48**
667	084	483	864	225	**47**
691	108	506	887	248	**46**
.57715	.59131	.60529	.61909	.63271	**45**
738	154	553	932	293	**44**
762	178	576	955	316	**43**
786	201	599	978	338	**42**
810	225	622	.62001	361	**41**
.57833	.59248	.60645	.62024	.63383	**40**
857	272	668	046	406	**39**
881	295	691	069	428	**38**
904	318	714	092	451	**37**
928	342	738	115	473	**36**
.57952	.59365	.60761	.62138	.63496	**35**
976	389	784	160	518	**34**
999	412	807	183	540	**33**
.58023	436	830	206	563	**32**
047	459	853	229	585	**31**
.58070	.59482	.60876	.62251	.63608	**30**
094	506	899	274	630	**29**
118	529	922	297	653	**28**
141	552	945	320	675	**27**
165	576	968	342	698	**26**
.58189	.59599	.60991	62365	.63720	**25**
212	622	.61015	388	742	**24**
236	646	038	411	765	**23**
260	669	061	433	787	**22**
283	693	084	456	810	**21**
.58307	.59716	.61107	.62479	.63832	**20**
330	739	130	502	854	**19**
354	763	153	524	877	**18**
378	786	176	547	899	**17**
401	809	199	570	922	**16**
.58425	.59832	.61222	.62592	.63944	**15**
449	856	245	615	966	**14**
472	879	268	638	989	**13**
496	902	291	660	.64011	**12**
519	926	314	683	033	**11**
.58543	.59949	.61337	.62706	.64056	**10**
567	972	360	728	078	**9**
590	995	383	751	100	**8**
614	.60019	406	774	123	**7**
637	042	429	796	145	**6**
.58661	.60065	.61451	.62819	.64167	**5**
684	089	474	842	190	**4**
708	112	497	864	212	**3**
731	135	520	887	234	**2**
755	158	543	909	256	**1**
.58779	.60182	.61566	.62932	.64279	**0**
54°	**53°**	**52°**	**51°**	**50°**	Cos
125°	126°	127°	128°	129°	

	139°	138°	137°	136°	135°
Sin	40°	41°	42°	43°	44°
0′	.64279	.65606	.66913	.68200	.69466
1	301	628	935	221	487
2	323	650	756	242	508
3	346	672	978	264	529
4	368	694	999	285	549
5	.64390	.65716	.67021	.68306	.69570
6	412	738	043	327	591
7	435	759	064	349	612
8	457	781	086	370	633
9	479	803	107	391	654
10	.64501	.65825	.67129	.68412	.69675
11	524	847	151	434	696
12	546	869	172	455	717
13	568	891	194	476	737
14	590	913	215	497	758
15	.64612	.65935	.67237	.68518	.69779
16	635	956	258	539	800
17	657	978	280	561	821
18	679	.66000	301	582	842
19	701	022	323	603	862
20	.64723	.66044	.67344	.68624	.69883
21	746	066	366	645	904
22	768	088	387	666	925
23	790	109	409	688	946
24	812	131	430	709	966
25	.64834	.66153	.67452	.68730	.69987
26	856	175	473	751	.70008
27	878	197	495	772	029
28	901	218	516	793	049
29	923	240	538	814	070
30	.64945	.66262	.67559	.68835	.70091
31	967	284	580	857	112
32	989	306	602	878	132
33	.65011	327	623	899	153
34	033	349	645	920	174
35	.65055	.66371	.67666	.68941	.70195
36	077	393	688	962	215
37	100	414	709	983	236
38	122	436	730	.69004	257
39	144	458	752	025	277
40	.65166	.66480	.67773	.69046	.70298
41	188	501	795	067	319
42	210	523	816	088	339
43	232	545	837	109	360
44	254	566	859	130	381
45	.65276	.66588	.67880	.69151	.70401
46	298	610	901	172	422
47	320	632	923	193	443
48	342	653	944	214	463
49	364	675	965	235	484
50	.65386	.66697	.67987	.69256	.70505
51	408	718	.68008	277	525
52	430	740	029	298	546
53	452	762	051	319	567
54	474	783	072	340	587
55	.65496	.66805	.68093	.69361	.70608
56	518	827	115	382	628
57	540	848	136	403	649
58	562	870	157	424	670
59	584	891	179	445	690
60	.65606	.66913	.68200	.69466	.70711
	49°	48°	47°	46°	45°
Cos	130°	131°	132°	133°	134°

134°	133°	132°	131°	130°	Sin
45°	**46°**	**47°**	**48°**	**49°**	
.70711	.71934	.73135	.74314	.75471	**60′**
731	954	155	334	490	**59**
752	974	175	353	509	**58**
772	995	195	373	528	**57**
793	.72015	215	392	547	**56**
.70813	.72035	.73234	.74412	.75566	**55**
834	055	254	431	585	**54**
855	075	274	451	604	**53**
875	095	294	470	623	**52**
896	116	314	489	642	**51**
.70916	.72136	.73333	.74509	.75661	**50**
937	156	353	528	680	**49**
957	176	373	548	700	**48**
978	196	393	567	719	**47**
998	216	413	586	738	**46**
.71019	.72236	.73432	.74606	.75756	**45**
039	257	452	625	775	**44**
059	277	472	644	794	**43**
080	297	491	664	813	**42**
100	317	511	683	832	**41**
.71121	.72337	.73531	.74703	.75851	**40**
141	357	551	722	870	**39**
162	377	570	741	889	**38**
182	397	590	760	908	**37**
203	417	610	780	927	**36**
.71223	.72437	.73629	.74799	.75946	**35**
243	457	649	818	965	**34**
264	477	669	838	984	**33**
284	497	688	857	.76003	**32**
305	517	708	876	022	**31**
.71325	.72537	.73728	.74896	.76041	**30**
345	557	747	915	059	**29**
366	577	767	934	078	**28**
386	597	787	953	097	**27**
407	617	806	973	116	**26**
.71427	.72637	.73826	.74992	.76135	**25**
447	657	846	.75011	154	**24**
468	677	865	030	173	**23**
488	697	885	050	192	**22**
508	717	904	069	210	**21**
.71529	.72737	.73924	.75088	.76229	**20**
549	757	944	107	248	**19**
569	777	963	126	267	**18**
590	797	983	146	286	**17**
610	817	.74002	165	304	**16**
.71630	.72837	.74022	.75184	.76323	**15**
650	857	041	203	342	**14**
671	877	061	222	361	**13**
691	897	080	241	380	**12**
711	917	100	261	398	**11**
.71732	.72937	.74120	.75280	.76417	**10**
752	957	139	299	436	**9**
772	976	159	318	455	**8**
792	996	178	337	473	**7**
813	.73016	198	356	492	**6**
.71833	.73036	.74217	.75375	.76511	**5**
853	056	237	395	530	**4**
873	076	256	414	548	**3**
894	096	276	433	567	**2**
914	116	295	452	586	**1**
.71934	.73135	.74314	.75471	.76604	**0**
44°	**43°**	**42°**	**41°**	**40°**	Cos
135°	136°	137°	138°	139°	

	129°	128°	127°	126°	125°
Sin	50°	51°	52°	53°	54°
0′	.76604	.77715	.78801	.79864	.80902
1	623	733	819	881	919
2	642	751	837	899	936
3	661	769	855	916	953
4	679	788	873	934	970
5	.76698	.77806	.78891	.79951	.80987
6	717	824	908	968	.81004
7	735	843	926	986	021
8	754	861	944	.80003	038
9	772	879	962	021	055
10	.76791	.77897	.78980	.80038	.81072
11	810	916	998	056	089
12	828	934	.79016	073	106
13	847	952	033	091	123
14	866	970	051	108	140
15	.76884	.77988	.79069	.80125	.81157
16	903	.78007	087	143	174
17	921	025	105	160	191
18	940	043	122	178	208
19	959	061	140	195	225
20	.76977	.78079	.79158	.80212	.81242
21	996	098	176	230	259
22	.77014	116	193	247	276
23	033	134	211	264	293
24	051	152	229	282	310
25	.77070	.78170	.79247	.80299	.81327
26	088	188	264	316	344
27	107	206	282	334	361
28	125	225	300	351	378
29	144	243	318	368	395
30	.77162	.78261	.79335	.80386	.81412
31	181	279	353	403	428
32	199	297	371	420	445
33	218	315	388	438	462
34	236	333	406	455	479
35	.77255	.78351	.79424	.80472	.81496
36	273	369	441	489	513
37	292	387	459	507	530
38	310	405	477	524	546
39	329	424	494	541	563
40	.77347	.78442	.79512	.80558	.81580
41	366	460	530	576	597
42	384	478	547	593	614
43	402	496	565	610	631
44	421	514	583	627	647
45	.77439	.78532	.79600	.80644	.81664
46	458	550	618	662	681
47	476	568	635	679	698
48	494	586	653	696	714
49	513	604	671	713	731
50	.77531	.78622	.79688	.80730	.81748
51	550	640	706	748	765
52	568	658	723	765	782
53	586	676	741	782	798
54	605	694	758	799	815
55	.77623	.78711	.79776	.80816	.81832
56	641	729	793	833	848
57	660	747	811	850	865
58	678	765	829	867	882
59	696	783	846	885	899
60	.77715	.78801	.79864	.80902	.81915
	39°	38°	37°	36°	35°
Cos	140°	141°	142°	143°	144°

124°	123°	122°	121°	120°	Sin
55°	**56°**	**57°**	**58°**	**59°**	
.81915	.82904	.83867	.84805	.85717	**60′**
932	920	883	820	732	**59**
949	936	899	836	747	**58**
965	953	915	851	762	**57**
982	969	930	866	777	**56**
.81999	.82985	.83946	.84882	.85792	**55**
.82015	.83001	962	897	806	**54**
032	017	978	913	821	**53**
048	034	994	928	836	**52**
065	050	.84009	943	851	**51**
.82082	.83066	.84025	.84959	.85866	**50**
098	082	041	974	881	**49**
115	098	057	989	896	**48**
132	115	072	.85005	911	**47**
148	131	088	020	926	**46**
.82165	.83147	.84104	.85035	.85941	**45**
181	163	120	051	956	**44**
198	179	135	066	970	**43**
214	195	151	081	985	**42**
231	212	167	096	.86000	**41**
.82248	.83228	.84182	.85112	.86015	**40**
264	244	198	127	030	**39**
281	260	214	142	045	**38**
297	276	230	157	059	**37**
314	292	245	173	074	**36**
.82330	.83308	.84261	.85188	.86089	**35**
347	324	277	203	104	**34**
363	340	292	218	119	**33**
380	356	308	234	133	**32**
396	373	324	249	148	**31**
.82413	.83389	.84339	.85264	.86163	**30**
429	405	355	279	178	**29**
446	421	370	294	192	**28**
462	437	386	310	207	**27**
478	453	402	325	222	**26**
.82495	.83469	.84417	.85340	.86237	**25**
511	485	433	355	251	**24**
528	501	448	370	266	**23**
544	517	464	385	281	**22**
561	533	480	401	295	**21**
.82577	.83549	.84495	.85416	.86310	**20**
593	565	511	431	325	**19**
610	581	526	446	340	**18**
626	597	542	461	354	**17**
643	613	557	476	369	**16**
.82659	.83629	.84573	.85491	.86384	**15**
675	645	588	506	398	**14**
692	660	604	521	413	**13**
708	676	619	536	427	**12**
724	692	635	551	442	**11**
.82741	.83708	.84650	.85567	.86457	**10**
757	724	666	582	471	**9**
773	740	681	597	486	**8**
790	756	697	612	501	**7**
806	772	712	627	515	**6**
.82822	.83788	.84728	.85642	.86530	**5**
839	804	743	657	544	**4**
855	819	759	672	559	**3**
871	835	774	687	573	**2**
887	851	789	702	588	**1**
.82904	.83867	.84805	.85717	.86603	**0**
34°	**33°**	**32°**	**31°**	**30°**	Cos
145°	146°	147°	148°	149°	

	119°	118°	117°	116°	115°
Sin	60°	61°	62°	63°	64°
0′	.86603	.87462	.88295	.89101	.89879
1	617	476	308	114	892
2	632	490	322	127	905
3	646	504	336	140	918
4	661	518	349	153	930
5	.86675	.87532	.88363	.89167	.89943
6	690	546	377	180	956
7	704	561	390	193	968
8	719	575	404	206	981
9	733	589	417	219	994
10	.86748	.87603	.88431	.89232	.90007
11	762	617	445	245	019
12	777	631	458	259	032
13	791	645	472	272	045
14	805	659	485	285	057
15	.86820	.87673	.88499	.89298	.90070
16	834	687	512	311	082
17	849	701	526	324	095
18	863	715	539	337	108
19	878	729	553	350	120
20	.86892	.87743	.88566	.89363	.90133
21	906	756	580	376	146
22	921	770	593	389	158
23	935	784	607	402	171
24	949	798	620	415	183
25	.86964	.87812	.88634	.89428	.90196
26	978	826	647	441	208
27	993	840	661	454	221
28	.87007	854	674	467	233
29	021	868	688	480	246
30	.87036	.87882	.88701	.89493	.90259
31	050	896	715	506	271
32	064	909	728	519	284
33	079	923	741	532	296
34	093	937	755	545	309
35	.87107	.87951	.88768	.89558	.90321
36	121	965	782	571	334
37	136	979	795	584	346
38	150	993	808	597	358
39	164	.88006	822	610	371
40	.87178	.88020	.88835	.89623	.90383
41	193	034	848	636	396
42	207	048	862	649	408
43	221	062	875	662	421
44	235	075	888	674	433
45	.87250	.88089	.88902	.89687	.90446
46	264	103	915	700	458
47	278	117	928	713	470
48	292	130	942	726	483
49	306	144	955	739	495
50	.87321	.88158	.88968	.89752	.90507
51	335	172	981	764	520
52	349	185	995	777	532
53	363	199	.89008	790	545
54	377	213	021	803	557
55	.87391	.88226	.89035	.89816	.90569
56	406	240	048	828	582
57	420	254	061	841	594
58	434	267	074	854	606
59	448	281	087	867	618
60	.87462	.88295	.89101	.89879	.90631
	29°	28°	27°	26°	25°
Cos	150°	151°	152°	153°	154°

114°	113°	112°	111°	110°	Sin
65°	**66°**	**67°**	**68°**	**69°**	
.90631	.91355	.92050	.92718	.93358	**60′**
643	366	062	729	368	**59**
655	378	073	740	379	**58**
668	390	085	751	389	**57**
680	402	096	762	400	**56**
.90692	.91414	.92107	.92773	.93410	**55**
704	425	119	784	420	**54**
717	437	130	794	431	**53**
729	449	141	805	441	**52**
741	461	152	816	452	**51**
.90753	.91472	.92164	.92827	.93462	**50**
766	484	175	838	472	**49**
778	496	186	849	483	**48**
790	508	198	859	493	**47**
802	519	209	870	503	**46**
.90814	.91531	.92220	.92881	.93514	**45**
826	543	231	892	524	**44**
839	555	243	902	534	**43**
851	566	254	913	544	**42**
863	578	265	924	555	**41**
.90875	.91590	.92276	.92935	.93565	**40**
887	601	287	945	575	**39**
899	613	299	956	585	**38**
911	625	310	967	596	**37**
924	636	321	978	606	**36**
.90936	.91648	.92332	.92988	.93616	**35**
948	660	343	999	626	**34**
960	671	355	.93010	637	**33**
972	683	366	020	647	**32**
984	694	377	031	657	**31**
.90996	.91706	.92388	.93042	.93667	**30**
.91008	718	399	052	677	**29**
020	729	410	063	688	**28**
032	741	421	074	698	**27**
044	752	432	084	708	**26**
.91056	.91764	.92444	.93095	.93718	**25**
068	775	455	106	728	**24**
080	787	466	116	738	**23**
092	799	477	127	748	**22**
104	810	488	137	759	**21**
.91116	.91822	.92499	.93148	.93769	**20**
128	833	510	159	779	**19**
140	845	521	169	789	**18**
152	856	532	180	799	**17**
164	868	543	190	809	**16**
.91176	.91879	.92554	.93201	.93819	**15**
188	891	565	211	829	**14**
200	902	576	222	839	**13**
212	914	587	232	849	**12**
224	925	598	243	859	**11**
.91236	.91936	.92609	.93253	.93869	**10**
248	948	620	264	879	**9**
260	959	631	274	889	**8**
272	971	642	285	899	**7**
283	982	653	295	909	**6**
.91295	.91994	.92664	.93306	.93919	**5**
307	.92005	675	316	929	**4**
319	016	686	327	939	**3**
331	028	697	337	949	**2**
343	039	707	348	959	**1**
.91355	.92050	.92718	.93358	.93969	**0**
24°	**23°**	**22°**	**21°**	**20°**	Cos
155°	156°	157°	158°	159°	

	109°	108°	107°	106°	105°
Sin	70°	71°	72°	73°	74°
0′	.93969	.94552	.95106	.95630	.96126
1	979	561	115	639	134
2	989	571	124	647	142
3	999	580	133	656	150
4	.94009	590	142	664	158
5	.94019	.94599	.95150	.95673	.96166
6	029	609	159	681	174
7	039	618	168	690	182
8	049	627	177	698	190
9	058	637	186	707	198
10	.94068	.94646	.95195	.95715	.96206
11	078	656	204	724	214
12	088	665	213	732	222
13	098	674	222	740	230
14	108	684	231	749	238
15	.94118	.94693	.95240	.95757	.96246
16	127	702	248	766	253
17	137	712	257	774	261
18	147	721	266	782	269
19	157	730	275	791	277
20	.94167	.94740	.95284	.95799	.96285
21	176	749	293	807	293
22	186	758	301	816	301
23	196	768	310	824	308
24	206	777	319	832	316
25	.94215	.94786	.95328	.95841	.96324
26	225	795	337	849	332
27	235	805	345	857	340
28	245	814	354	865	347
29	254	823	363	874	355
30	.94264	.94832	.95372	.95882	.96363
31	274	842	380	890	371
32	284	851	389	898	379
33	293	860	398	907	386
34	303	869	407	915	394
35	.94313	.94878	.95415	.95923	.96402
36	322	888	424	931	410
37	332	897	433	940	417
38	342	906	441	948	425
39	351	915	450	956	433
40	.94361	.94924	.95459	.95964	.96440
41	370	933	467	972	448
42	380	943	476	981	456
43	390	952	485	989	463
44	399	961	493	997	471
45	.94409	.94970	.95502	.96005	.96479
46	418	979	511	013	486
47	428	988	519	021	494
48	438	997	528	029	502
49	447	.95006	536	037	509
50	.94457	.95015	.95545	.96046	.96517
51	466	024	554	054	524
52	476	033	562	062	532
53	485	043	571	070	540
54	495	052	579	078	547
55	.94504	.95061	.95588	.96086	.96555
56	514	070	596	094	562
57	523	079	605	102	570
58	533	088	613	110	578
59	542	097	622	118	585
60	.94552	.95106	.95630	.96126	.96593
	19°	18°	17°	16°	15°
Cos	160°	161°	162°	163°	164°

104°	103°	102°	101°	100°	Sin
75°	**76°**	**77°**	**78°**	**79°**	
.96593	.97030	.97437	.97815	.98163	**60′**
600	037	444	821	168	**59**
608	044	450	827	174	**58**
615	051	457	833	179	**57**
623	058	463	839	185	**56**
.96630	.97065	.97470	.97845	.98190	**55**
638	072	476	851	196	**54**
645	079	483	857	201	**53**
653	086	489	863	207	**52**
660	093	496	869	212	**51**
.96667	.97100	.97502	.97875	.98218	**50**
675	106	508	881	223	**49**
682	113	515	887	229	**48**
690	120	521	893	234	**47**
697	127	528	899	240	**46**
.96705	.97134	.97534	.97905	.98245	**45**
712	141	541	910	250	**44**
719	148	547	916	256	**43**
727	155	553	922	261	**42**
734	162	560	928	267	**41**
.96742	.97169	.97566	.97934	.98272	**40**
749	176	573	940	277	**39**
756	182	579	946	283	**38**
764	189	585	952	288	**37**
771	196	592	958	294	**36**
.96778	.97203	.97598	.97963	.98299	**35**
786	210	604	969	304	**34**
793	217	611	975	310	**33**
800	223	617	981	315	**32**
807	230	623	987	320	**31**
.96815	.97237	.97630	.97992	.98325	**30**
822	244	636	998	331	**29**
829	251	642	.98004	336	**28**
837	257	648	010	341	**27**
844	264	655	016	347	**26**
.96851	.97271	.97661	.98021	.98352	**25**
858	278	667	027	357	**24**
866	284	673	033	362	**23**
873	291	680	039	368	**22**
880	298	686	044	373	**21**
.96887	.97304	.97692	.98050	.98378	**20**
894	311	698	056	383	**19**
902	318	705	061	389	**18**
909	325	711	067	394	**17**
916	331	717	073	399	**16**
.96923	.97338	.97723	.98079	.98404	**15**
930	345	729	084	409	**14**
937	351	735	090	414	**13**
945	358	742	096	420	**12**
952	365	748	101	425	**11**
.96959	.97371	.97754	.98107	.98430	**10**
966	378	760	112	435	**9**
973	384	766	118	440	**8**
980	391	772	124	445	**7**
987	398	778	129	450	**6**
.96994	.97404	.97784	.98135	.98455	**5**
.97001	411	791	140	461	**4**
008	417	797	146	466	**3**
015	424	803	152	471	**2**
023	430	809	157	476	**1**
.97030	.97437	.97815	.98163	.98481	**0**
14°	**13°**	**12°**	**11°**	**10°**	Cos
165°	166°	167°	168°	169°	

	99°	98°	97°	96°	95°
Sin	80°	81°	82°	83°	84°
0′	.98481	.98769	.99027	.99255	.99452
1	486	773	031	258	455
2	491	778	035	262	458
3	496	782	039	265	461
4	501	787	043	269	464
5	.98506	.98791	.99047	.99272	.99467
6	511	796	051	276	470
7	516	800	055	279	473
8	521	805	059	283	476
9	526	809	063	286	479
10	.98531	.98814	.99067	.99290	.99482
11	536	818	071	293	485
12	541	823	075	297	488
13	546	827	079	300	491
14	551	832	083	303	494
15	.98556	.98836	.99087	.99307	.99497
16	561	841	091	310	500
17	565	845	094	314	503
18	570	849	098	317	506
19	575	854	102	320	508
20	.98580	.98858	.99106	.99324	.99511
21	585	863	110	327	514
22	590	867	114	331	517
23	595	871	118	334	520
24	600	876	122	337	523
25	.98604	.98880	.99125	.99341	.99526
26	609	884	129	344	528
27	614	889	133	347	531
28	619	893	137	351	534
29	624	897	141	354	537
30	.98629	.98902	.99144	.99357	.99540
31	633	906	148	360	542
32	638	910	152	364	545
33	643	914	156	367	548
34	648	919	160	370	551
35	.98652	.98923	.99163	.99374	.99553
36	657	927	167	377	556
37	662	931	171	380	559
38	667	936	175	383	562
39	671	940	178	386	564
40	.98676	.98944	.99182	.99390	.99567
41	681	948	186	393	570
42	686	953	189	396	572
43	690	957	193	399	575
44	695	961	197	402	578
45	.98700	.98965	.99200	.99406	.99580
46	704	969	204	409	583
47	709	973	208	412	586
48	714	978	211	415	588
49	718	982	215	418	591
50	.98723	.98986	.99219	.99421	.99594
51	728	990	222	424	596
52	732	994	226	428	599
53	737	998	230	431	602
54	741	.99002	233	434	604
55	.98746	.99006	.99237	.99437	.99607
56	751	011	240	440	609
57	755	015	244	443	612
58	760	019	248	446	614
59	764	023	251	449	617
60	.98769	.99027	.99255	.99452	.99619
	9°	8°	7°	6°	5°
Cos	170°	171°	172°	173°	174°

94°	93°	92°	91°	90°	Sin
85°	**86°**	**87°**	**88°**	**89°**	
.99619	.99756	.99863	.99939	.99985	**60′**
622	758	864	940	985	**59**
625	760	866	941	986	**58**
627	762	867	942	986	**57**
630	764	869	943	987	**56**
.99632	.99766	.99870	.99944	.99987	**55**
635	768	872	945	988	**54**
637	770	873	946	988	**53**
639	772	875	947	989	**52**
642	774	876	948	989	**51**
.99644	.99776	.99878	.99949	.99989	**50**
647	778	879	950	990	**49**
649	780	881	951	990	**48**
652	782	882	952	991	**47**
654	784	883	952	991	**46**
.99657	.99786	.99885	.99953	.99991	**45**
659	788	886	954	992	**44**
661	790	888	955	992	**43**
664	792	889	956	993	**42**
666	793	890	957	993	**41**
.99668	.99795	.99892	.99958	.99993	**40**
671	797	893	959	994	**39**
673	799	894	959	994	**38**
676	801	896	960	994	**37**
678	803	897	961	995	**36**
.99680	.99804	.99898	.99962	.99995	**35**
683	806	900	963	995	**34**
685	808	901	963	995	**33**
687	810	902	964	996	**32**
689	812	904	965	996	**31**
.99692	.99813	.99905	.99966	.99996	**30**
694	815	906	966	996	**29**
696	817	907	967	997	**28**
699	819	909	968	997	**27**
701	821	910	969	997	**26**
.99703	.99822	.99911	.99969	.99997	**25**
705	824	912	970	998	**24**
708	826	913	971	998	**23**
710	827	915	972	998	**22**
712	829	916	972	998	**21**
.99714	.99831	.99917	.99973	.99998	**20**
716	833	918	974	998	**19**
719	834	919	974	999	**18**
721	836	921	975	999	**17**
723	838	922	976	999	**16**
.99725	.99839	.99923	.99976	.99999	**15**
727	841	924	977	999	**14**
729	842	925	977	999	**13**
731	844	926	978	999	**12**
734	846	927	979	999	**11**
.99736	.99847	.99929	.99979	1.00000	**10**
738	849	930	980	000	**9**
740	851	931	980	000	**8**
742	852	932	981	000	**7**
744	854	933	982	000	**6**
.99746	.99855	.99934	.99982	1.00000	**5**
748	857	935	983	000	**4**
750	858	936	983	000	**3**
752	860	937	984	000	**2**
754	861	938	984	000	**1**
.99756	.99863	.99939	.99985	1.00000	**0**
4°	**3°**	**2°**	**1°**	**0°**	Cos
175°	176°	177°	178°	179°	

	179°	178°	177°	176°	175°
Tan	**0°**	**1°**	**2°**	**3°**	**4°**
0′	.00000	.01746	.03492	.05241	.06993
1	029	775	521	270	.07022
2	058	804	550	299	051
3	087	833	579	328	080
4	116	862	609	357	110
5	.00145	.01891	.03638	.05387	.07139
6	175	920	667	416	168
7	204	949	696	445	197
8	233	978	725	474	227
9	262	.02007	754	503	256
10	.00291	.02036	.03783	.05533	.07285
11	320	066	812	562	314
12	349	095	842	591	344
13	378	124	871	620	373
14	407	153	900	649	402
15	.00436	.02182	.03929	.05678	.07431
16	465	211	958	708	461
17	495	240	987	737	490
18	524	269	.04016	766	519
19	553	298	046	795	548
20	.00582	.02328	.04075	.05824	.07578
21	611	357	104	854	607
22	640	386	133	883	636
23	669	415	162	912	665
24	698	444	191	941	695
25	.00727	.02473	.04220	.05970	.07724
26	756	502	250	999	753
27	785	531	279	.06029	782
28	815	560	308	058	812
29	844	589	337	087	841
30	.00873	.02619	.04366	.06116	.07870
31	902	648	395	145	899
32	931	677	424	175	929
33	960	706	454	204	958
34	989	735	483	233	987
35	.01018	.02764	.04512	.06262	.08017
36	047	793	541	291	046
37	076	822	570	321	075
38	105	851	599	350	104
39	135	881	628	379	134
40	.01164	.02910	.04658	.06408	.08163
41	193	939	687	437	192
42	222	968	716	467	221
43	251	997	745	496	251
44	280	.03026	774	525	280
45	.01309	.03055	.04803	.06554	.08309
46	338	084	833	584	339
47	367	114	862	613	368
48	396	143	891	642	397
49	425	172	920	671	427
50	.01455	.03201	.04949	.06700	.08456
51	484	230	978	730	485
52	513	259	.05007	759	514
53	542	288	037	788	544
54	571	317	066	817	573
55	.01600	.03346	.05095	.06847	.08602
56	629	376	124	876	632
57	658	405	153	905	661
58	687	434	182	934	690
59	716	463	212	963	720
60	.01746	.03492	.05241	.06993	.08749
	89°	**88°**	**87°**	**86°**	**85°**
Cot	90°	91°	92°	93°	94°

174°	173°	172°	171°	170°	Tan
5°	6°	7°	8°	9°	
.08749	.10510	.12278	.14054	.15838	60′
778	540	308	084	868	59
807	569	338	113	898	58
837	599	367	143	928	57
866	628	397	173	958	56
.08895	.10657	.12426	.14202	.15988	55
925	687	456	232	.16017	54
954	716	485	262	047	53
983	746	515	291	077	52
.09013	775	544	321	107	51
.09042	.10805	.12574	.14351	.16137	50
071	834	603	381	167	49
101	863	633	410	196	48
130	893	662	440	226	47
159	922	692	470	256	46
.09189	.10952	.12722	.14499	.16286	45
218	981	751	529	316	44
247	.11011	781	559	346	43
277	040	810	588	376	42
306	070	840	618	405	41
.09335	.11099	.12869	.14648	.16435	40
365	128	899	678	465	39
394	158	929	707	495	38
423	187	958	737	525	37
453	217	988	767	555	36
.09482	.11246	.13017	.14796	.16585	35
511	276	047	826	615	34
541	305	076	856	645	33
570	335	106	886	674	32
600	364	136	915	704	31
.09629	.11394	.13165	.14945	.16734	30
658	423	195	975	764	29
688	452	224	.15005	794	28
717	482	254	034	824	27
746	511	284	064	854	26
.09776	.11541	.13313	.15094	.16884	25
805	570	343	124	914	24
834	600	372	153	944	23
864	629	402	183	974	22
893	659	432	213	.17004	21
.09923	.11688	.13461	.15243	.17033	20
952	718	491	272	063	19
981	747	521	302	093	18
.10011	777	550	332	123	17
040	806	580	362	153	16
.10069	.11836	.13609	.15391	.17183	15
099	865	639	421	213	14
128	895	669	451	243	13
158	924	698	481	273	12
187	954	728	511	303	11
.10216	.11983	.13758	.15540	.17333	10
246	.12013	787	570	363	9
275	042	817	600	393	8
305	072	846	630	423	7
334	101	876	660	453	6
.10363	.12131	.13906	.15689	.17483	5
393	160	935	719	513	4
422	190	965	749	543	3
452	219	995	779	573	2
481	249	.14024	809	603	1
.10510	.12278	.14054	.15838	.17633	0
84°	83°	82°	81°	80°	Cot
95°	96°	97°	98°	99°	

	169°	168°	167°	166°	165°
Tan	**10°**	**11°**	**12°**	**13°**	**14°**
0′	.17633	.19438	.21256	.23087	.24933
1	663	468	286	117	964
2	693	498	316	148	995
3	723	529	347	179	.25026
4	753	559	377	209	056
5	.17783	.19589	.21408	.23240	.25087
6	813	619	438	271	118
7	843	649	469	301	149
8	873	680	499	332	180
9	903	710	529	363	211
10	.17933	.19740	.21560	.23393	.25242
11	963	770	590	424	273
12	993	801	621	455	304
13	.18023	831	651	485	335
14	053	861	682	516	366
15	.18083	.19891	.21712	.23547	.25397
16	113	921	743	578	428
17	143	952	773	608	459
18	173	982	804	639	490
19	203	.20012	834	670	521
20	.18233	.20042	.21864	.23700	.25552
21	263	073	895	731	583
22	293	103	925	762	614
23	323	133	956	793	645
24	353	164	986	823	676
25	.18384	.20194	.22017	.23854	.25707
26	414	224	047	885	738
27	444	254	078	916	769
28	474	285	108	946	800
29	504	315	139	977	831
30	.18534	.20345	.22169	.24008	.25862
31	564	376	200	039	893
32	594	406	231	069	924
33	624	436	261	100	955
34	654	466	292	131	986
35	.18684	.20497	.22322	.24162	.26017
36	714	527	353	193	048
37	745	557	383	223	079
38	775	588	414	254	110
39	805	618	444	285	141
40	.18835	.20648	.22475	.24316	.26172
41	865	679	505	347	203
42	895	709	536	377	235
43	925	739	567	408	266
44	955	770	597	439	297
45	.18986	.20800	.22628	.24470	.26328
46	.19016	830	658	501	359
47	046	861	689	532	390
48	076	891	719	562	421
49	106	921	750	593	452
50	.19136	.20952	.22781	.24624	.26483
51	166	982	811	655	515
52	197	.21013	842	686	546
53	227	043	872	717	577
54	257	073	903	747	608
55	.19287	.21104	.22934	.24778	.26639
56	317	134	964	809	670
57	347	164	995	840	701
58	378	195	.23026	871	733
59	408	225	056	902	764
60	.19438	.21256	.23087	.24933	.26795
	79°	**78°**	**77°**	**76°**	**75°**
Cot	100°	101°	102°	103°	104°

164°	163°	162°	161°	160°	Tan
15°	**16°**	**17°**	**18°**	**19°**	
.26795	.28675	.30573	.32492	.34433	**60′**
826	706	605	524	465	**59**
857	738	637	556	498	**58**
888	769	669	588	530	**57**
920	801	700	621	563	**56**
.26951	.28832	.30732	.32653	.34596	**55**
982	864	764	685	628	**54**
.27013	895	796	717	661	**53**
044	927	828	749	693	**52**
076	958	860	782	726	**51**
.27107	.28990	.30891	.32814	.34758	**50**
138	.29021	923	846	791	**49**
169	053	955	878	824	**48**
201	084	987	911	856	**47**
232	116	.31019	943	889	**46**
.27263	.29147	.31051	.32975	.34922	**45**
294	179	083	.33007	954	**44**
326	210	115	040	987	**43**
357	242	147	072	.35020	**42**
388	274	178	104	052	**41**
.27419	.29305	.31210	.33136	.35085	**40**
451	337	242	169	118	**39**
482	368	274	201	150	**38**
513	400	306	233	183	**37**
545	432	338	266	216	**36**
.27576	.29463	.31370	.33298	.35248	**35**
607	495	402	330	281	**34**
638	526	434	363	314	**33**
670	558	466	395	346	**32**
701	590	498	427	379	**31**
.27732	.29621	.31530	.33460	.35412	**30**
764	653	562	492	445	**29**
795	685	594	524	477	**28**
826	716	626	557	510	**27**
858	748	658	589	543	**26**
.27889	.29780	.31690	.33621	.35576	**25**
921	811	722	654	608	**24**
952	843	754	686	641	**23**
983	875	786	718	674	**22**
.28015	906	818	751	707	**21**
.28046	.29938	.31850	.33783	.35740	**20**
077	970	882	816	772	**19**
109	.30001	914	848	805	**18**
140	033	946	881	838	**17**
172	065	978	913	871	**16**
.28203	.30097	.32010	.33945	.35904	**15**
234	128	042	978	937	**14**
266	160	074	.34010	969	**13**
297	192	106	043	.36002	**12**
329	224	139	075	035	**11**
.28360	.30255	.32171	.34108	.36068	**10**
391	287	203	140	101	**9**
423	319	235	173	134	**8**
454	351	267	205	167	**7**
486	382	299	238	199	**6**
.28517	.30414	.32331	.34270	.36232	**5**
549	446	363	303	265	**4**
580	478	396	335	298	**3**
612	509	428	368	331	**2**
643	541	460	400	364	**1**
.28675	.30573	.32492	.34433	.36397	**0**
74°	**73°**	**72°**	**71°**	**70°**	Cot
105°	106°	107°	108°	109°	

	159°	158°	157°	156°	155°
Tan	20°	21°	22°	23°	24°
0′	.36397	.38386	.40403	.42447	.44523
1	430	420	436	482	558
2	463	453	470	516	593
3	496	487	504	551	627
4	529	520	538	585	662
5	.36562	.38553	.40572	.42619	.44697
6	595	587	606	654	732
7	628	620	640	688	767
8	661	654	674	722	802
9	694	687	707	757	837
10	.36727	.38721	.40741	.42791	.44872
11	760	754	775	826	907
12	793	787	809	860	942
13	826	821	843	894	977
14	859	854	877	929	.45012
15	.36892	.38888	.40911	.42963	.45047
16	925	921	945	998	082
17	958	955	979	.43032	117
18	991	988	.41013	067	152
19	.37024	.39022	047	101	187
20	.37057	.39055	.41081	.43136	.45222
21	090	089	115	170	257
22	123	122	149	205	292
23	157	156	183	239	327
24	190	190	217	274	362
25	.37223	.39223	.41251	.43308	.45397
26	256	257	285	343	432
27	289	290	319	378	467
28	322	324	353	412	502
29	355	357	387	447	538
30	.37388	.39391	.41421	.43481	.45573
31	422	425	455	516	608
32	455	458	490	550	643
33	488	492	524	585	678
34	521	526	558	620	713
35	.37554	.39559	.41592	.43654	.45748
36	588	593	626	689	784
37	621	626	660	724	819
38	654	660	694	758	854
39	687	694	728	793	889
40	.37720	.39727	.41763	.43828	.45924
41	754	761	797	862	960
42	787	795	831	897	995
43	820	829	865	932	.46030
44	853	862	899	966	065
45	.37887	.39896	.41933	.44001	.46101
46	920	930	968	036	136
47	953	963	.42002	071	171
48	986	997	036	105	206
49	.38020	.40031	070	140	242
50	.38053	.40065	.42105	.44175	.46277
51	086	098	139	210	312
52	120	132	173	244	348
53	153	166	207	279	383
54	186	200	242	314	418
55	.38220	.40234	.42276	.44349	.46454
56	253	267	310	384	489
57	286	301	345	418	525
58	320	335	379	453	560
59	353	369	413	488	595
60	.38386	.40403	.42447	.44523	.46631
	69°	68°	67°	66°	65°
Cot	110°	111°	112°	113°	114°

154°	153°	152°	151°	150°	Tan
25°	26°	27°	28°	29°	
.46631	.48773	.50953	.53171	.55431	60′
666	809	989	208	469	59
702	845	.51026	246	507	58
737	881	063	283	545	57
772	917	099	320	583	56
.46808	.48953	.51136	.53358	.55621	55
843	989	173	395	659	54
879	.49026	209	432	697	53
914	062	246	470	736	52
950	098	283	507	774	51
.46985	.49134	.51319	.53545	.55812	50
.47021	170	356	582	850	49
056	206	393	620	888	48
092	242	430	657	926	47
128	278	467	694	964	46
.47163	.49315	.51503	.53732	.56003	45
199	351	540	769	041	44
234	387	577	807	079	43
270	423	614	844	117	42
305	459	651	882	156	41
.47341	.49495	.51688	.53920	.56194	40
377	532	724	957	232	39
412	568	761	995	270	38
448	604	798	.54032	309	37
483	640	835	070	347	36
.47519	.49677	.51872	.54107	.56385	35
555	713	909	145	424	34
590	749	946	183	462	33
626	786	983	220	501	32
662	822	.52020	258	539	31
.47698	.49858	.52057	.54296	.56577	30
733	894	094	333	616	29
769	931	131	371	654	28
805	967	168	409	693	27
840	.50004	205	446	731	26
.47876	.50040	.52242	.54484	.56769	25
912	076	279	522	808	24
948	113	316	560	846	23
984	149	353	597	885	22
.48019	185	390	635	923	21
.48055	.50222	.52427	.54673	.56962	20
091	258	464	711	.57000	19
127	295	501	748	039	18
163	331	538	786	078	17
198	368	575	824	116	16
.48234	.50404	.52613	.54862	.57155	15
270	441	650	900	193	14
306	477	687	938	232	13
342	514	724	975	271	12
378	550	761	.55013	309	11
.48414	.50587	.52798	.55051	.57348	10
450	623	836	089	386	9
486	660	873	127	425	8
521	696	910	165	464	7
557	733	947	203	503	6
.48593	.50769	.52985	.55241	.57541	5
629	806	.53022	279	580	4
665	843	059	317	619	3
701	879	096	355	657	2
737	916	134	393	696	1
.48773	.50953	.53171	.55431	.57735	0
64°	63°	62°	61°	60°	Cot
115°	116°	117°	118°	119°	

	149°	148°	147°	146°	145°
Tan	30°	31°	32°	33°	34°
0′	.57735	.60086	.62487	.64941	.67451
1	774	126	527	982	493
2	813	165	568	.65024	536
3	851	205	608	065	578
4	890	245	649	106	620
5	.57929	.60284	.62689	.65148	.67663
6	968	324	730	189	705
7	.58007	364	770	231	748
8	046	403	811	272	790
9	085	443	852	314	832
10	.58124	.60483	.62892	.65355	.67875
11	162	522	933	397	917
12	201	562	973	438	960
13	240	602	.63014	480	.68002
14	279	642	055	521	045
15	.58318	.60681	.63095	.65563	.68088
16	357	721	136	604	130
17	396	761	177	646	173
18	435	801	217	688	215
19	474	841	258	729	258
20	.58513	.60881	.63299	.65771	.68301
21	552	921	340	813	343
22	591	960	380	854	386
23	631	.61000	421	896	429
24	670	040	462	938	471
25	.58709	.61080	.63503	.65980	.68514
26	748	120	544	.66021	557
27	787	160	584	063	600
28	826	200	625	105	642
29	865	240	666	147	685
30	.58905	.61280	.63707	.66189	.68728
31	944	320	748	230	771
32	983	360	789	272	814
33	.59022	400	830	314	857
34	061	440	871	356	900
35	.59101	.61480	.63912	.66398	.68942
36	140	520	953	440	985
37	179	561	994	482	.69028
38	218	601	.64035	524	071
39	258	641	076	566	114
40	.59297	.61681	.64117	.66608	.69157
41	336	721	158	650	200
42	376	761	199	692	243
43	415	801	240	734	286
44	454	842	281	776	329
45	.59494	.61882	.64322	.66818	.69372
46	533	922	363	860	416
47	573	962	404	902	459
48	612	.62003	446	944	502
49	651	043	487	986	545
50	.59691	.62083	.64528	.67028	.69588
51	730	124	569	071	631
52	770	164	610	113	675
53	809	204	652	155	718
54	849	245	693	197	761
55	.59888	.62285	.64734	.67239	.69804
56	928	325	775	282	847
57	967	366	817	324	891
58	.60007	406	858	366	934
59	046	446	899	409	977
60	.60086	.62487	.64941	.67451	.70021
	59°	58°	57°	56°	55°
Cot	120°	121°	122°	123°	124°

144°	143°	142°	141°	140°	Tan
35°	36°	37°	38°	39°	
.70021	.72654	.75355	.78129	.80978	60′
064	699	401	175	.81027	59
107	743	447	222	075	58
151	788	492	269	123	57
194	832	538	316	171	56
.70238	.72877	.75584	.78363	.81220	55
281	921	629	410	268	54
325	966	675	457	316	53
368	.73010	721	504	364	52
412	055	767	551	413	51
.70455	.73100	.75812	.78598	.81461	50
499	144	858	645	510	49
542	189	904	692	558	48
586	234	950	739	606	47
629	278	996	786	655	46
.70673	.73323	.76042	.78834	.81703	45
717	368	088	881	752	44
760	413	134	928	800	43
804	457	180	975	849	42
848	502	226	.79022	898	41
.70891	.73547	.76272	.79070	.81946	40
935	592	318	117	995	39
979	637	364	164	.82044	38
.71023	681	410	212	092	37
066	726	456	259	141	36
.71110	.73771	.76502	.79306	.82190	35
154	816	548	354	238	34
198	861	594	401	287	33
242	906	640	449	336	32
285	951	686	496	385	31
.71329	.73996	.76733	.79544	.82434	30
373	.74041	779	591	483	29
417	086	825	639	531	28
461	131	871	686	580	27
505	176	918	734	629	26
.71549	.74221	.76964	.79781	.82678	25
593	267	.77010	829	727	24
637	312	057	877	776	23
681	357	103	924	825	22
725	402	149	972	874	21
.71769	.74447	.77196	.80020	.82923	20
813	492	242	067	972	19
857	538	289	115	.83022	18
901	583	335	163	071	17
946	628	382	211	120	16
.71990	.74674	.77428	.80258	.83169	15
.72034	719	475	306	218	14
078	764	521	354	268	13
122	810	568	402	317	12
167	855	615	450	366	11
.72211	.74900	.77661	.80498	.83415	10
255	946	708	546	465	9
299	991	754	594	514	8
344	.75037	801	642	564	7
388	082	848	690	613	6
.72432	.75128	.77895	.80738	.83662	5
477	173	941	786	712	4
521	219	988	834	761	3
565	264	.78035	882	811	2
610	310	082	930	860	1
.72654	.75355	.78129	.80978	.83910	0
54°	53°	52°	51°	50°	Cot
125°	126°	127°	128°	129°	

	139°	138°	137°	136°	135°
Tan	40°	41°	42°	43°	44°
0′	.83910	.86929	.90040	.93252	.96569
1	960	980	093	306	625
2	.84009	.87031	146	360	681
3	059	082	199	415	738
4	108	133	251	469	794
5	.84158	.87184	.90304	.93524	.96850
6	208	236	357	578	907
7	258	287	410	633	963
8	307	338	463	688	.97020
9	357	389	516	742	076
10	.84407	.87441	.90569	.93797	.97133
11	457	492	621	852	189
12	507	543	674	906	246
13	556	595	727	961	302
14	606	646	781	.94016	359
15	.84656	.87698	.90834	.94071	.97416
16	706	749	887	125	472
17	756	801	940	180	529
18	806	852	993	235	586
19	856	904	.91046	290	643
20	.84906	.87955	.91099	.94345	.97700
21	956	.88007	153	400	756
22	.85006	059	206	455	813
23	057	110	259	510	870
24	107	162	313	565	927
25	.85157	.88214	.91366	.94620	.97984
26	207	265	419	676	.98041
27	257	317	473	731	098
28	308	369	526	786	155
29	358	421	580	841	213
30	.85408	.88473	.91633	.94896	.98270
31	458	524	687	952	327
32	509	576	740	.95007	384
33	559	628	794	062	441
34	609	680	847	118	499
35	.85660	.88732	.91901	.95173	.98556
36	710	784	955	229	613
37	761	836	.92008	284	671
38	811	888	062	340	728
39	862	940	116	395	786
40	.85912	.88992	.92170	.95451	.98843
41	963	.89045	224	506	901
42	.86014	097	277	562	958
43	064	149	331	618	.99016
44	115	201	385	673	073
45	.86166	.89253	.92439	.95729	.99131
46	216	306	493	785	189
47	267	358	547	841	247
48	318	410	601	897	304
49	368	463	655	952	362
50	.86419	.89515	.92709	.96008	.99420
51	470	567	763	064	478
52	521	620	817	120	536
53	572	672	872	176	594
54	623	725	926	232	652
55	.86674	.89777	.92980	.96288	.99710
56	725	830	.93034	344	768
57	776	883	088	400	826
58	827	935	143	457	884
59	878	988	197	513	942
60	.86929	.90040	.93252	.96569	1.00000
	49°	48°	47°	46°	45°
Cot	130°	131°	132°	133°	134°

134°	133°	132°	131°	130°	Tan
45°	**46°**	**47°**	**48°**	**49°**	
1.00000	1.03553	1.07237	1.11061	1.15037	**60′**
0058	3613	7299	1126	5104	**59**
0116	3674	7362	1191	5172	**58**
0175	3734	7425	1256	5240	**57**
0233	3794	7487	1321	5308	**56**
1.00291	1.03855	1.07550	1.11387	1.15375	**55**
0350	3915	7613	1452	5443	**54**
0408	3976	7676	1517	5511	**53**
0467	4036	7738	1582	5579	**52**
0525	4097	7801	1648	5647	**51**
1.00583	1.04158	1.07864	1.11713	1.15715	**50**
0642	4218	7927	1778	5783	**49**
0701	4279	7990	1844	5851	**48**
0759	4340	8053	1909	5919	**47**
0818	4401	8116	1975	5987	**46**
1.00876	1.04461	1.08179	1.12041	1.16056	**45**
0935	4522	8243	2106	6124	**44**
0994	4583	8306	2172	6192	**43**
1053	4644	8369	2238	6261	**42**
1112	4705	8432	2303	6329	**41**
1.01170	1.04766	1.08496	1.12369	1.16398	**40**
1229	4827	8559	2435	6466	**39**
1288	4888	8622	2501	6535	**38**
1347	4949	8686	2567	6603	**37**
1406	5010	8749	2633	6672	**36**
1.01465	1.05072	1.08813	1.12699	1.16741	**35**
1524	5133	8876	2765	6809	**34**
1583	5194	8940	2831	6878	**33**
1642	5255	9003	2897	6947	**32**
1702	5317	9067	2963	7016	**31**
1.01761	1.05378	1.09131	1.13029	1.17085	**30**
1820	5439	9195	3096	7154	**29**
1879	5501	9258	3162	7223	**28**
1939	5562	9322	3228	7292	**27**
1998	5624	9386	3295	7361	**26**
1.02057	1.05685	1.09450	1.13361	1.17430	**25**
2117	5747	9514	3428	7500	**24**
2176	5809	9578	3494	7569	**23**
2236	5870	9642	3561	7638	**22**
2295	5932	9706	3627	7708	**21**
1.02355	1.05994	1.09770	1.13694	1.17777	**20**
2414	6056	9834	3761	7846	**19**
2474	6117	9899	3828	7916	**18**
2533	6179	9963	3894	7986	**17**
2593	6241	1.10027	3961	8055	**16**
1.02653	1.06303	1.10091	1.14028	1.18125	**15**
2713	6365	0156	4095	8194	**14**
2772	6427	0220	4162	8264	**13**
2832	6489	0285	4229	8334	**12**
2892	6551	0349	4296	8404	**11**
1.02952	1.06613	1.10414	1.14363	1.18474	**10**
3012	6676	0478	4430	8544	**9**
3072	6738	0543	4498	8614	**8**
3132	6800	0607	4565	8684	**7**
3192	6862	0672	4632	8754	**6**
1.03252	1.06925	1.10737	1.14699	1.18824	**5**
3312	6987	0802	4767	8894	**4**
3372	7049	0867	4834	8964	**3**
3433	7112	0931	4902	9035	**2**
3493	7174	0996	4969	9105	**1**
1.03553	1.07237	1.11061	1.15037	1.19175	**0**
44°	**43°**	**42°**	**41°**	**40°**	Cot
135°	136°	137°	138°	139°	

	129°	128°	127°	126°	125°
Tan	50°	51°	52°	53°	54°
0′	1.19175	1.23490	1.27994	1.32704	1.37638
1	9246	3563	8071	2785	7722
2	9316	3637	8148	2865	7807
3	9387	3710	8225	2946	7891
4	9457	3784	8302	3026	7976
5	1.19528	1.23858	1.28379	1.33107	1.38060
6	9599	3931	8456	3187	8145
7	9669	4005	8533	3268	8229
8	9740	4079	8610	3349	8314
9	9811	4153	8687	3430	8399
10	1.19882	1.24227	1.28764	1.33511	1.38484
11	9953	4301	8842	3592	8568
12	1.20024	4375	8919	3673	8653
13	0095	4449	8997	3754	8738
14	0166	4523	9074	3835	8824
15	1.20237	1.24597	1.29152	1.33916	1.38909
16	0308	4672	9229	3998	8994
17	0379	4746	9307	4079	9079
18	0451	4820	9385	4160	9165
19	0522	4895	9463	4242	9250
20	1.20593	1.24969	1.29541	1.34323	1.39336
21	0665	5044	9618	4405	9421
22	0736	5118	9696	4487	9507
23	0808	5193	9775	4568	9593
24	0879	5268	9853	4650	9679
25	1.20951	1.25343	1.29931	1.34732	1.39764
26	1023	5417	1.30009	4814	9850
27	1094	5492	0087	4896	9936
28	1166	5567	0166	4978	1.40022
29	1238	5642	0244	5060	0109
30	1.21310	1.25717	1.30323	1.35142	1.40195
31	1382	5792	0401	5224	0281
32	1454	5867	0480	5307	0367
33	1526	5943	0558	5389	0454
34	1598	6018	0637	5472	0540
35	1.21670	1.26093	1.30716	1.35554	1.40627
36	1742	6169	0795	5637	0714
37	1814	6244	0873	5719	0800
38	1886	6319	0952	5802	0887
39	1959	6395	1031	5885	0974
40	1.22031	1.26471	1.31110	1.35968	1.41061
41	2104	6546	1190	6051	1148
42	2176	6622	1269	6134	1235
43	2249	6698	1348	6217	1322
44	2321	6774	1427	6300	1409
45	1.22394	1.26849	1.31507	1.36383	1.41497
46	2467	6925	1586	6466	1584
47	2539	7001	1666	6549	1672
48	2612	7077	1745	6633	1759
49	2685	7153	1825	6716	1847
50	1.22758	1.27230	1.31904	1.36800	1.41934
51	2831	7306	1984	6883	2022
52	2904	7382	2064	6967	2110
53	2977	7458	2144	7050	2198
54	3050	7535	2224	7134	2286
55	1.23123	1.27611	1.32304	1.37218	1.42374
56	3196	7688	2384	7302	2462
57	3270	7764	2464	7386	2550
58	3343	7841	2544	7470	2638
59	3416	7917	2624	7554	2726
60	1.23490	1.27994	1.32704	1.37638	1.42815
	39°	38°	37°	36°	35°
Cot	140°	141°	142°	143°	144°

124°	123°	122°	121°	120°	Tan
55°	**56°**	**57°**	**58°**	**59°**	
1.42815	1.48256	1.53986	1.60033	1.66428	**60′**
2903	8349	4085	0137	6538	**59**
2992	8442	4183	0241	6647	**58**
3080	8536	4281	0345	6757	**57**
3169	8629	4379	0449	6867	**56**
1.43258	1.48722	1.54478	1.60553	1.66978	**55**
3347	8816	4576	0657	7088	**54**
3436	8909	4675	0761	7198	**53**
3525	9003	4774	0865	7309	**52**
3614	9097	4873	0970	7419	**51**
1.43703	1.49190	1.54972	1.61074	1.67530	**50**
3792	9284	5071	1179	7641	**49**
3881	9378	5170	1283	7752	**48**
3970	9472	5269	1388	7863	**47**
4060	9566	5368	1493	7974	**46**
.44149	1.49661	1.55467	1.61598	1.68085	**45**
4239	9755	5567	1703	8196	**44**
4329	9849	5666	1808	8308	**43**
4418	9944	5766	1914	8419	**42**
4508	1.50038	5866	2019	8531	**41**
1.44598	1.50133	1.55966	1.62125	1.68643	**40**
4688	0228	6065	2230	8754	**39**
4778	0322	6165	2336	8866	**38**
4868	0417	6265	2442	8979	**37**
4958	0512	6366	2548	9091	**36**
1.45049	1.50607	1.56466	1.62654	1.69203	**35**
5139	0702	6566	2760	9316	**34**
5229	0797	6667	2866	9428	**33**
5320	0893	6767	2972	9541	**32**
5410	0988	6868	3079	9653	**31**
1.45501	1.51084	1.56969	1.63185	1.69766	**30**
5592	1179	7069	3292	9879	**29**
5682	1275	7170	3398	9992	**28**
5773	1370	7271	3505	1.70106	**27**
5864	1466	7372	3612	0219	**26**
1.45955	1.51562	1.57474	1.63719	1.70332	**25**
6046	1658	7575	3826	0446	**24**
6137	1754	7676	3934	0560	**23**
6229	1850	7778	4041	0673	**22**
6320	1946	7879	4148	0787	**21**
1.46411	1.52043	1.57981	1.64256	1.70901	**20**
6503	2139	8083	4363	1015	**19**
6595	2235	8184	4471	1129	**18**
6686	2332	8286	4579	1244	**17**
6778	2429	8388	4687	1358	**16**
1.46870	1.52525	1.58490	1.64795	1.71473	**15**
6962	2622	8593	4903	1588	**14**
7053	2719	8695	5011	1702	**13**
7146	2816	8797	5120	1817	**12**
7238	2913	8900	5228	1932	**11**
1.47330	1.53010	1.59002	1.65337	1.72047	**10**
7422	3107	9105	5445	2163	**9**
7514	3205	9208	5554	2278	**8**
7607	3302	9311	5663	2393	**7**
7699	3400	9414	5772	2509	**6**
1.47792	1.53497	1.59517	1.65881	1.72625	**5**
7885	3595	9620	5990	2741	**4**
7977	3693	9723	6099	2857	**3**
8070	3791	9826	6209	2973	**2**
8163	3888	9930	6318	3089	**1**
1.48256	1.53986	1.60033	1.66428	1.73205	**0**
34°	**33°**	**32°**	**31°**	**30°**	Cot
145°	146°	147°	148°	149°	

	119°	118°	117°	116°	115°
Tan	60°	61°	62°	63°	64°
0′	1.73205	1.80405	1.88073	1.96261	2.05030
1	3321	0529	8205	6402	5182
2	3438	0653	8337	6544	5333
3	3555	0777	8469	6685	5485
4	3671	0901	8602	6827	5637
5	1.73788	1.81025	1.88734	1.96969	2.05790
6	3905	1150	8867	7111	5942
7	4022	1274	9000	7253	6094
8	4140	1399	9133	7395	6247
9	4257	1524	9266	7538	6400
10	1.74375	1.81649	1.89400	1.97681	2.06553
11	4492	1774	9533	7823	6706
12	4610	1899	9667	7966	6860
13	4728	2025	9801	8110	7014
14	4846	2150	9935	8253	7167
15	1.74964	1.82276	1.90069	1.98396	2.07321
16	5082	2402	0203	8540	7476
17	5200	2528	0337	8684	7630
18	5319	2654	0472	8828	7785
19	5437	2780	0607	8972	7939
20	1.75556	1.82906	1.90741	1.99116	2.08094
21	5675	3033	0876	9261	8250
22	5794	3159	1012	9406	8405
23	5913	3286	1147	9550	8560
24	6032	3413	1282	9695	8716
25	1.76151	1.83540	1.91418	1.99841	2.08872
26	6271	3667	1554	9986	9028
27	6390	3794	1690	2.00131	9184
28	6510	3922	1826	0277	9341
29	6629	4049	1962	0423	9498
30	1.76749	1.84177	1.92098	2.00569	2.09654
31	6869	4305	2235	0715	9811
32	6990	4433	2371	0862	9969
33	7110	4561	2508	1008	2.10126
34	7230	4689	2645	1155	0284
35	1.77351	1.84818	1.92782	2.01302	2.10442
36	7471	4946	2920	1449	0600
37	7592	5075	3057	1596	0758
38	7713	5204	3195	1743	0916
39	7834	5333	3332	1891	1075
40	1.77955	1.85462	1.93470	2.02039	2.11233
41	8077	5591	3608	2187	1392
42	8198	5720	3746	2335	1552
43	8319	5850	3885	2483	1711
44	8441	5979	4023	2631	1871
45	1.78563	1.86109	1.94162	2.02780	2.12030
46	8685	6239	4301	2929	2190
47	8807	6369	4440	3078	2350
48	8929	6499	4579	3227	2511
49	9051	6630	4718	3376	2671
50	1.79174	1.86760	1.94858	2.03526	2.12832
51	9296	6891	4997	3675	2993
52	9419	7021	5137	3825	3154
53	9542	7152	5277	3975	3316
54	9665	7283	5417	4125	3477
55	1.79788	1.87415	1.95557	2.04276	2.13639
56	9911	7546	5698	4426	3801
57	1.80034	7677	5838	4577	3963
58	0158	7809	5979	4728	4125
59	0281	7941	6120	4879	4288
60	1.80405	1.88073	1.96261	2.05030	2.14451
	29°	28°	27°	26°	25°
Cot	150°	151°	152°	153°	154°

114°	113°	112°	111°	110°	Tan
65°	**66°**	**67°**	**68°**	**69°**	
2.14451	2.24604	2.35585	2.47509	2.60509	**60′**
4614	4780	5776	7716	0736	**59**
4777	4956	5967	7924	0963	**58**
4940	5132	6158	8132	1190	**57**
5104	5309	6349	8340	1418	**56**
2.15268	2.25486	2.36541	2.48549	2.61646	**55**
5432	5663	6733	8758	1874	**54**
5596	5840	6925	8967	2103	**53**
5760	6018	7118	9177	2332	**52**
5925	6196	7311	9386	2561	**51**
2.16090	2.26374	2.37504	2.49597	2.62791	**50**
6255	6552	7697	9807	3021	**49**
6420	6730	7891	2.50018	3252	**48**
6585	6909	8084	0229	3483	**47**
6751	7088	8279	0440	3714	**46**
2.16917	2.27267	2.38473	2.50652	2.63945	**45**
7083	7447	8668	0864	4177	**44**
7249	7626	8863	1076	4410	**43**
7416	7806	9058	1289	4642	**42**
7582	7987	9253	1502	4875	**41**
2.17749	2.28167	2.39449	2.51715	2.65109	**40**
7916	8348	9645	1929	5342	**39**
8084	8528	9841	2142	5576	**38**
8251	8710	2.40038	2357	5811	**37**
8419	8891	0235	2571	6046	**36**
2.18587	2.29073	2.40432	2.52786	2.66281	**35**
8755	9254	0629	3001	6516	**34**
8923	9437	0827	3217	6752	**33**
9092	9619	1025	3432	6989	**32**
9261	9801	1223	3648	7225	**31**
2.19430	2.29984	2.41421	2.53865	2.67462	**30**
9599	2.30167	1620	4082	7700	**29**
9769	0351	1819	4299	7937	**28**
9938	0534	2019	4516	8175	**27**
2.20108	0718	2218	4734	8414	**26**
2.20278	2.30902	2.42418	2.54952	2.68653	**25**
0449	1086	2618	5170	8892	**24**
0619	1271	2819	5389	9131	**23**
0790	1456	3019	5608	9371	**22**
0961	1641	3220	5827	9612	**21**
2.21132	2.31826	2.43422	2.56046	2.69853	**20**
1304	2012	3623	6266	2.70094	**19**
1475	2197	3825	6487	0335	**18**
1647	2383	4027	6707	0577	**17**
1819	2570	4230	6928	0819	**16**
2.21992	2.32756	2.44433	2.57150	2.71062	**15**
2164	2943	4636	7371	1305	**14**
2337	3130	4839	7593	1548	**13**
2510	3317	5043	7815	1792	**12**
2683	3505	5246	8038	2036	**11**
2.22857	2.33693	2.45451	2.58261	2.72281	**10**
3030	3881	5655	8484	2526	**9**
3204	4069	5860	8708	2771	**8**
3378	4258	6065	8932	3017	**7**
3553	4447	6270	9156	3263	**6**
2.23727	2.34636	2.46476	2.59381	2.73509	**5**
3902	4825	6682	9606	3756	**4**
4077	5015	6888	9831	4004	**3**
4252	5205	7095	2.60057	4251	**2**
4428	5395	7302	0283	4499	**1**
2.24604	2.35585	2.47509	2.60509	2.74748	**0**
24°	**23°**	**22°**	**21°**	**20°**	Cot
155°	156°	157°	158°	159°	

	109°	108°	107°	106°	105°
Tan	70°	71°	72°	73°	74°
0′	2.74748	2.90421	3.07768	3.27085	3.48741
1	74997	90696	08073	27426	49125
2	75246	90971	08379	27767	49509
3	75496	91246	08685	28109	49894
4	75746	91523	08991	28452	50279
5	2.75996	2.91799	3.09298	3.28795	3.50666
6	76247	92076	09606	29139	51053
7	76498	92354	09914	29483	51441
8	76750	92632	10223	29829	51829
9	77002	92910	10532	30174	52219
10	2.77254	2.93189	3.10842	3.30521	3.52609
11	77507	93468	11153	30868	53001
12	77761	93748	11464	31216	53393
13	78014	94028	11775	31565	53785
14	78269	94309	12087	31914	54179
15	2.78523	2.94591	3.12400	3.32264	3.54573
16	78778	94872	12713	32614	54968
17	79033	95155	13027	32965	55364
18	79289	95437	13341	33317	55761
19	79545	95721	13656	33670	56159
20	2.79802	2.96004	3.13972	3.34023	3.56557
21	80059	96288	14288	34377	56957
22	80316	96573	14605	34732	57357
23	80574	96858	14922	35087	57758
24	80833	97144	15240	35443	58160
25	2.81091	2.97430	3.15558	3.35800	3.58562
26	81350	97717	15877	36158	58966
27	81610	98004	16197	36516	59370
28	81870	98292	16517	36875	59775
29	82130	98580	16838	37234	60181
30	2.82391	2.98868	3.17159	3.37594	3.60588
31	82653	99158	17481	37955	60996
32	82914	99447	17804	38317	61405
33	83176	99738	18127	38679	61814
34	83439	3.00028	18451	39042	62224
35	2.83702	3.00319	3.18775	3.39406	3.62636
36	83965	00611	19100	39771	63048
37	84229	00903	19426	40136	63461
38	84494	01196	19752	40502	63874
39	84758	01489	20079	40869	64289
40	2.85023	3.01783	3.20406	3.41236	3.64705
41	85289	02077	20734	41604	65121
42	85555	02372	21063	41973	65538
43	85822	02667	21392	42343	65957
44	86089	02963	21722	42713	66376
45	2.86356	3.03260	3.22053	3.43084	3.66796
46	86624	03556	22384	43456	67217
47	86892	03854	22715	43829	67638
48	87161	04152	23048	44202	68061
49	87430	04450	23381	44576	68485
50	2.87700	3.04749	3.23714	3.44951	3.68909
51	87970	05049	24049	45327	69335
52	88240	05349	24383	45703	69761
53	88511	05649	24719	46080	70188
54	88783	05950	25055	46458	70616
55	2.89055	3.06252	3.25392	3.46837	3.71046
56	89327	06554	25729	47216	71476
57	89600	06857	26067	47596	71907
58	89873	07160	26406	47977	72338
59	90147	07464	26745	48359	72771
60	2.90421	3.07768	3.27085	3.48741	3.73205
	19°	18°	17°	16°	15°
Cot	160°	161°	162°	163°	164°

104°	103°	102°	101°	100°	Tan
75°	76°	77°	78°	79°	
3.73205	4.01078	4.33148	4.70463	5.14455	60′
73640	01576	33723	71137	15256	59
74075	02074	34300	71813	16058	58
74512	02574	34879	72490	16863	57
74950	03076	35459	73170	17671	56
3.75388	4.03578	4.36040	4.73851	5.18480	55
75828	04081	36623	74534	19293	54
76268	04586	37207	75219	20107	53
76709	05092	37793	75906	20925	52
77152	05599	38381	76595	21744	51
3.77595	4.06107	4.38969	4.77286	5.22566	50
78040	06616	39560	77978	23391	49
78485	07127	40152	78673	24218	48
78931	07639	40745	79370	25048	47
79378	08152	41340	80068	25880	46
3.79827	4.08666	4.41936	4.80769	5.26715	45
80276	09182	42534	81471	27553	44
80726	09699	43134	82175	28393	43
81177	10216	43735	82882	29235	42
81630	10736	44338	83590	30080	41
3.82083	4.11256	4.44942	4.84300	5.30928	40
82537	11778	45548	85013	31778	39
82992	12301	46155	85727	32631	38
83449	12825	46764	86444	33487	37
83906	13350	47374	87162	34345	36
3.84364	4.13877	4.47986	4.87882	5.35206	35
84824	14405	48600	88605	36070	34
85284	14934	49215	89330	36936	33
85745	15465	49832	90056	37805	32
86208	15997	50451	90785	38677	31
3.86671	4.16530	4.51071	4.91516	5.39552	30
87136	17064	51693	92249	40429	29
87601	17600	52316	92984	41309	28
88068	18137	52941	93721	42192	27
88536	18675	53568	94460	43077	26
3.89004	4.19215	4.54196	4.95201	5.43966	25
89474	19756	54826	95945	44857	24
89945	20298	55458	96690	45751	23
90417	20842	56091	97438	46648	22
90890	21387	56726	98188	47548	21
3.91364	4.21933	4.57363	4.98940	5.48451	20
91839	22481	58001	99695	49356	19
92316	23030	58641	5.00451	50264	18
92793	23580	59283	01210	51176	17
93271	24132	59927	01971	52090	16
3.93751	4.24685	4.60572	5.02734	5.53007	15
94232	25239	61219	03499	53927	14
94713	25795	61868	04267	54851	13
95196	26352	62518	05037	55777	12
95680	26911	63171	05809	56706	11
3.96165	4.27471	4.63825	5.06584	5.57638	10
96651	28032	64480	07360	58573	9
97139	28595	65138	08139	59511	8
97627	29159	65797	08921	60452	7
98117	29724	66458	09704	61397	6
3.98607	4.30291	4.67121	5.10490	5.62344	5
99099	30860	67786	11279	63295	4
99592	31430	68452	12069	64248	3
4.00086	32001	69121	12862	65205	2
00582	32573	69791	13658	66165	1
4.01078	4.33148	4.70463	5.14455	5.67128	0
14°	13°	12°	11°	10°	Cot
165°	166°	167°	168°	169°	

	99°	98°	97°	96°	95°
Tan	80°	81°	82°	83°	84°
0′	5.67128	6.31375	7.11537	8.14435	9.51436
1	68094	32566	13042	16398	54106
2	69064	33761	14553	18370	56791
3	70037	34961	16071	20352	59490
4	71013	36165	17594	22344	62205
5	5.71992	6.37374	7.19125	8.24345	9.64935
6	72974	38587	20661	26355	67680
7	73960	39804	22204	28376	70441
8	74949	41026	23754	30406	73217
9	75941	42253	25310	32446	76009
10	5.76937	6.43484	7.26873	8.34496	9.78817
11	77936	44720	28442	36555	81641
12	78938	45961	30018	38625	84482
13	79944	47206	31600	40705	87338
14	80953	48456	33190	42795	90211
15	5.81966	6.49710	7.34786	8.44896	9.93101
16	82982	50970	36389	47007	96007
17	84001	52234	37999	49128	98931
18	85024	53503	39616	51259	10.0187
19	86051	54777	41240	53402	0.0483
20	5.87080	6.56055	7.42871	8.55555	10.0780
21	88114	57339	44509	57718	0.1080
22	89151	58627	46154	59893	0.1381
23	90191	59921	47806	62078	0.1683
24	91236	61219	49465	64275	0.1988
25	5.92283	6.62523	7.51132	8.66482	10.2294
26	93335	63831	52806	68701	0.2602
27	94390	65144	54487	70931	0.2913
28	95448	66463	56176	73172	0.3224
29	96510	67787	57872	75425	0.3538
30	5.97576	6.69116	7.59575	8.77689	10.3854
31	98646	70450	61287	79964	0.4172
32	99720	71789	63005	82252	0.4491
33	6.00797	73133	64732	84551	0.4813
34	01878	74483	66466	86862	0.5136
35	6.02962	6.75838	7.68208	8.89185	10.5462
36	04051	77199	69957	91520	0.5789
37	05143	78564	71715	93867	0.6118
38	06240	79936	73480	96227	0.6450
39	07340	81312	75254	98598	0.6783
40	6.08444	6.82694	7.77035	9.00983	10.7119
41	09552	84082	78825	03379	0.7457
42	10664	85475	80622	05789	0.7797
43	11779	86874	82428	08211	0.8139
44	12899	88278	84242	10646	0.8483
45	6.14023	6.89688	7.86064	9.13093	10.8829
46	15151	91104	87895	15554	0.9178
47	16283	92525	89734	18028	0.9529
48	17419	93952	91582	20516	0.9882
49	18559	95385	93438	23016	1.0237
50	6.19703	6.96823	7.95302	9.25530	11.0594
51	20851	98268	97176	28058	1.0954
52	22003	99718	99058	30599	1.1316
53	23160	7.01174	8.00948	33155	1.1681
54	24321	02637	02848	35724	1.2048
55	6.25486	7.04105	8.04756	9.38307	11.2417
56	26655	05579	06674	40904	1.2789
57	27829	07059	08600	43515	1.3163
58	29007	08546	10536	46141	1.3540
59	30189	10038	12481	48781	1.3919
60	6.31375	7.11537	8.14435	9.51436	11.4301
	9°	8°	7°	6°	5°
Cot	170°	171°	172°	173°	174°

94°	93°	92°	91°	90°	Tan
85°	86°	87°	88°	89°	
11.4301	14.3007	19.0811	28.6363	57.2900	60′
1.4685	4.3607	9.1879	8.8771	8.2612	59
1.5072	4.4212	9.2959	9.1220	9.2659	58
1.5461	4.4823	9.4051	9.3711	60.3058	57
1.5853	4.5438	9.5156	9.6245	1.3829	56
11.6248	14.6059	19.6273	29.8823	62.4992	55
1.6645	4.6685	9.7403	30.1446	3.6567	54
1.7045	4.7317	9.8546	0.4116	4.8580	53
1.7448	4.7954	9.9702	0.6833	6.1055	52
1.7853	4.8596	20.0872	0.9599	7.4019	51
11.8262	14.9244	20.2056	31.2416	68.7501	50
1.8673	4.9898	0.3253	1.5284	70.1533	49
1.9087	5.0557	0.4465	1.8205	1.6151	48
1.9504	5.1222	0.5691	2.1181	3.1390	47
1.9923	5.1893	0.6932	2.4213	4.7292	46
12.0346	15.2571	20.8188	32.7303	76.3900	45
2.0772	5.3254	0.9460	3.0452	8.1263	44
2.1201	5.3943	1.0747	3.3662	9.9434	43
2.1632	5.4638	1.2049	3.6935	81.8470	42
2.2067	5.5340	1.3369	4.0273	3.8435	41
12.2505	15.6048	21.4704	34.3678	85.9398	40
2.2946	5.6762	1.6056	4.7151	8.1436	39
2.3390	5.7483	1.7426	5.0695	90.4633	38
2.3838	5.8211	1.8813	5.4313	2.9085	37
2.4288	5.8945	2.0217	5.8006	5.4895	36
12.4742	15.9687	22.1640	36.1776	98.2179	35
2.5199	6.0435	2.3081	6.5627	101.107	34
2.5660	6.1190	2.4541	6.9560	04.171	33
2.6124	6.1952	2.6020	7.3579	07.426	32
2.6591	6.2722	2.7519	7.7686	10.892	31
12.7062	16.3499	22.9038	38.1885	114.589	30
2.7536	6.4283	3.0577	8.6177	18.540	29
2.8014	6.5075	3.2137	9.0568	22.774	28
2.8496	6.5874	3.3718	9.5059	27.321	27
2.8981	6.6681	3.5321	9.9655	32.219	26
12.9469	16.7496	23.6945	40.4358	137.507	25
2.9962	6.8319	3.8593	0.9174	43.237	24
3.0458	6.9150	4.0263	1.4106	49.465	23
3.0958	6.9990	4.1957	1.9158	56.259	22
3.1461	7.0837	4.3675	2.4335	63.700	21
13.1969	17.1693	24.5418	42.9641	171.885	20
3.2480	7.2558	4.7185	3.5081	80.932	19
3.2996	7.3432	4.8978	4.0661	90.984	18
3.3515	7.4314	5.0798	4.6386	202.219	17
3.4039	7.5205	5.2644	5.2261	14.858	16
13.4566	17.6106	25.4517	45.8294	229.182	15
3.5098	7.7015	5.6418	6.4489	45.552	14
3.5634	7.7934	5.8348	7.0853	64.441	13
3.6174	7.8863	6.0307	7.7395	86.478	12
3.6719	7.9802	6.2296	8.4121	312.521	11
13.7267	18.0750	26.4316	49.1039	343.774	10
3.7821	8.1708	6.6367	9.8157	381.971	9
3.8378	8.2677	6.8450	50.5485	429.718	8
3.8940	8.3655	7.0566	1.3032	491.106	7
3.9507	8.4645	7.2715	2.0807	572.957	6
14.0079	18.5645	27.4899	52.8821	687.549	5
4.0655	8.6656	7.7117	3.7086	859.436	4
4.1235	8.7678	7.9372	4.5613	1145.92	3
4.1821	8.8711	8.1664	5.4415	1718.87	2
4.2411	8.9755	8.3994	6.3506	3437.75	1
14.3007	19.0811	28.6363	57.2900	Infinite	0
4°	3°	2°	1°	0°	Cot
175°	176°	177°	178°	179°	

VI. CONVERSION FACTORS.

Angles.

1 rad. = 57.2958 deg. = 3437.75 min. = 206,265 sec.

Areas.

1 sq. mile = 640 acres = 258.999 hectares.

1 hectare = 100 ares = 10,000 sq. meters = 2.471 acres.

1 acre = 10 sq. chains = 43,560 sq. ft.

1 sq. yd. = 9 sq. ft. = 0.836 sq. meter.

1 sq. meter = 10.764 sq. ft. = 1.196 sq. yd.

Densities.

1 lb. per cu. ft. = 16.018 kg. per cu. meter.

1 lb. per cu. in. = 27.680 g. per cu. cm.

1 kg. per cu. meter = 0.06243 lb. per cu. ft.

1 g. per cu. cm. = 0.03613 lb. per cu. in.

Discharge.

1 cu. ft. per sec. = 448.9 gal. per min. = 1.9835 acre-ft. per day.

1 acre-ft. per day = 0.5042 cu. ft. per sec.

1,000,000 gal. per day = 3.0689 acre-ft. per day = 1.547 cu. ft. per sec.

1 cu. ft. per sec. = 40 miner's inches.

1 miner's inch = 1.5 cu. ft. per min. = 11.22 gal. per min.

1 in. of rainfall per hr. = 1.008 cu. ft. per sec. per acre.

Energy.

1 ft-lb. = 1.356 joules or watt-sec.

1 joule = 10^7 ergs = 10^7 dyne-cm.

1 horse-power-hr. = 1.98×10^6 ft-lb. = 0.7457 kw-hr. = 2544 Btu.

1 kw-hr. = 1.341 horse-power-hr. = 3411 Btu. = 2.654×10^6 ft-lb.

1 Btu. = 778.4 ft-lb. = 0.252 kg-cal.

1 meter-kilogram = 7.233 ft-lb.

Force.

1 lb. = 0.4536 kg. = 444,822 dynes.
1 kg. = 2.2046 lb. = 980,665 dynes.
1,000,000 dynes = 2.2481 lb. = 1.020 kg.

Length.

1 mile = 5280 ft. = 80 chains = 320 rods = 1.6094 kilometers.
1 meter = 39.37 inches = 3.2808 ft. = 1.0936 yd.
1 in. = 2.54 cm. = 25.4 mm.
1 yd. = 0.9144 meter.
1 ft. = 30.48 cm. = 0.3048 meter.

Power.

1 horse-power = 33,000 ft-lb. per min. = 550 ft-lb. per sec.
1 horse-power = 0.7457 kw. = 0.7066 Btu. per sec.
1 kw. = 1.341 horse-power = 737.5 ft-lb. per sec.
1 horse-power = 1.0139 metric horse-power.

Pressure.

1 ft. of water = 62.4 lb. per sq. ft. = 0.433 lb. per sq. in.
1 in. of mercury = 1.134 ft. of water = 0.4912 lb. per sq. in.
1 atmosphere = 14.697 lb. per sq. in. = 33.9 ft. of water.
1 lb. per sq. ft. = 4.8824 kg. per sq. meter.
1 lb. per sq. in. = 0.07031 kg. per sq. cm.
1 kg. per sq. cm. = 14.223 lb. per sq. in. = 32.8 ft. of water.
1 ton per sq. ft. = 13.889 lb. per sq. in.

Temperature.

Deg. C. = (deg. F. − 32) × 0.55556.
Deg. F. = (1.8 × deg. C.) + 32.

Velocity.

1 rad. per sec. = 9.5496 rev. per min. = 0.15916 rev. per sec.

1 rev. per min. = 6.0000 deg. per sec.

1 ft. per sec. = 0.6818 miles per hr.

1 mile per hr. = 88 ft. per min. = 1.4667 ft. per sec.

Volume.

1 cu. yd. = 27 cu. ft. = 21.696 bushels.

1 cu. meter = 1000 liters = 1.308 cu. yds.

1 bu. = 8 gal. (dry) = 1.2445 cu. ft. = 2150.4 cu. in.

1 gal. (dry measure) = 1.1637 gal. (liquid measure).

1 cu. ft. = 7.481 gal. (liquid measure).

Weight.

1 lb. Avoir. = 1.2153 lb. Troy or Apoth.

1 lb. Avoir. = 16 oz. = 7000 grains = 0.4536 kg.

1 kg. = 2.2046 lb. Avoir.

1 short ton = 2000 lb. = 0.90718 metric ton.

1 long ton = 2240 lb. = 1.120 short tons.

1 metric ton = 1000 kg. = 2204.6 lb.

Pressure, In. of Mercury.	Temp., ° F.	Volume of 1 Lb. in Cu. Ft.	Thermal Head in B.t.u.		Latent Heat, B.t.u.
			of liquid.	of vapor.	
p	t	v''	i'	i''	r
1	79.1	652	47.1	1095.0	1047.9
1.2	84.7	549	52.7	97.6	44.9
1.4	89.5	474.3	57.6	99.8	42.3
1.6	93.8	418.2	61.8	1101.8	40.0
1.8	97.7	374.3	65.7	03.5	37.9
2	101.2	338.9	69.2	1105.1	1036.0
3	115.1	231.4	83.0	11.4	28.3
4	125.4	176.5	93.4	15.9	22.5
6	140.8	120.7	108.7	22.6	13.9
8	152.3	92.1	120.2	27.5	07.4
10	161.5	74.8	129.4	1131.4	1002.1
15	179.1	51.1	147.0	38.8	991.7
20	192.4	39.1	160.3	44.1	83.8
25	203.1	31.7	170.1	48.3	77.3
29.92	212.0	26.8	180.0	51.7	71.7
lb. per sq. in.					
15	213.0	26.30	181.0	1152.2	971.2
16	216.3	24.76	184.3	53.4	69.1
17	219.4	23.40	187.5	54.6	67.1
18	222.4	22.18	190.5	55.7	65.2
19	225.2	21.09	193.3	56.7	63.4
20	228.0	20.10	196.0	1157.7	961.7
22	233.1	18.38	201.2	59.6	58.4
24	237.8	16.95	206.0	61.3	55.3
26	242.2	15.73	210.4	62.8	52.4
28	246.4	14.67	214.6	64.3	49.7
30	250.3	13.76	218.6	1165.7	947.1
32	254.0	12.95	222.4	66.9	44.6
34	257.6	12.24	225.9	68.1	42.2
36	260.9	11.60	229.4	69.2	39.9
38	264.2	11.03	232.6	70.3	37.7
40	267.2	10.51	235.8	1171.3	935.5
42	270.2	10.04	238.8	72.2	33.5
44	273.0	9.61	241.7	73.2	31.5
46	275.8	9.22	244.5	74.0	29.6
48	278.4	8.86	247.2	74.8	27.7
50	281.0	8.53	249.8	1175.6	925.9
52	283.5	8.22	252.3	76.4	24.1
54	285.9	7.93	254.7	77.1	22.4
56	288.2	7.67	257.1	77.8	20.7
58	290.5	7.42	259.5	78.5	19.0
60	292.7	7.18	261.7	1179.1	917.4
62	294.9	6.97	263.9	79.7	15.8
64	296.9	6.76	266.1	80.3	14.3
66	299.0	6.57	268.2	80.9	12.7
68	301.0	6.39	270.2	81.5	11.2

SATURATED STEAM (Goodenough).

Energy in B.t.u.		Entropy			Pressure, In. of Mercury.
of vaporization.	of vapor.	of liquid.	of vaporization.	of vapor.	
ρ	u''	s'	$\frac{r}{T}$	s''	p
988.7	1035.8	0.0915	1.9455	2.0370	1
85.0	37.7	.1019	.9198	.0217	1.2
81.9	39.4	.1108	.8980	.0087	1.4
79.1	40.9	.1185	.8791	1.9976	1.6
76.6	42.3	.1254	.8624	.9878	1.8
974.3	1043.5	0.1316	1.8474	1.9790	2
65.2	48.2	.1561	.7893	.9454	3
58.3	51.7	.1739	.7478	.9217	4
48.1	56.8	.1998	.6888	.8886	6
40.4	60.5	.2187	.6464	.8651	8
934.1	1063.5	0.2336	1.6134	1.8470	10
22.0	69.0	.2617	.5526	.8143	15
12.7	73.1	.2822	.5089	.7912	20
05.2	76.2	.2986	.4747	.7733	25
898.8	78.8	.3120	.4469	.7589	29.92
					lb. per sq. in.
898.1	1079.1	0.3135	1.4438	1.7573	15
95.8	80.0	.3184	.4337	.7521	16
93.5	80.9	.3230	.4242	.7473	17
91.4	81.7	.3274	.4153	.7427	18
89.3	82.5	.3316	.4068	.7384	19
887.3	1083.3	0.3356	1.3987	1.7343	20
83.6	84.7	.3430	.3837	.7267	22
80.1	85.9	.3499	.3698	.7197	24
76.8	87.1	.3563	.3570	.7133	26
73.7	88.2	.3622	.3452	.7074	28
870.7	1089.2	0.3679	1.3340	1.7019	30
67.9	90.2	.3731	.3236	.6967	32
65.2	91.0	.3781	.3137	.6918	34
62.7	91.9	.3829	.3044	.6873	36
60.2	92.7	.3874	.2956	.6830	38
857.8	1093.4	0.3917	1.2871	1.6788	40
55.5	94.2	.3958	.2791	.6749	42
53.3	94.8	.3998	.2714	.6712	44
51.2	95.5	.4036	.2640	.6676	46
49.1	96.1	.4072	.2570	.6642	48
847.1	1096.7	0.4108	1.2501	1.6609	50
45.1	97.2	.4142	.2436	.6577	52
43.2	97.8	.4174	.2373	.6547	54
41.4	98.3	.4206	.2311	.6517	56
39.5	98.8	.4237	.2252	.6489	58
837.8	1099.3	0.4267	1.2195	1.6462	60
36.0	99.7	.4296	.2139	.6435	62
34.3	1100.2	.4324	.2085	.6409	64
31.1	01.0	.4379	.1981	.6360	66
32.7	00.6	.4352	.2032	.6384	68

Pressure, Lb. per Sq. In.	Temp., ° F.	Volume of 1 Lb. in Cu. Ft.	Thermal Head in B.t.u. of liquid.	Thermal Head in B.t.u. of vapor.	Latent Heat, B.t.u.
p	t	v''	i'	i''	r
70	302.9	6.22	272.2	1182.0	909.8
72	304.8	6.05	274.2	82.5	08.3
74	306.7	5.90	276.1	83.0	06.9
76	308.5	5.75	278.0	83.5	05.5
78	310.3	5.61	279.8	84.0	04.2
80	312.0	5.48	281.6	1184.4	902.8
82	313.7	5.35	283.4	84.9	01.5
84	315.4	5.23	285.1	85.3	900.2
86	317.1	5.12	286.8	85.7	898.9
88	318.7	5.01	288.5	86.1	97.7
90	320.3	4.905	290.1	1186.5	896.4
92	321.8	4.805	291.7	86.9	95.2
94	323.3	4.709	293.3	87.3	94.0
96	324.8	4.617	294.8	87.7	92.8
98	326.3	4.528	296.4	88.0	91.6
100	327.8	4.442	297.9	1188.4	890.5
105	331.4	4.240	301.6	89.2	87.6
110	334.8	4.057	305.1	90.0	84.8
115	338.1	3.889	308.6	90.7	82.1
120	341.3	3.735	311.9	91.4	79.5
125	344.4	3.593	315.1	1192.0	876.9
130	347.4	3.461	318.2	92.6	74.4
135	350.3	3.340	321.2	93.2	72.0
140	353.1	3.226	324.2	93.7	69.6
145	355.8	3.120	327.0	94.2	67.2
150	358.5	3.020	329.8	1194.7	864.9
155	361.1	2.927	332.5	95.2	62.7
160	363.6	2.839	335.2	95.7	60.5
165	366.1	2.757	337.8	96.1	58.3
170	368.5	2.679	340.3	96.5	56.2
175	370.8	2.605	342.8	1196.9	854.1
180	373.1	2.536	345.2	97.2	52.0
185	375.4	2.470	347.6	97.6	49.9
190	377.6	2.408	350.0	97.9	47.9
195	379.7	2.348	352.2	98.2	46.0
200	381.9	2.292	354.5	1198.5	844.0
210	386.0	2.186	358.8	99.0	40.2
220	390.0	2.090	363.0	99.5	36.5
230	393.8	2.002	367.1	99.9	32.8
240	397.5	1.921	371.0	1200.3	29.3
250	401.1	1.846	374.9	1200.6	825.8
260	404.5	1.777	378.6	01.0	22.4
270	407.9	1.713	382.2	01.2	19.1
280	411.2	1.654	385.7	01.5	15.8
300	417.5	1.545	392.4	01.9	09.4

SATURATED STEAM (Goodenough).

Energy in B.t.u.		Entropy			Pressure, Lb. per Sq. In.
of vaporization.	of vapor.	of liquid.	of vaporization.	of vapor.	
ρ	u''	s'	$\frac{r}{T}$	s''	p
829.5	1101.4	0.4405	1.1931	1.6336	70
27.9	01.8	.4431	.1883	.6313	72
26.4	02.2	.4456	.1835	.6291	74
24.9	02.6	.4480	.1789	.6269	76
23.4	02.9	.4504	.1744	.6248	78
821.9	1103.2	0.4527	1.1700	1.6227	80
20.5	03.6	.4550	.1657	.6207	82
19.1	03.9	.4572	.1615	.6187	84
17.7	04.2	.4594	.1574	.6168	86
16.3	04.5	.4615	.1534	.6149	88
815.0	1104.8	0.4636	1.1495	1.6131	90
13.7	05.1	.4657	.1456	.6113	92
12.4	05.4	.4677	.1419	.6096	94
11.1	05.6	.4697	.1381	.6079	96
09.8	05.9	.4717	.1345	.6062	98
808.6	1106.2	0.4736	1.1309	1.6045	100
05.5	06.8	.4782	.1222	.6004	105
02.6	07.3	.4827	.1138	.5965	110
799.7	07.9	.4870	.1058	.5928	115
96.9	08.4	.4911	.0982	.5893	120
794.2	1108.8	0.4950	1.0908	1.5858	125
91.6	09.3	.4989	.0836	.5825	130
89.0	09.7	.5026	.0767	.5793	135
86.4	10.1	.5062	.0700	.5762	140
84.0	10.5	.5097	.0636	.5733	145
781.6	1110.9	0.5131	1.0573	1.5704	150
79.2	11.2	.5164	.0512	.5676	155
76.9	11.5	.5196	.0453	.5649	160
74.6	11.8	.5227	.0395	.5622	165
72.4	12.1	.5258	.0339	.5597	170
770.2	1112.4	0.5287	1.0284	1.5572	175
68.0	12.7	.5316	.0231	.5547	180
65.9	12.9	.5344	.0179	.5523	185
63.9	13.2	.5372	.0128	.5500	190
61.8	13.4	.5399	.0079	.5478	195
759.8	1113.6	0.5426	1.0030	1.5456	200
55.9	14.0	.5477	.9936	.5413	210
52.1	14.3	.5526	.9846	.5372	220
48.3	14.6	.5573	.9760	.5333	230
44.7	14.9	.5619	.9676	.5295	240
741.2	1115.2	0.5663	0.9595	1.5258	250
37.7	15.4	.5706	.9517	.5223	260
34.4	15.6	.5747	.9442	.5189	270
31.1	15.8	.5787	.9369	.5156	280
24.7	16.0	.5863	.9229	.5092	300

Pressure Lb. per Sq. In.	1.50 x	1.50 i	1.50 v	1.55 x	1.55 i	1.55 v
300	0.990	1194	1.53	474	1239	1.70
280	0.983	1188	1.63	458	1232	1.80
260	0.977	1182	1.74	442	1225	1.90
240	0.970	1175	1.86	425	1218	2.02
220	0.962	1168	2.01	407	1210	2.16
200	0.955	1160	2.19	388	1202	2.32
190	0.951	1156	2.29	378	1198	2.41
180	0.947	1152	2.40	0.995	1193	2.52
170	0.942	1147	2.53	0.991	1188	2.65
160	0.938	1142	2.66	0.986	1183	2.80
150	0.933	1137	2.82	0.981	1178	2.96
140	0.929	1132	3.00	0.976	1172	3.15
130	0.924	1126	3.20	0.970	1166	3.36
120	0.919	1120	3.43	0.964	1160	3.60
110	0.913	1113	3.71	0.958	1153	3.89
100	0.908	1106	4.03	0.952	1145	4.23
95	0.905	1102	4.22	0.949	1141	4.42
90	0.902	1098	4.42	0.945	1137	4.64
85	0.898	1094	4.65	0.942	1133	4.87
80	0.895	1090	4.90	0.938	1128	5.13
76	0.892	1086	5.13	0.935	1124	5.37
72	0.890	1082	5.38	0.932	1120	5.64
68	0.887	1078	5.67	0.928	1116	5.93
64	0.883	1074	5.97	0.925	1112	6.25
60	0.880	1069	6.32	0.921	1107	6.62
56	0.877	1064	6.73	0.917	1102	7.04
52	0.873	1059	7.18	0.913	1096	7.51
48	0.869	1054	7.70	0.909	1090	8.05
44	0.865	1048	8.32	0.905	1084	8.70
40	0.861	1041	9.05	0.900	1077	9.46
36	0.856	1034	9.93	0.895	1070	10.38
32	0.851	1027	11.03	0.889	1062	11.52
28	0.846	1018	12.42	0.883	1053	12.97
24	0.840	1008	14.23	0.876	1043	14.85
20	0.833	997	16.73	0.868	1031	17.45
18	0.829	990	18.38	0.864	1024	19.17
16	0.824	983	20.41	0.859	1017	21.27
In. Hg						
30	0.821	978	21.96	0.856	1012	22.88
24	0.813	965	26.80	0.847	998	27.91
20	0.807	954	31.54	0.840	987	32.84
16	0.800	942	38.51	0.832	974	40.06
12	0.791	926	49.8	0.822	957	51.83
10	0.785	916	58.7	0.816	947	61.0
8	0.778	904	71.7	0.809	935	74.5
6	0.770	889	92.9	0.800	919	96.5
5	0.765	880	109.5	0.794	910	113.7
4	0.759	869	133.9	0.787	898	138.9
3	0.751	855	173.8	0.779	884	180.3
2	0.741	837	251.0	0.768	865	260.2
1	0.724	806	472	0.750	833	489

Pressure Lb. per Sq. In.	1.60			1.65		
	x	i	v	x	i	v
300	554	1287	1.91	648	1340	2.13
280	538	1280	2.01	630	1332	2.25
260	520	1272	2.13	612	1324	2.38
240	502	1264	2.26	592	1315	2.53
220	483	1256	2.42	571	1305	2.70
200	462	1246	2.60	548	1295	2.91
190	450	1241	2.70	536	1289	3.02
180	439	1236	2.82	524	1283	3.15
170	427	1231	2.94	511	1277	3.29
160	414	1226	3.08	497	1271	3.44
150	401	1220	3.23	482	1264	3.62
140	387	1213	3.41	467	1258	3.81
130	372	1207	3.60	451	1251	4.03
120	356	1200	3.83	434	1243	4.28
110	340	1193	4.09	416	1235	4.58
100	0.996	1185	4.42	396	1226	4.92
95	0.992	1181	4.63	386	1221	5.12
90	0.988	1176	4.85	375	1216	5.33
85	0.985	1172	5.10	364	1211	5.57
80	0.981	1167	5.37	352	1206	5.83
76	0.977	1163	5.62	342	1202	6.07
72	0.974	1159	5.89	332	1197	6.32
68	0.970	1154	6.20	321	1192	6.59
64	0.966	1149	6.53	310	1187	6.90
60	0.962	1144	6.91	298	1182	7.24
56	0.958	1139	7.35	0.999	1177	7.66
52	0.954	1134	7.84	0.994	1171	8.17
48	0.949	1128	8.41	0.989	1164	8.76
44	0.944	1121	9.08	0.983	1158	9.45
40	0.939	1114	9.87	0.978	1150	10.28
36	0.933	1106	10.82	0.971	1142	11.26
32	0.927	1098	12.01	0.965	1134	12.50
28	0.920	1089	13.51	0.957	1124	14.06
24	0.913	1078	15.47	0.949	1113	16.09
20	0.904	1065	18.17	0.940	1100	18.89
18	0.899	1058	19.95	0.935	1092	20.73
16	0.894	1051	22.14	0.929	1084	23.00
In. Hg						
30	0.890	1045	23.81	0.925	1079	24.73
24	0.881	1031	29.03	0.915	1064	30.14
20	0.873	1019	34.13	0.907	1052	35.43
16	0.864	1006	41.62	0.897	1038	43.18
12	0.854	989	53.8	0.885	1020	55.8
10	0.847	978	63.3	0.878	1009	65.7
8	0.839	965	77.3	0.869	996	80.1
6	0.829	949	100.1	0.859	979	103.6
5	0.823	939	117.9	0.852	969	122.1
4	0.816	928	144.0	0.845	957	149.0
3	0.807	913	186.7	0.835	942	193.2
2	0.795	893	269	0.822	921	279
1	0.775	860	506	0.801	887	522

Pressure Lb. per Sq. In.	1.70 x	1.70 i	1.70 v	1.75 x	1.75 i	1.75 v
150	577	1314	4.03	......	......	......
140	560	1306	4.25	......	......	......
130	543	1298	4.50	......	......	......
120	524	1290	4.78	626	1341	5.32
110	504	1281	5.11	604	1331	5.69
100	483	1271	5.50	581	1320	6.12
96	474	1267	5.67	571	1316	6.32
92	464	1263	5.86	561	1311	6.52
88	455	1258	6.06	550	1306	6.75
84	445	1253	6.27	539	1301	6.99
80	434	1249	6.51	528	1296	7.26
76	423	1244	6.77	516	1290	7.55
72	412	1239	7.06	504	1284	7.87
68	400	1233	7.37	491	1278	8.22
64	388	1228	7.72	478	1272	8.61
60	375	1222	8.11	463	1266	9.04
58	368	1219	8.32	456	1262	9.28
56	361	1216	8.54	448	1259	9.53
54	354	1212	8.78	441	1255	9.80
52	347	1209	9.03	433	1252	10.08
50	340	1206	9.30	424	1248	10.38
48	332	1202	9.59	416	1244	10.71
46	324	1199	9.91	407	1240	11.06
44	316	1195	10.25	398	1236	11.45
42	307	1191	10.61	388	1231	11.86
40	298	1187	11.01	379	1227	12.31
38	289	1183	11.45	368	1222	12.80
36	279	1179	11.93	358	1217	13.33
34	269	1174	12.46	347	1212	13.93
32	258	1169	13.04	335	1207	14.58
30	0.999	1164	13.74	323	1202	15.31
28	0.995	1159	14.60	310	1196	16.13
26	0.990	1154	15.58	296	1190	17.06
24	0.986	1148	16.71	282	1183	18.12
22	0.981	1141	18.03	267	1176	19.37
20	0.976	1134	19.61	250	1169	20.81
18	0.970	1127	21.52	233	1161	22.54
16	0.964	1118	23.86	0.999	1152	24.73
In. Hg						
30	0.960	1112	25.66	0.994	1146	26.58
24	0.948	1097	31.25	0.982	1130	32.36
20	0.940	1085	36.72	0.973	1117	38.02
16	0.929	1070	44.74	0.962	1102	46.30
12	0.917	1052	57.78	0.948	1083	59.77
10	0.909	1040	68.0	0.940	1071	70.3
8	0.900	1027	82.9	0.930	1057	85.7
6	0.888	1009	107.2	0.918	1039	110.7
5	0.881	999	126.2	0.911	1029	130.4
4	0.873	986	154.1	0.902	1015	159.1
3	0.863	970	200	0.891	999	206
2	0.849	949	288	0.876	977	297
1	0.827	914	539	0.853	940	556

FOR STEAM (Goodenough).

Pressure Lb. per Sq. In.	1.80			1.85		
	x	i	v	x	i	v
90	663	1362	7.38	......	......	......
88	657	1359	7.50	......	......	......
86	651	1356	7.64	......	......	......
84	645	1354	7.78	......	......	......
82	639	1351	7.92	......	......	......
80	633	1348	8.08	......	......	......
78	627	1345	8.24	......	......	......
76	620	1341	8.40	......	......	......
74	614	1338	8.57	......	......	......
72	607	1335	8.75	......	......	......
70	600	1332	8.94	......	......	......
68	593	1329	9.14	......	......	......
66	585	1325	9.35	......	......	......
64	578	1322	9.58	......	......	......
62	571	1318	9.82	......	......	......
60	563	1314	10.06	......	......	......
58	555	1311	10.33	......	......	......
56	546	1307	10.61	......	......	......
54	538	1303	10.91	......	......	......
52	529	1299	11.23	......	......	......
50	520	1294	11.57	626	1346	12.87
48	511	1290	11.93	616	1341	13.28
46	501	1286	12.33	605	1336	13.72
44	491	1281	12.76	595	1331	14.20
42	480	1276	13.22	583	1326	14.71
40	470	1271	13.72	572	1320	15.27
38	458	1266	14.26	559	1314	15.89
36	447	1260	14.86	546	1308	16.56
34	434	1255	15.52	533	1302	17.30
32	422	1249	16.26	519	1295	18.12
30	408	1243	17.08	504	1288	19.03
28	394	1236	18.00	488	1281	20.05
26	379	1230	19.04	472	1274	21.23
24	363	1222	20.23	455	1266	22.55
22	346	1214	21.62	436	1257	24.10
20	328	1206	23.23	416	1248	25.92
18	308	1197	25.17	394	1238	28.08
16	287	1187	27.52	370	1227	30.69
In. Hg						
30	272	1181	29.34	354	1219	32.73
24	234	1163	34.78	312	1200	38.81
20	204	1150	40.0	279	1185	44.6
16	0.994	1134	47.9	240	1167	52.9
12	0.980	1114	61.8	193	1146	65.9
10	0.971	1102	72.6	166	1133	75.3
8	0.961	1088	88.5	0.991	1118	91.3
6	0.948	1069	114.3	0.977	1099	117.9
5	0.940	1058	134.6	0.969	1088	138.7
4	0.930	1045	164.2	0.959	1074	169.2
3	0.919	1028	212.6	0.947	1057	219.1
2	0.903	1005	306	0.930	1033	315
1	0.878	967	573	0.904	994	589

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